ON THE DYNAMICS OF UNSTABLE QUARK-GLUON PLASMA*

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Since the quark-gluon plasma, which is unstable due to anisotropic momentum distribution, evolves fast in time, plasma's characteristics have to be studied as initial value problems. The chromodynamic fluctuations and the momentum broadening of a fast parton traversing the plasma are discussed here. The two quantities are shown to exponentially grow in time.

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1. Introduction

The Quark-Gluon Plasma (QGP), which is produced at the early stage of relativistic heavy-ion collisions, is most probably unstable due to anisotropic momentum distribution of quarks and gluons (partons), see the review [1]. The instability makes the system's state strongly time dependent, and consequently various plasma characteristics must be found as solutions of initial value problem. Two such characteristics are discussed in my lecture. I start with the fluctuations of chromodynamic fields and then, the correlation functions of the fields are used to compute the momentum broadening of a fast parton traversing the plasma. The two characteristics exponentially grow in time due to unstable modes. Throughout my lecture, which is based on two recent publications [2, 3], the plasma is assumed to be weakly coupled.

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2. Chromodynamic fluctuations

In the Quark-Gluon Plasma (QGP), which is on average locally colorless, chromodynamic fields, color charges and currents experience random fluctuations. In the equilibrium plasmas there are characteristic stationary spectra of fluctuations which can be found by means of the fluctuation-dissipation relations. Fluctuations in nonequilibrium systems evolve in time, and their characteristics usually depend on an initial state of the system.

Fluctuations can be theoretically studied by means of several methods reviewed in the classical monographs [4, 5]. The method, which was chosen in [2], is clearly exposed in the handbook [6]. It is applicable to both equilibrium and nonequilibrium plasmas but the initial plasma state is assumed to be on average charge neutral, stationary and homogeneous. It will be shown that when the plasma state is stable, the initial fluctuations exponentially decay and in the long time limit one finds a stationary spectrum of the fluctuations. When the initial state is unstable, the memory of initial fluctuations is not lost, as the unstable modes, which are usually present in the initial fluctuation spectrum, exponentially grow.

The chromodynamic fluctuations are studied here using the transport theory of weakly coupled QGP which is formulated in terms of particles and classical fields. The particles — quarks, antiquarks and gluons — should be understood as sufficiently hard quasiparticle excitations of QCD quantum fields while the classical fields are highly populated soft gluonic modes. The transport equation of quarks reads

$$\big(D^0 + \boldsymbol{v}\cdot\mathbf{D}\big)\, Q(t,\boldsymbol{r},\boldsymbol{p}) - \frac{g}{2}\{\boldsymbol{E}(t,\boldsymbol{r}) + \boldsymbol{v}\times\boldsymbol{B}(t,\boldsymbol{r}), \nabla_p Q(t,\boldsymbol{r},\boldsymbol{p})\} = 0\,, \quad (1)$$

where $Q(t,\boldsymbol{r},\boldsymbol{p})$ is the on-mass-shell quark distribution function which is $N_c \times N_c$ Hermitian matrix belonging to the fundamental representation of the $\mathrm{SU}(N_c)$ group; the covariant derivative in the four-vector notation reads $D^\mu \equiv \partial^\mu - ig[A^\mu(x), \cdots\,]$ and $\boldsymbol{E}(t,\boldsymbol{r})$ and $\boldsymbol{B}(t,\boldsymbol{r})$ are the chromoelectric and chromomagnetic fields. The symbol $\{\ldots,\ldots\}$ denotes the anticommutator. Since the fluctuations of interest are assumed to be of the time scale, which is much shorter than that of inter-parton collisions, the collision terms are absent in Eq. (1). There are analogous transport equations for antiquark $(\bar{Q}(t,\boldsymbol{r},\boldsymbol{p}))$ and gluon $(G(t,\boldsymbol{r},\boldsymbol{p}))$ distribution functions.

