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ELLIPTIC FLOW FLUCTUATIONS

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We suggest to perform systematic measurements of the elliptic flow fluctuations which are sensitive to the early stage dynamics of heavy-ion collisions at high-energies. Significant flow fluctuations are shown to be generated due to the formation of topological clusters and development of the filamentation instability. The statistical noise and hydrodynamic fluctuations are also estimated.

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1. Introduction

A high-energy collision of heavy ions is often called the Little Bang because of its similarity to the cosmological Big Bang. Both phenomena are violent explosions and both have attracted attention of experimentalists who have gathered unprecedented amount of data, limited basically by the data processing technology. The experiments provided a lot of valuable information about the system's evolution. In particular, small variation in the temperature of background radiation have revealed mean dipole component, caused by the motion of the Solar System relative to the Big Bang heat bath.

Tiny ($\sim 10^{-5}$) fluctuations on top of the dipole contribution, which have been recently decomposed into angular harmonics with l up to about 2000, show peaks due to frozen sound exited 15 billions of years ago. A study of harmonic fluctuations in the Little Bang may possibly reveal something interesting like frozen modulations as well.

One of the most spectacular experimental results obtained by now in relativistic heavy-ion collisions at RHIC is strong elliptic flow quantified by mean value of the second angular harmonics v_2 [1–5]. The phenomenon, which is sensitive to the collision early stage [6] when the interaction zone is of the almond shape, is naturally explained within a hydrodynamics as a result of large density gradients [7–12]. Since, the hydrodynamic description is applicable for a system which is in local thermodynamic equilibrium, the large elliptic flow suggests a surprisingly short, below 1 fm/c [13], equilibration time which is difficult to reconcile with dynamic calculations, at least those performed within the perturbative QCD, see *e.g.* [14], where the early rapid expansion is closer to free streaming than to hydrodynamic evolution. We note here that the hydrodynamic model [15], which assumes the equilibration of only transverse degrees of freedom, has appeared rather unsuccessful in describing the experimental data [1–5]. There have also been attempts [16,17] to explain the large elliptic flow within models which do not invoke thermodynamic equilibrium. Finally, at large transverse momentum $p_{\rm T} = 2$–$10\,$GeV the magnitude of v_2 seem to be well described by the surface emission model [26].

In such a situation, it is certainly desirable to look for experimental observables which can shed more light on the system dynamics at a collision early stage. We propose to go beyond measuring the *mean* elliptic flow magnitude, and study its (and higher harmonics, if it ever be possible) *fluctuations on the event-by-event basis*[1]. To be specific, we suggest to measure v_2 in every collision and then to analyze the variance of v_2. The first attempt of such an measurement has been undertaken in a very recent study by STAR Collaboration [3]. The result is however rather inconclusive. Our aim here is to motivate the work in order to improve experimental procedures.

The elliptic flow fluctuations are shown to be sensitive even to somewhat exotic phenomena which have been argued to occur at the collision early stage. We consider here the filamentation instability [18–20] initiated due to the strong momentum anisotropy of the parton system, and the generation and subsequent explosions of the topological clusters [21]. To detect these dynamical phenomena of interest one needs, however, a reliable estimate of the usual fluctuation. Therefore, a magnitude of the statistical noise is

[1] This is similar idea to the above mentioned measurement of the angular fluctuations of T. In cosmology, there is, of course, only one event but one can study angular fluctuations in various regions of the sky.

discussed in detail. We take into account the interference of various Fourier harmonics and finite resolution of the reaction plane reconstruction. We also study the fluctuations caused by the impact parameter and particle multiplicity variation.

2. Formulation of the problem

Two methods have been developed to quantify the ellipticity, as well as higher harmonics, in the azimuthal angle distribution. One possibility is to work directly with the correlation functions of 2, 4, 6 or even more particles [22, 23]. Then, only the *relative* emission angles of particles matter and there is no need to determine a direction of the impact parameter ψ_{R}. Using the cumulants, one can partially eliminate detector effects, and consequently even detectors with relatively small acceptance can be used. The resulting Fourier coefficients include, however, not only the effects associated with the ellipticity of events but are contaminated by any 2-, 4-, or, in general, n-body correlations caused by resonances, jets, quantum statistics *etc.* However, it is hoped that these *nonflow* correlations are dominated by the two-body effects and thus the genuine four-particle correlations provide rather clean information about the flow. Going to the six-particle correlations, the procedure can be further improved.

