SUPERSYMMETRIC QED PLASMA*

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We systematically compare the $\mathcal{N}=1$ SUSY QED plasma to its non-supersymmetric counterpart which is QED plasma of electrons, positrons and photons. Collective excitations and collisional processes in the two systems are confronted to each other in a regime of small coupling. The collective and collisional characteristics of supersymmetric plasma are both very similar to those of QED plasma.

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1. Introduction

Supersymmetry is a good candidate to be a symmetry of Nature at sufficiently high energies and if true, SUSY plasmas existed in the early Universe. Experiments at the Large Hadron Collider can soon provide an evidence of supersymmetry. However, independently of its ontological status, supersymmetric field theories are worth studying because of their unique features. A discovery of the AdS/CFT duality of the five-dimensional gravity in anti de Sitter geometry and the conformal field theories, see the review [1], stimulated a great interest in the $\mathcal{N}=4$ supersymmetric Yang–Mills theory which is both classically and quantum mechanically conformally invariant. The duality offers a unique tool to study strongly coupled field theories, as the gravitational constant is inversely proportional to the coupling constant of dual conformal field theory and thus some problems of strongly coupled field theories can be solved via weakly coupled gravity. Some intriguing results have been obtained, see the reviews [2, 3], but relevance of the results for non-supersymmetric theories, which are of our actual interest, remains an

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open issue. One asks how properties of the plasma governed by $\mathcal{N}=4$ Super Yang–Mills theory are related to those of the usual quark-gluon plasma experimentally studied in relativistic heavy-ion collisions. While such a comparison of the two systems is a difficult task, some comparative analyses have been done in the domain of weak coupling, where perturbative methods are applicable [4, 5, 6, 7, 8].

Our aim is to systematically compare the $\mathcal{N}=4$ Super Yang–Mills plasma to the QCD one with a particular emphasis on non-equilibrium characteristics which has not attracted much attention yet. We started, however, with the supersymmetric $\mathcal{N}=1$ QED plasma which is noticeably simpler than that of $\mathcal{N}=4$ Super Yang–Mills. We first studied collective excitations of ultrarelativistic $\mathcal{N}=1$ SUSY QED plasma which were confronted with those of an electromagnetic plasma of electrons, positrons and photons [9]. In the subsequent paper [10], we focused on collisional processes which control plasma transport properties. Here we summarize our all findings. Throughout the paper we use a natural system of units with $c=\hbar=k_B=1$ and the metric tensor of the signature $(+---)$.

2. $\mathcal{N}=1$ SUSY QED

We start our considerations by writing down the Lagrangian of $\mathcal{N}=1$ SUSY QED which is known, see *e.g.* [11], to be

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{i}{2}\bar{\Lambda}\partial\!\!\!/\,\Lambda+i\bar{\Psi}D\!\!\!\!/\,\Psi+\left(D_\mu\phi_{\rm L}\right)^*\left(D^\mu\phi_{\rm L}\right)+\left(D^*_\mu\phi_{\rm R}\right)\left(D^\mu\phi^*_{\rm R}\right)\\ & +\sqrt{2}e\left(\bar{\Psi}P_{\rm R}\lambda\phi_{\rm L}-\bar{\Psi}P_{\rm L}\lambda\phi^*_{\rm R}+\phi^*_{\rm L}\bar{\Lambda}P_{\rm L}\Psi-\phi_{\rm R}\bar{\Lambda}P_{\rm R}\Psi\right)\\ & -\frac{e^2}{2}\left(\phi^*_{\rm L}\phi_{\rm L}-\phi^*_{\rm R}\phi_{\rm R}\right)^2\,, \end{aligned} \tag{1}$$

where the strength tensor $F^{\mu\nu}$ is expressed through the electromagnetic four-potential A^μ as $F^{\mu\nu}\equiv\partial^\mu A^\nu-\partial^\nu A^\mu$ and the covariant derivative is $D^\mu\equiv\partial^\mu+ieA^\mu$; Λ is the Majorana bispinor photino field, Ψ is the Dirac bispinor electron field, $\phi_{\rm L}$ and $\phi_{\rm R}$ are the scalar left selectron and right selectron fields; the projectors $P_{\rm L}$ and $P_{\rm R}$ are defined in a standard way $P_{\rm L}\equiv\frac{1}{2}(1-\gamma_5)$ and $P_{\rm R}\equiv\frac{1}{2}(1+\gamma_5)$. Since we are interested in ultrarelativistic plasmas, the mass terms are neglected in the Lagrangian.