The transport equations are supplemented by the Yang–Mills equations describing a self-consistent generation of the chromoelectric and chromomagnetic fields by the color four-current $j^\mu = (\rho, \boldsymbol{j})$

$$j^\mu_a(t,\boldsymbol{r}) = -g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^\mu}{E_{\boldsymbol{p}}}\mathrm{Tr}\left[\tau^a(Q(t,\boldsymbol{r},\boldsymbol{p}) - \bar{Q}(t,\boldsymbol{r},\boldsymbol{p})) + T^a G(t,\boldsymbol{r},\boldsymbol{p})\right]\,,$$

where τ^a, T^a with $a = 1, \ldots, N_c^2 - 1$ are the $\mathrm{SU}(N_c)$ group generators in the fundamental and adjoint representations.

I consider small deviations from a stationary homogeneous state which is globally and locally colorless; there are no currents as well. The quark distribution function of this state is $Q^0_{nm}(\boldsymbol{p}) = n(\boldsymbol{p})\,\delta^{nm}$. Due to the absence of color charges and currents in the stationary and homogeneous state, the chromoelectric $\boldsymbol{E}(t,\boldsymbol{r})$ and chromomagnetic $\boldsymbol{B}(t,\boldsymbol{r})$ fields are expected to vanish while the potentials $A^0(t,\boldsymbol{r}), \boldsymbol{A}(t,\boldsymbol{r})$ are of pure gauge only. Since the plasma under considerations is assumed to be weakly coupled with the perturbative vacuum state, the potentials can be gauged away to vanish.

The quark distribution function is written down as $Q(t,\boldsymbol{r},\boldsymbol{p}) = Q^0(\boldsymbol{p}) + \delta Q(t,\boldsymbol{r},\boldsymbol{p})$, and we assume that $|Q^0| \gg |\delta Q|$ and $|\nabla_p Q^0| \gg |\nabla_p \delta Q|$ with the analogous formulas for antiquarks and gluons. The transport (1) and Yang–Mills equations are linearized in deviations from the stationary homogeneous state. We assume that δQ, $\boldsymbol{E}$, $\boldsymbol{B}$, A^0 and $\boldsymbol{A}$ are all of the same order. The linearized transport equation is

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\delta Q(t,\boldsymbol{r},\boldsymbol{p}) - g(\boldsymbol{E}(t,\boldsymbol{r}) + \boldsymbol{v}\times\boldsymbol{B}(t,\boldsymbol{r}))\nabla_p n(\boldsymbol{p}) = 0\,.$$

After the linearization the Yang–Mills equations get the familiar form of Maxwell equations of multi-component electrodynamics.

The linearized transport and Maxwell equations are solved with the initial conditions $\delta Q(t=0,\boldsymbol{r},\boldsymbol{p}) = \delta Q_0(\boldsymbol{r},\boldsymbol{p})$, $\boldsymbol{E}(t=0,\boldsymbol{r}) = \boldsymbol{E}_0(\boldsymbol{r})$ and $\boldsymbol{B}(t=0,\boldsymbol{r}) = \boldsymbol{B}_0(\boldsymbol{r})$, by means of the one-sided Fourier transformation defined as

$$f(\omega,\boldsymbol{k}) = \int_0^\infty dt \int d^3r e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} f(t,\boldsymbol{r})\,.$$

The chromoelectric field, which solves the equations, is found as

$$\begin{aligned}
&\left[-\boldsymbol{k}^2\delta^{ij} + k^ik^j + \omega^2\varepsilon^{ij}(\omega,\boldsymbol{k})\right] E^j_a(\omega,\boldsymbol{k}) \\
&= -i\frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\,\frac{v^i(\boldsymbol{v}\times\boldsymbol{B}_{a0}(\boldsymbol{k}))^j\nabla^j_p f(\boldsymbol{p})}{\omega - \boldsymbol{v}\cdot\boldsymbol{k}} \\
&-g\omega\int \frac{d^3p}{(2\pi)^3}\,\frac{v^i}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}}\,\delta N^a_0(\boldsymbol{k},\boldsymbol{p}) + i\omega E^i_{a0}(\boldsymbol{k}) - i(\boldsymbol{k}\times\boldsymbol{B}_{a0}(\boldsymbol{k}))^i\,, \quad (2)
\end{aligned}$$