In our considerations, we will refer to another method [24, 25], which was formulated earlier and is usually called a standard one. The method focuses on the angular distributions relative to direction of the impact parameter. The experimental procedure splits in two steps which should be as independent from each other as possible. In the first step, one uses all available *multi-body* information about an event in order to determine the impact parameter direction ψ_{R}. In the second step, one constructs the distribution of the azimuthal angle relative to ψ_{R} of "selected particles" and one computes the Fourier coefficients. The sets of particles used at these two steps are different from each other, and we will call their numbers as M and N, respectively. In order to reduce non-flow correlations, the particles of both sets (subevents) are usually separated by a rapidity gap. They are still correlated by the flow because the direction of the impact parameter ψ_{R} is a global feature of an event, like the magnitude of the impact parameter itself[2]. In practice, it is desired to use the cumulant and standard method simultaneously. Comparing the results, one can eventually separate the "flow" (global correlation between all secondaries) from "local correlations" involv-

[2] In principle, the impact parameter magnitude can be reconstructed from 2-, 4-, 6-, ..., n-particle correlators. However, it would be very difficult and presumably rather inaccurate method. Using total multiplicity or forward calorimeter signal does the job very well.

ing a few particles only. Such a comparison made by STAR [3] has shown that unless one goes to high $p_{\rm T}$ or very central collisions, the flow dominates and 2-body correlations contribute to v_2 obtained from the standard method only at the 10–15 % level.

Since the one-particle distribution in a single event can be written as

$$P(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \psi_{\rm R}))\right] \Theta(\phi)\,\Theta(2\pi - \phi)\,, \tag{1}$$

the n-th Fourier amplitude is determined as

$$v_n = \overline{\cos(n(\phi - \psi_{\rm R}))}\,,$$

with $\overline{\cdots}$ denoting averaging over one-particle distribution in a single event.

The reaction plane is never reconstructed precisely and the real reaction plane angle $\psi_{\rm R}$ deviates from the estimated angle $\psi_{\rm E}$. One observes that

$$v_n = \frac{1}{R_n}\,\overline{\cos(n(\phi - \psi_{\rm E}))}\,,$$

where $R_n \equiv \cos(n(\psi_{\rm R} - \psi_{\rm E}))$ is the reaction plane resolution factor.

Let us now think about the ensemble of events with every event representing a single nucleus–nucleus collision. The angular harmonics v_n are measured for each event. It should be stressed that this is not only the angle $\psi_{\rm R}$ (and $\psi_{\rm E}$) which varies form event to event but the amplitudes of Fourier harmonics can also vary due to dynamical reasons. According to [25], the average over events of the harmonic's amplitude is not defined as

$$\langle v_n \rangle \stackrel{\rm def}{=} \left\langle \frac{\overline{\cos(n(\phi - \psi_{\rm E}))}}{R_n} \right\rangle, \tag{2}$$

but

$$\langle v_n \rangle \stackrel{\rm def}{=} \frac{\left\langle \overline{\cos(n(\phi - \psi_{\rm E}))} \right\rangle}{\langle R_n \rangle}\,, \tag{3}$$

where $\langle\cdots\rangle$ denotes averaging over events. Since the procedure of determining the angle $\psi_{\rm E}$ is arranged to be maximally independent from that of computing of $\left\langle \overline{\cos(n(\phi - \psi_{\rm E}))} \right\rangle$, it is expected that the event averaging of R_n and of $\overline{\cos(n(\phi - \psi_{\rm E}))}$ are independent from each other. If so,

$$\left\langle \frac{\overline{\cos(n(\phi - \psi_{\rm E}))}}{R_n} \right\rangle = \left\langle \frac{1}{R_n} \right\rangle \left\langle \overline{\cos(n(\phi - \psi_{\rm E}))} \right\rangle \approx \frac{\left\langle \overline{\cos(n(\phi - \psi_{\rm E}))} \right\rangle}{\langle R_n \rangle}\,,$$

where the approximate equality holds if the resolution factor $\cos(n(\psi_{\rm R}-\psi_{\rm E}))$ does not much vary from event to event. Thus, the definitions (2), (3) are approximately equivalent to each other, provided $\psi_{\rm R}$ is reconstructed sufficiently well (which requires $M \gg 1$).