3. Collective excitations

We consider collective excitations of $\mathcal{N}=1$ SUSY QED plasma which is homogeneous but the momentum distribution is, in general, different from the equilibrium one. The excitations are determined by the dispersion equations which for (quasi-)photons, electrons, photinos and selectrons read

$$\det\left[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)\right] = 0\,, \tag{2}$$

$$\det\left[\not{k} - \Sigma(k)\right] = 0\,, \tag{3}$$

$$\det\left[\not{k} - \tilde{\Pi}(k)\right] = 0\,, \tag{4}$$

$$k^2 + \tilde{\Sigma}_{\mathrm{L,R}}(k) = 0\,, \tag{5}$$

where $\Pi^{\mu\nu}(k)$, $\Sigma(k)$, $\tilde{\Pi}(k)$, $\tilde{\Sigma}_{\mathrm{L,R}}(k)$ are the retarded self-energies of photons, electrons, photinos and left or right selectrons. Since the plasma under consideration is generally out of equilibrium, the self energies were computed using the Keldysh–Schwinger formalism. We were interested in collective modes which occur, when wavelength of a quasi-particle is much bigger than a characteristic interparticle distance in the plasma, therefore we worked in the Hard Loop Approach, see the review [12], which had been generalized to anisotropic systems in [13, 14]. We also assumed that the plasma is electrically neutral and that the distribution functions of particles and antiparticles are equal to each other. The distribution functions of left and right selectrons are assumed to be the same as well.

Self-energies are usually defined by means of a Dyson–Schwinger equation which for the case of polarization tensor Π has the following symbolic form $\mathcal{D} = D - D\,\Pi\,\mathcal{D}$, where $\mathcal{D}$ and D is the interacting and free photon propagator, respectively. We computed the self energies perturbatively at one-loop level. Within the Keldysh–Schwinger formalism one first finds the contour self-energies and further on the retarded self-energies are extracted. Using the Hard Loop Approximation, the polarization tensor was found as

$$\begin{aligned}\Pi^{\mu\nu}(k) \;=\; & 4e^2 \int \frac{d^3p}{(2\pi)^3}\, \frac{f_e(\boldsymbol{p}) + f_s(\boldsymbol{p})}{E_p} \\ & \times \frac{k^2 p^\mu p^\nu - \left(p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)\right)(k\cdot p)}{(k\cdot p + i0^+)^2}\,,\end{aligned} \tag{6}$$

where f_e and f_s are the distribution functions of electrons and selectrons. As seen, Π vanishes in the vacuum limit ($f_e, f_s \to 0$) which is a genuine feature of supersymmetric theories. In the non-supersymmetric counterpart, the vacuum contribution to Π diverges and it requires a special treatment.

Up to the vacuum contribution, the polarization tensor of supersymmetric plasma and of its non-supersymmetric counterpart have the same structure. Therefore, the spectra of collective excitations of gauge bosons in the two systems are identical. In equilibrium plasma we have the longitudinal (plasmon) mode and the transverse one which are discussed in *e.g.* the textbook [15]. When the plasma is out of equilibrium, there is a whole variety of possible collective excitations. In particular, there are unstable modes, see *e.g.* the review [16], which exponentially grow in time and strongly influence the system's dynamics.

The one-loop electron self-energy, which was found as

$$\Sigma(k) \;=\; e^2 \int \frac{d^3p}{(2\pi)^3} \, \frac{f_\gamma(\boldsymbol{p}) + f_e(\boldsymbol{p}) + f_{\tilde\gamma}(\boldsymbol{p}) + f_s(\boldsymbol{p})}{E_p} \, \frac{\not p}{k\cdot p + i0^+}\,, \tag{7}$$

with f_γ and $f_{\tilde\gamma}$ being the distribution functions of photons and photinos, has the same structure for the SUSY QED and QED plasma. Therefore, we have identical spectrum of excitations of charged fermions in the two systems. In equilibrium plasma there are two modes, see in *e.g.* the textbook [15], of opposite helicity over chirality ratio. One mode corresponds to the positive energy fermion, another one, sometimes called a plasmino, is a specific medium effect. Out of equilibrium the spectrum changes but no unstable modes have been found even for an extremely anisotropic momentum distribution [17, 18].