where $f(\boldsymbol{p}) \equiv n(\boldsymbol{p}) + \bar{n}(\boldsymbol{p}) + 2N_c n_g(\boldsymbol{p})$ and $\delta N^a_0(\boldsymbol{r},\boldsymbol{p}) \equiv \mathrm{Tr}[\tau^a(\delta Q_0(\boldsymbol{r},\boldsymbol{p}) - \delta\bar{Q}_0(\boldsymbol{r},\boldsymbol{p})) + T^a\delta G_0(\boldsymbol{r},\boldsymbol{p})]$; $\varepsilon^{ij}(\omega,\boldsymbol{k})$ is the chromodielectric tensor of, in general, anisotropic plasma in the collisionless limit; $\varepsilon^{ij}(\omega,\boldsymbol{k})$ does not carry any color indices, as it corresponds to a colorless state of the plasma.

When the plasma stationary state is isotropic, the dielectric tensor can be expressed through its longitudinal ($\varepsilon_{\rm L}(\omega,\boldsymbol{k})$) and transverse ($\varepsilon_{\rm T}(\omega,\boldsymbol{k})$) components and the matrix $\Sigma^{ij}(\omega,\boldsymbol{k}) \equiv -\boldsymbol{k}^2\delta^{ij} + k^ik^j + \omega^2\varepsilon^{ij}(\omega,\boldsymbol{k})$ from the left-hand side of Eq. (2) can be inverted as

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\boldsymbol{k}) = \frac{1}{\omega^2\varepsilon_{\rm L}(\omega,\boldsymbol{k})}\frac{k^ik^j}{\boldsymbol{k}^2} + \frac{1}{\omega^2\varepsilon_{\rm T}(\omega,\boldsymbol{k}) - \boldsymbol{k}^2}\left(\delta^{ij} - \frac{k^ik^j}{\boldsymbol{k}^2}\right).$$

Then, Eq. (2) provides an explicit expression of the chromoelectric field. Using the Maxwell equations, the chromomagnetic field, color current and color density can be all expressed through the chromoelectric field.

The correlation functions $\langle E_a^i(t_1,\boldsymbol{r}_1)E_b^j(t_2,\boldsymbol{r}_2)\rangle$, $\langle B_a^i(t_1,\boldsymbol{r}_1)B_b^j(t_2,\boldsymbol{r}_2)\rangle$, where $\langle\cdots\rangle$ denotes averaging over statistical ensemble, are determined by the initial correlations such as $\langle \delta N_0^a(\boldsymbol{r}_1,\boldsymbol{p}_1)\delta N_0^b(\boldsymbol{r}_2,\boldsymbol{p}_2)\rangle$, $\langle E_{a0}^i(\boldsymbol{r}_1)E_{b0}^j(\boldsymbol{r}_2)\rangle$, $\langle \delta N_0^a(\boldsymbol{r}_1,\boldsymbol{p}_1)E_{b0}^j(\boldsymbol{r}_2)\rangle$ which can be all expressed, using the Maxwell equations, through the correlation function of the distribution functions. The latter one is identified with the respective correlation function of the classical system of free quarks, antiquarks and gluons which on average is stationary and homogeneous. For quarks the free correlation function is

$$\begin{aligned}&\langle \delta Q^{mn}(t_1,\boldsymbol{r}_1,\boldsymbol{p}_1)\delta Q^{pr}(t_2,\boldsymbol{r}_2,\boldsymbol{p}_2)\rangle_{\rm free}\\ &= \delta^{mr}\delta^{np}(2\pi)^3\delta^{(3)}(\boldsymbol{p}_1-\boldsymbol{p}_2)\times\delta^{(3)}(\boldsymbol{r}_2-\boldsymbol{r}_1-\boldsymbol{v}_1(t_2-t_1))\,n(\boldsymbol{p}_1).\end{aligned}$$

In the case of equilibrium plasma, where all collective modes are damped, I consider the times which are much longer than the decay time of collective excitations. Then, the correlation function of the chromoelectric fields equals

$$\left\langle E_a^i(t_1,\boldsymbol{r}_1)E_b^j(t_2,\boldsymbol{r}_2)\right\rangle_\infty = \int\frac{d\omega}{2\pi}\frac{d^3k}{(2\pi)^3}e^{-i\left(\omega(t_1-t_2)-\boldsymbol{k}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)\right)}\left\langle E_a^iE_b^j\right\rangle_k,$$

where the fluctuation spectrum is

$$\left\langle E_a^iE_b^j\right\rangle_k = 2\delta^{ab}T\omega^3\left[\frac{k^ik^j}{\boldsymbol{k}^2}\frac{{\rm Im}\varepsilon_{\rm L}(\omega,\boldsymbol{k})}{|\omega^2\varepsilon_{\rm L}(\omega,\boldsymbol{k})|^2} + \left(\delta^{ij} - \frac{k^ik^j}{\boldsymbol{k}^2}\right)\frac{{\rm Im}\varepsilon_{\rm T}(\omega,\boldsymbol{k})}{|\omega^2\varepsilon_{\rm T}(\omega,\boldsymbol{k})-\boldsymbol{k}^2|^2}\right].$$

As seen, the fluctuation spectrum has strong peaks corresponding to the collective modes determined by the equations $\varepsilon_{\rm L}(\omega,\boldsymbol{k})=0$ and $\omega^2\varepsilon_{\rm T}(\omega,\boldsymbol{k})-\boldsymbol{k}^2=0$.

As an example of a nonequilibrium situation, I discuss fluctuations of longitudinal chromoelectric fields in the two-stream system which is unstable with respect to longitudinal modes. Nonequlibrium calculations are much more difficult than the equilibrium ones. The first problem is to invert the matrix $\Sigma^{ij}(\omega,\boldsymbol{k})$. In the case of longitudinal electric field, which is discussed here, the matrix is replaced by the scalar function.

The distribution function of the two-stream system is chosen to be

$$f(\boldsymbol{p}) = (2\pi)^3 n \left[\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) + \delta^{(3)}(\boldsymbol{p}+\boldsymbol{q})\right],$$

where n is the effective parton density in a single stream. There are four roots $\pm\omega_\pm(\boldsymbol{k})$ of the dispersion equation $\varepsilon_{\rm L}(\omega,\boldsymbol{k})=0$. The solution $\omega_+(\boldsymbol{k})$ represents the stable modes and $\omega_-(\boldsymbol{k})$ corresponds to the well-known two-stream electrostatic instability for $\boldsymbol{k}\cdot\boldsymbol{u}\neq 0$ and $\boldsymbol{k}^2(\boldsymbol{k}\cdot\boldsymbol{u})^2 < 2\mu^2(\boldsymbol{k}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2)$ where $\boldsymbol{u}\equiv \boldsymbol{q}/|\boldsymbol{q}|$ is the stream velocity and $\mu^2 \equiv g^2 n/2|\boldsymbol{q}|$. Then, $\omega_-(\boldsymbol{k}) = i\gamma_{\boldsymbol{k}}$ with $0\leq \gamma_{\boldsymbol{k}} \in R$.

The correlation function of longitudinal chromoelectric fields generated by the unstable modes is found as

$$\begin{aligned}
&\left\langle E_a^i(t_1,\boldsymbol{r}_1) E_b^j(t_2,\boldsymbol{r}_2)\right\rangle_{\rm unstable} \\
&= \frac{g^2}{2}\,\delta^{ab} n \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\boldsymbol{k}(\boldsymbol{r}_1-\boldsymbol{r}_2)}}{\boldsymbol{k}^4}\frac{k^i k^j\left(\gamma_{\boldsymbol{k}}^2+(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)^2}{\left(\omega_+^2-\omega_-^2\right)^2\gamma_{\boldsymbol{k}}^2} \\
&\times\left[\left(\gamma_{\boldsymbol{k}}^2+(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\cosh\left(\gamma_{\boldsymbol{k}}(t_1+t_2)\right)+\left(\gamma_{\boldsymbol{k}}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\cosh(\gamma_{\boldsymbol{k}}(t_1-t_2))\right]. \qquad (3)
\end{aligned}$$