The photino self-energy was computed as

$$\tilde\Pi(k) = e^2 \int \frac{d^3p}{(2\pi)^3} \, \frac{f_s(\boldsymbol{p}) + f_e(\boldsymbol{p})}{E_p} \, \frac{\not p}{k\cdot p + i0^+} \tag{8}$$

and its structure coincides with the electron self-energy (7). The spectra of collective excitations are also identical. When the plasma momentum distribution is anisotropic and unstable photon modes occur, the photino modes remain stable. Therefore, the supersymmetry does not induce an instability in the photino sector, as one could naively expect.

Finally, we present the one-loop retarded self-energy of selectron

$$\tilde\Sigma(k) = -2e^2 \int \frac{d^3p}{(2\pi)^3} \, \frac{f_e(\boldsymbol{p}) + f_\gamma(\boldsymbol{p}) + f_s(\boldsymbol{p}) + f_{\tilde\gamma}(\boldsymbol{p})}{E_p} \tag{9}$$

which is the same for left and right selectron fields. Because of supersymmetry it vanishes in the vacuum limit when all the distributions functions are zero. This is also effect of supersymmetry that the distribution functions of electrons and selectrons and of photons and photions enter Eq. (9) with the same coefficients.

The selectron self-energy (9) is independent of k, it is negative and real. Therefore, $\tilde\Sigma$ can be written as $\tilde\Sigma = -m_{\rm eff}^2$, where $m_{\rm eff}$ is the effective selectron mass. Then, the solutions of dispersion equation (5) are $E_k = \pm\sqrt{m_{\rm eff}^2 + \boldsymbol{k}^2}$.

4. Effective action

The Hard Loop Approach can be formulated in an elegant and compact way by using the effective action, the form of which is dictated by structure of self energies. Since the self energy of a given field is the second functional derivative of the action with respect to the field, the action of, say, electromagnetic field is of the form

$$\mathcal{L}_2^{(A)}(x) = \frac{1}{2}\int d^4y\, A_\mu(x)\Pi^{\mu\nu}(x-y)A_\nu(y)\,, \tag{10}$$

where the polarization tensor is given by Eq. (6). The subscript '2' indicates that the above effective action generates only the two-point function. To generate n-point functions the action $\mathcal{L}_2$ needs to be extended to a gauge invariant form by replacing the ordinary derivatives by the covariants. Repeating the calculations described in detail in [14], one finds the Hard Loop effective actions of $\mathcal{N}=1$ SUSY QED as

$$\mathcal{L}_{\rm HL}^{(A)} = 4e^2\int\frac{d^3p}{(2\pi)^3}\frac{f_1(\boldsymbol{p})}{E_p}\,F_{\mu\nu}(x)\frac{p^\nu p^\rho}{(p\cdot\partial)^2}F_\rho{}^\mu(x)\,, \tag{11}$$

$$\mathcal{L}_{\rm HL}^{(\Psi)} = ie^2\int\frac{d^3p}{(2\pi)^3}\frac{f_2(\boldsymbol{p})}{E_p}\,\bar{\Psi}(x)\frac{p\cdot\gamma}{p\cdot D}\Psi(x)\,, \tag{12}$$

$$\mathcal{L}_{\rm HL}^{(\Lambda)} = ie^2\int\frac{d^3p}{(2\pi)^3}\frac{f_1(\boldsymbol{p})}{E_p}\,\bar{\Lambda}(x)\frac{p\cdot\gamma}{p\cdot\partial}\Lambda(y)\,, \tag{13}$$

$$\mathcal{L}_{\rm HL}^{(\phi_{\rm L,R})} = -2e^2\int\frac{d^3p}{(2\pi)^3}\frac{f_2(\boldsymbol{p})}{E_p}\,\phi^*_{\rm L,R}(x)\phi_{\rm L,R}(x)\,, \tag{14}$$

where $f_1(\mathbf{p}) \equiv f_e(\boldsymbol{p}) + f_s(\boldsymbol{p})$ and $f_2(\mathbf{p}) \equiv f_e(\boldsymbol{p}) + f_\gamma(\boldsymbol{p}) + f_s(\boldsymbol{p}) + f_{\tilde{\gamma}}(\boldsymbol{p})$.