As seen, the correlation function of the unstable system is invariant with respect to space translations — it depends on the difference $(\boldsymbol{r}_1-\boldsymbol{r}_2)$ only. The plasma state, which is initially on average homogeneous, remains like this in course of the system's temporal evolution. The time dependence of the correlation function is very different from the space dependence. The electric fields exponentially grow and so does the correlation function both in (t_1+t_2) and (t_1-t_2). The fluctuation spectrum also evolves in time, as the growth rate of unstable modes is wave-vector dependent and after a sufficiently long time the fluctuation spectrum is dominated by the fastest growing modes.

3. Momentum broadening of a fast parton

When a highly energetic parton travels through dense QCD matter, it receives random kicks from elastic interactions with constituents of the plasma. The average transverse momentum transfer per unit path length is related to the radiative energy loss of the parton [7]. The parameter describing the average amount of transverse momentum broadening per unit length is called $\hat{q}$ and is defined as

$$\hat{q} \equiv d\left\langle \Delta\boldsymbol{p}_{\rm T}^2\right\rangle/dz\,, \qquad (4)$$

when the fast parton flies along the direction z. The values of $\hat{q}$ extracted from experimental data on relativistic heavy-ion collisions vary in a rather broad range 0.5–15 GeV^2/fm depending on the model of hard particle propagation in strongly interacting matter produced in nuclear collisions [8, 9].

The calculations of $\hat{q}$ for the case of perturbative QGP in equilibrium are well understood (see [10, 11] for recent work). For such a plasma the value of $\hat{q}$ is predicted to lie at the lower end of the range of values deduced from experiments [9]. However, the plasma momentum distribution is initially anisotropic and recently $\hat{q}$ has been computed [12, 13] for such a plasma. However, the fact that the anisotropic plasma as an unstable system evolves fast in time has not been taken into account. It seems rather unjustified, as the numerical simulations [14, 15] clearly indicate that $\hat{q}$ receives a sizable contribution from the unstable growing modes.

An analytic approach to compute $\hat{q}$ in unstable plasma has been developed in [3]. It formulates the transverse momentum fluctuations in terms of classical Langevin problem. $\hat{q}$ is computed by treating the parton as an energetic classical particle with SU(3) color charge moving in the presence of the fluctuating color fields. Then, $\hat{q}$ is expressed through the correlation function of chromodynamic fields computed in [2]. For the equilibrium plasma the Langevin approach recovers the known result obtained within the standard thermal field theory [16].

Let me consider a classical parton which moves across a QGP. Its motion is described by the Wong equations [17]

$$\frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau)\,, \tag{5}$$

$$\frac{dp^\mu(\tau)}{d\tau} = gQ^a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_\nu(\tau)\,, \tag{6}$$

$$\frac{dQ_a(\tau)}{d\tau} = -gf^{abc}p_\mu(\tau)\, A_b^\mu(x(\tau))\, Q_c(\tau)\,, \tag{7}$$

where τ, $x^\mu(\tau)$, $u^\mu(\tau)$ and $p^\mu(\tau)$ are, respectively, the parton's proper time, its trajectory, four-velocity and four-momentum; $F_a^{\mu\nu}$ and A_a^μ denote the chromodynamic field strength tensor and four-potential, respectively, and Q^a is the classical color charge of the parton.

We look for a solution of the Wong equations in a specific gauge assuming that the potential vanishes along the parton's trajectory *i.e.* our gauge condition is

$$p_\mu(\tau)\, A_a^\mu(x(\tau)) = 0\,.$$

Then, Eq. (7) simply tells that Q_a is constant as a function of τ.

One solves Eqs. (5, 6) assuming that the parton's momentum $\boldsymbol{p}$ is so high that its changes $\Delta\boldsymbol{p}$ caused by the interactions with the medium are small compared with $\boldsymbol{p}$. The changes of the velocity vector $\boldsymbol{v}$ are then negligible, and we can consider the parton to move along a straight-line path with constant velocity.

Assuming that the parton moves with the speed of light in the *positive* z-direction that is $x^\mu(t) = (t, 0, 0, t)$, one finds

$$p^\mu(t) = p^\mu(0) + gQ_a \int_0^t dt' \left[F_a^{\mu 0}(t') - F_a^{\mu 3}(t')\right] , \tag{8}$$

where $F_a^{\mu\nu}(t)$ should be understood as a short hand notation of $F_a^{\mu\nu}(x(t))$.

In the spirit of Langevin approach, we consider the ensemble average $\langle p^\mu(t) p^\nu(t)\rangle$ indicated by the angular brackets. The ensemble average involves averaging over color charges which is performed by means of the relation

$$\int dQ\, Q_a Q_b = C_2 \delta^{ab} ,$$

where $C_2 = 1/2$ for particles (quarks) in fundamental representation of the $\mathrm{SU}(N_c)$ group and $C_2 = N_c$ for particles (gluons) in adjoint representation. Then, we find

$$\begin{aligned} \langle p^\mu(t) p^\nu(t)\rangle &= \langle p^\mu(0) p^\nu(0)\rangle \\ &+ g^2 \frac{C_R}{N_c^2 - 1} \int_0^t dt_1 \int_0^t dt_2 \left\langle \left(F_a^{\mu 0}(t_1) - F_a^{\mu 3}(t_1)\right) \left(F_a^{\nu 0}(t_2) - F_a^{\mu 3}(t_2)\right)\right\rangle , \end{aligned} \tag{9}$$

where C_R $(R = F, A)$ is the eigenvalue of the quadratic Casimir operator, $C_F = (N_c^2 - 1)/2N_c$ and $C_A = N_c$.

Introducing $\Delta \boldsymbol{p}_{\rm T}(t) \equiv \boldsymbol{p}_{\rm T}(t) - \boldsymbol{p}_{\rm T}(0)$ and $\langle \Delta \boldsymbol{p}_{\rm T}^2(t)\rangle \equiv \langle \Delta \boldsymbol{p}_{\rm T}(t) \cdot \Delta \boldsymbol{p}_{\rm T}(t)\rangle$, we have

$$\begin{aligned} \left\langle \Delta \boldsymbol{p}_{\rm T}^2(t)\right\rangle \; &= \; g^2 \frac{C_R}{N_c^2 - 1} \int_0^t dt_1 \int_0^t dt_2 \Big[\langle E_a^x(t_1) E_a^x(t_2)\rangle + \langle E_a^y(t_1) E_a^y(t_2)\rangle \\ &\quad - \langle E_a^x(t_1) B_a^y(t_2)\rangle + \langle E_a^y(t_1) B_a^x(t_2)\rangle - \langle B_a^y(t_1) E_a^x(t_2)\rangle \\ &\quad + \langle B_a^x(t_1) E_a^y(t_2)\rangle + \langle B_a^x(t_1) B_a^x(t_2)\rangle + \langle B_a^y(t_1) B_a^y(t_2)\rangle\Big] , \end{aligned} \tag{10}$$

where, say, $E_a^x(t)$ should be understood as $E_a^x(t, \boldsymbol{r}(t))$ with $\boldsymbol{r}(t) \equiv (0, 0, t)$ that is only the fields at the parton's trajectory enter Eq. (10).

Eq. (10) has been derived in the specific gauge and the right-hand side of the equation is, in general, gauge dependent. However, the field correlation functions derived in [2] are gauge independent within the Hard-Loop Approximation, as discussed in detail in Sec. VIIIA of [2]. Therefore, Eq. (10) is gauge independent within the used approximations.