The actions (11)–(14) are obtained from the self-energies but the reasoning can be turned around. As argued in [19, 20], the actions of gauge bosons (11), charged fermions (12) and charged scalars (14) are of unique gauge invariant form. Therefore, the respective self-energies can be, in principle, inferred from the known QED self-energies with some help of supersymmetry arguments. An explicit computation of photino self-energy seems to be unavoidable.

5. Collisional processes

As seen in the Lagrangian (2), there is a self-interaction of selectron field due to the terms $(\phi^*_{\rm L}\phi_{\rm L})^2$, $(\phi^*_{\rm R}\phi_{\rm R})^2$ and $-2\phi^*_{\rm L}\phi_{\rm L}\phi^*_{\rm R}\phi_{\rm R}$, there is also a four-boson coupling $\phi^*_{\rm L,R}\phi_{\rm L,R}A^\mu A_\mu$. Such a contact interaction is qualitatively different than that caused by an exchange of massless boson. The scattering cross section in the absence of other interactions is isotropic in the center-of-mass frame of colliding particles and the energy and momentum transfers are bigger than that in electromagnetic interactions. Therefore, one expects that transport properties of supersymmetric $\mathcal{N} = 1$ QED plasma differ from those of QED plasma of electrons, positrons and photons. To test this expectation, we computed the cross sections of all binary processes which occur in $\mathcal{N} = 1$ SUSY QED plasma in the lowest non-trivial order of $\alpha \equiv e^2/4\pi$. The complete list of processes is presented in [10].

There are five processes where only electrons, positrons and photons take part. These processes occur in both the supersymmetric QED plasma and usual electromagnetic one. There are twenty eight other processes which are characteristic for the $\mathcal{N} = 1$ SUSY QED. Among those processes there are eight of special interest. Compton scattering of selectrons is an example, the cross section of which is independent of momentum transfer t or u. The matrix element of such a process is simply a number. These processes are qualitatively different from those in electromagnetic plasmas which are dominated by an interaction with small momentum transfer. We note that for each plasma particle e, γ, $\tilde{e}$, $\tilde{\gamma}$ such a process exists.

It should be remembered that the temperature T is the only dimensional parameter which characterizes an equilibrium ultrarelativistic plasma. Consequently, the parametric form of transport coefficients can be determined by dimensional arguments. For example, the shear viscosity must be proportional to T^3/α^2 and it is thus hard to expect that the viscosity of supersymmetric plasma is qualitatively different than that of electromagnetic one. Indeed, the shear viscosity of an $\mathcal{N} = 4$ Super Yang–Mills plasma is rather similar to that of a quark-gluon plasma [21].

We considered two transport characteristics of the $\mathcal{N} = 1$ QED plasma which are not so constrained by dimensional arguments. Specifically, we computed the collisional energy loss and momentum broadening of a particle traversing the equilibrium plasma. The dimensional argument does not work here because the two quantities depend not only on the plasma temperature T but on the test particle energy E as well.

When the matrix element equals $|\mathcal{M}|^2 = 4e^4$, as in the case of scattering of selectron on photons, the energy loss of high energy particle ($E \gg T$) in equilibrium $\mathcal{N} = 1$ SUSY QED plasma was found to be

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2 \tag{15}$$

which should be confronted with the energy loss of an energetic muon in ultrarelativistic QED plasma of electrons, positrons and photons [22]

$$\frac{dE}{dx} = -\frac{e^4}{48\pi^3}\,T^2\left(\ln\frac{E}{eT} + 2.031\right). \tag{16}$$

As seen, the formulas (15), (16) are similar to each other up the logarithm term discussed in [10]. The similarity is rather surprising, if one realizes how different are the differential cross sections of interest.

The energy loss can be estimated as $\frac{dE}{dx} \sim \langle\Delta E\rangle/\lambda$, where $\langle\Delta E\rangle$ is the typical change of particle's energy in a single collision and λ is the particle's mean free path given as $\lambda^{-1} = \rho\,\sigma$ with $\rho \sim T^3$ being the density of scatterers and σ denoting the cross section. For the differential cross section $\frac{d\sigma}{dt} \sim e^4/s^2$, the total cross section is $\sigma \sim e^4/s$. When a highly energetic particle with energy E scatters on massless plasma particle, $s \sim ET$ and consequently $\sigma \sim e^4/(ET)$. The inverse mean free path is thus estimated as $\lambda^{-1} \sim e^4T^2/E$. When the scattering process is independent of momentum transfer, $\langle\Delta E\rangle$ is of the order of E and we finally find $-\frac{dE}{dx} \sim e^4T^2$. When compared to the case of Coulomb scattering, the energy transfer in a single collision is much bigger but the cross section is smaller in the same proportion. Consequently, the two interactions corresponding to very different differential cross sections lead to very similar energy losses.