Let us now assume that the QGP is translationally invariant in space and time and, hence, the field correlators depend only on the difference of the field's arguments. This assumption, which is relevant for equilibrium plasmas, will be not adopted for the two-stream system as the growth of unstable modes break the translational invariance in time. Making use of the translational invariance, one introduces the fluctuation spectrum

$$\left\langle E_a^i E_a^j \right\rangle_k \equiv \int dt \int d^3 r e^{i(\omega t - \boldsymbol{k}\boldsymbol{r})} \left\langle E_a^i(t,\boldsymbol{r}) E_a^j(0,\boldsymbol{0}) \right\rangle , \qquad (11)$$

which allows us to write down Eq. (10) in the form

$$\begin{aligned} \left\langle \Delta \boldsymbol{p}_{\mathrm{T}}^2(t) \right\rangle \;=\; & g^2 \frac{C_R}{N_c^2 - 1} \int_0^t dt_1 \int_0^t dt_2 \int \frac{d^4 k}{(2\pi)^4}\, e^{i(\omega - k_z)(t_1 - t_2)} \\ & \times \Big[\langle E_a^x E_a^x \rangle_k + \langle E_a^y E_a^y \rangle_k - \langle E_a^x B_a^y \rangle_k + \langle E_a^y B_a^x \rangle_k \\ & - \langle B_a^y E_a^x \rangle_k + \langle B_a^x E_a^y \rangle_k + \langle B_a^x B_a^x \rangle_k + \langle B_a^y B_a^y \rangle_k \Big] . \qquad (12) \end{aligned}$$

Since the double integral over t_1 and t_2 tends to a delta function of $(\omega - k_z)/2$ in the long-time limit

$$\int_0^t dt_1 \int_0^t dt_2 e^{i(\omega - k_z)(t_1 - t_2)} = \frac{4 \sin\left(\frac{(\omega - k_z)t}{2}\right)}{(\omega - k_z)^2} \overset{t \to \infty}{\longrightarrow} \pi t \delta\left(\frac{\omega - k_z}{2}\right) ,$$

we find the transport coefficient $\hat{q} = d\langle \Delta \boldsymbol{p}_{\mathrm{T}}^2(t) \rangle / dt$ as

$$\begin{aligned} \hat{q} = 2\pi g^2 \frac{C_R}{N_c^2 - 1} \int \frac{d^4 k}{(2\pi)^4}\, \delta(\omega - k_z) \Big[\langle E_a^x E_a^x \rangle_k + \langle E_a^y E_a^y \rangle_k \\ - \langle E_a^x B_a^y \rangle_k + \langle E_a^y B_a^x \rangle_k - \langle B_a^y E_a^x \rangle_k + \langle B_a^x E_a^y \rangle_k + \langle B_a^x B_a^x \rangle_k + \langle B_a^y B_a^y \rangle_k \Big] . \quad (13) \end{aligned}$$

Using the equilibrium field correlators derived in [2], Eq. (13) gives

$$\hat{q} = 2 g^2 C_R T \int \frac{d^3 k}{(2\pi)^3}\, \frac{k_{\mathrm{T}}^2}{k_z \boldsymbol{k}^2} \left[\frac{\mathrm{Im}\varepsilon_{\mathrm{L}}(k_z, \boldsymbol{k})}{|\varepsilon_{\mathrm{L}}(k_z, \boldsymbol{k})|^2} + \frac{k_z^2 k_{\mathrm{T}}^2 \,\mathrm{Im}\varepsilon_{\mathrm{T}}(k_z, \boldsymbol{k})}{|k_z^2 \varepsilon_{\mathrm{T}}(k_z, \boldsymbol{k}) - \boldsymbol{k}^2|^2} \right] . \qquad (14)$$

The classical formula (14) holds for the inverse wave vectors of the fields which are much longer than the de Broglie wavelength of plasma particles. It requires $|\boldsymbol{k}| \ll T$ where T is the plasma temperature. For larger wave vectors a quantum approach discussed in [3] is needed.

Let us now consider the momentum broadening of a fast parton in unstable anisotropic plasmas. For the sake of analytical tractability, the two-stream plasma is discussed and only longitudinal electric fields are taken into account. Substituting the correlation function (3) into Eq. (10), one finds

$$\begin{aligned}\left\langle \Delta\boldsymbol{p}_{\rm T}^2(t)\right\rangle &= g^2\frac{C_R}{N_c^2-1}\int_0^t dt_1\int_0^t dt_2\left[\langle E_a^x(t_1)\,E_a^x(t_2)\rangle+\langle E_a^y(t_1)\,E_a^y(t_2)\rangle\right]\\ &= \frac{g^4}{4}C_R n\int\frac{d^3k}{(2\pi)^3}\frac{k_{\rm T}^2}{\boldsymbol{k}^4\left(\omega_+^2-\omega_-^2\right)^2}\frac{\left(\gamma_{\boldsymbol{k}}^2+(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)^2}{\gamma_{\boldsymbol{k}}^2\left(k_z^2+\gamma_{\boldsymbol{k}}^2\right)}\\ &\times\Bigg[\left(\gamma_{\boldsymbol{k}}^2+(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\left(|e^{(ik_z+\gamma_{\boldsymbol{k}})t}-1|^2+|e^{(ik_z-\gamma_{\boldsymbol{k}})t}-1|^2\right)\\ &+4\left(\gamma_{\boldsymbol{k}}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\frac{k_z^2-\gamma_{\boldsymbol{k}}^2}{k_z^2+\gamma_{\boldsymbol{k}}^2}\Bigg]\,.\end{aligned}\tag{15}$$

When only the fastest growing mode is included, Eq. (15) changes to

$$\hat{q}=\frac{d\langle\boldsymbol{p}_{\rm T}^2(t)\rangle}{dt}\approx\frac{g^4}{2}C_R\,n\int\frac{d^3k}{(2\pi)^3}\,e^{2\gamma_{\boldsymbol{k}}t}\,\frac{k_{\rm T}^2\left(\gamma_{\boldsymbol{k}}^2+(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)^3}{\boldsymbol{k}^4\left(\omega_+^2-\omega_-^2\right)^2\gamma_{\boldsymbol{k}}\left(k_z^2+\gamma_{\boldsymbol{k}}^2\right)}\,.\tag{16}$$

It should be noted that Eq. (16), in contrast to Eq. (15), suffers from divergence when $\gamma_{\boldsymbol{k}}\to 0$. The momentum broadening (16) grows exponentially in time, as the exponentially growing fields exert an exponentially growing influence on the propagating parton. This effect is missing in the previous results [12, 13] obtained for the anisotropic plasma treated as a stationary medium.

4. Discussion and outlook

If the momentum distribution of partons is anisotropic, the quark–gluon is unstable with respect the chromomagnetic modes. The instability growth rate γ is of order gT [1], where T is here the characteristic parton momentum corresponding to temperature of equilibrium plasma. The time scale of unstable mode growth γ^{-1} is the shortest dynamical time scale in weakly coupled plasmas. Therefore, the unstable plasma cannot be treated as a static medium whenever a characteristics, which involves color degrees of freedom, is studied.

In this paper I discussed the fluctuations of chromodynamic fields in the plasma, and then, the field correlation functions were used to compute the momentum broadening of a fast parton flying across the plasma. Both quantities were found as solutions of initial value problem. In the case of unstable plasma, the field fluctuations as well as the momentum broadening were explicitly shown to exponentially grow in time.

An analytic treatment of unstable plasma is rather difficult. Therefore, instead of the plasma with momentum distribution relevant for relativistic heavy-ion collisions, a toy model representing the two stream system was discussed. The model grasps some important features of unstable systems, but it cannot be used for any quantitative predictions on the QGP produced in high-energy nucleus–nucleus collisions. An analysis the chromodynamic fluctuations and momentum broadening in such plasma is in progress but many other plasma characteristics, in particular transport coefficients, need to be studied to understand QGP from early stage of relativistic heavy-ion collisions.

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