We also computed the momentum broadening, which is usually denoted as $\hat{q}$, due to the scattering which is momentum-transfer independent. $\hat{q}$ determines a magnitude of radiative energy loss of a highly energetic particle in a plasma medium [23]. When the matrix element equals $|\mathcal{M}|^2 = 4e^4$, the momentum broadening of a highly energetic particle is

$$\hat{q} = \frac{e^4\zeta(3)}{12\pi^3}\,T^3 \tag{17}$$

and it should be compared to the momentum broadening driven by one-photon exchange which is of the order of $e^4\ln(1/e)\,T^3$ [24]. As seen, the momentum broadening and consequently the radiative energy loss of a highly energetic particle in SUSY QED and QED plasma are similar (up to the logarithm term) to each other.

6. Conclusions

Collective modes in ultrarelativistic $\mathcal{N} = 1$ SUSY QED plasma are essentially the same as in the electromagnetic plasma of electrons, positrons and photons. Although there are binary processes in supersymmetric plasma, the cross sections of which are independent of the momentum transfer, transport properties of $\mathcal{N} = 1$ SUSY QED plasma are very similar to those of QED one.

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REFERENCES

[1] O. Aharony *et al.*, *Phys. Rep.* **323**, 183 (2000).
[2] D.T. Son, A.O. Starinets, *Annu. Rev. Nucl. Part. Sci.* **57**, 95 (2007).
[3] R.A. Janik, *Lect. Notes Phys.* **828**, 147 (2011).
[4] S. Caron-Huot *et al.*, *J. High Energy Phys.* **0612**, 015 (2006).
[5] S. Caron-Huot, G.D. Moore, *J. High Energy Phys.* **0802**, 081 (2008).
[6] J.P. Blaizot, E. Iancu, U. Kraemmer, A. Rebhan, *J. High Energy Phys.* **0706**, 035 (2007).
[7] P.M. Chesler, A. Vuorinen, *J. High Energy Phys.* **0611**, 037 (2006).
[8] P.M. Chesler, A. Gynther, A. Vuorinen, *J. High Energy Phys.* **0909**, 003 (2009).
[9] A. Czajka, S. Mrówczyński, *Phys. Rev.* **D83**, 045001 (2011).
[10] A. Czajka, S. Mrówczyński, *Phys. Rev.* **D84**, 105020 (2011).
[11] T. Binoth, E.W.N. Glover, P. Marquard, J.J. van der Bij, *J. High Energy Phys.* **0205**, 060 (2002).
[12] U. Kraemmer, A. Rebhan, *Rep. Prog. Phys.* **67**, 351 (2004).
[13] S. Mrówczyński, M.H. Thoma, *Phys. Rev.* **D62**, 036011 (2000).
[14] S. Mrówczyński, A. Rebhan, M. Strickland, *Phys. Rev.* **D70**, 025004 (2004).
[15] M. Le Bellac, *Thermal Field Theory*, Cambridge University Press, Cambridge 2000.
[16] S. Mrówczyński, M.H. Thoma, *Annu. Rev. Nucl. Part. Sci.* **57**, 61 (2007).
[17] S. Mrówczyński, *Phys. Rev.* **D65**, 117501 (2002).
[18] B. Schenke, M. Strickland, *Phys. Rev.* **D74**, 065004 (2006).
[19] J. Frenkel, J.C. Taylor, *Nucl. Phys.* **B374**, 156 (1992).
[20] E. Braaten, R.D. Pisarski, *Phys. Rev.* **D45**, 1827 (1992).
[21] S. Caron-Huot, S. Jeon, G.D. Moore, *Phys. Rev. Lett.* **98**, 172303 (2007).
[22] E. Braaten, M.H. Thoma, *Phys. Rev.* **D44**, 1298 (1991).
[23] R. Baier *et al.*, *Nucl. Phys.* **B484**, 265 (1997).
[24] P.B. Arnold, W. Xiao, *Phys. Rev.* **D78**, 125008 (2008).