While the definition (2) can be uniquely extended to the second moment, there is an ambiguity how to generalize the definition (3). Therefore, we define

$$\langle v_n^2\rangle \stackrel{\rm def}{=} \frac{1}{\langle R_n\rangle^2}\left\langle \overline{\cos(n(\phi-\psi_{\rm E}))}^2\right\rangle ,$$

and the fluctuations as

$$\begin{aligned}\mathsf{Var}(v_n) &\stackrel{\rm def}{=} \langle v_n^2\rangle - \langle v_n\rangle^2 \\ &= \frac{1}{\langle R_n\rangle^2}\left(\left\langle \overline{\cos(n(\phi-\psi_{\rm E}))}^2\right\rangle - \left\langle \overline{\cos(n(\phi-\psi_{\rm E}))}\right\rangle^2\right) ,\end{aligned} \tag{4}$$

where $\langle R_n\rangle^2$ enters as a multiplicative factor. However, R_n generates event-by-event fluctuations of observed v_n. To see the effect, let us consider the single-particle azimuthal distribution in a given event of the form (1) with the amplitudes v_n being exactly the same in *all* events. Then, $\langle v_n\rangle = v_n$ but

$$\mathsf{Var}(v_n) = \frac{\langle R_n^2\rangle - \langle R_n\rangle^2}{\langle R_n\rangle^2}\, v_n^2 .$$

Thus, the fluctuations of R_n contribute to $\mathsf{Var}(v_n)$. We will often use the symbol $\delta v_n \equiv \sqrt{\mathsf{Var}(v_n)}$.

In the following sections, we will focus our attention on the second harmonics and consider several sources of the v_2 fluctuations.

3. Statistical noise

We start our discussions of the fluctuations of v_2 with those caused by the finite number N of particles which are used at the step 2 of the standard method when the Fourier amplitudes are determined. We assume here that v_n do *not* change from event to event. We also assume that the only correlations in the system are those due to the flow. Then, the azimuthal distribution of N particles is a product of N single particle distributions. Namely,

$$P_N(\phi_1,\phi_2,\cdots,\phi_N) = \mathcal{P}_N P(\phi_1)\, P(\phi_2)\ \cdots\ P(\phi_N) ,$$

where $\mathcal{P}_N$ is the multiplicity distribution while all distributions $P(\phi_i)$ are given by Eq. (1). The single particle distributions $P(\phi_i)$ are correlated to each other because of the common angle $\psi_{\rm R}$.

In a single event, the ellipticity is found as

$$v_2 = \frac{1}{R_2}\,\frac{1}{N}\sum_{i=1}^{N} \cos(2(\phi_i - \psi_{\rm E}))\,,$$

where ϕ_i is the azimuthal angle of i-th particle and N is the event's multiplicity. According to the definition (3) the ensemble average of v_2 then equals

$$\begin{aligned}
\langle v_2 \rangle &= \frac{1}{\langle R_2 \rangle}\left\langle \frac{1}{N}\sum_{i=1}^{N} \cos(2(\phi_i - \psi_{\rm E}))\right\rangle \\
&= \sum_{N=1}^{\infty} \mathcal{P}_N\,\frac{1}{N}\int_0^{2\pi} d\phi_1 P(\phi_1)\int_0^{2\pi} d\phi_2 P(\phi_2) \\
&\quad \cdots \int_0^{2\pi} d\phi_N P(\phi_N)\sum_{i=1}^{N} \cos(2(\phi_i - \psi_{\rm E})) \\
&= v_2\,.
\end{aligned} \tag{5}$$

We note that the event-by-event averaging of R_2 and of $\cos(2(\phi_i - \psi_{\rm E}))$ are assumed to be independent from each other.

The second moment is

$$\begin{aligned}
\langle v_2^2 \rangle &= \frac{1}{\langle R_2 \rangle^2}\left\langle \left(\frac{1}{N}\sum_{i=1}^{N} \cos(2(\phi_i - \psi_{\rm E}))\right)^2 \right\rangle \\
&= \frac{1}{\langle R_2 \rangle^2}\sum_{N=1}^{\infty} \mathcal{P}_N\,\frac{1}{N^2}\int_0^{2\pi} d\phi_1 P(\phi_1)\int_0^{2\pi} d\phi_2 P(\phi_2) \\
&\quad \cdots \int_0^{2\pi} d\phi_N P(\phi_N)\left(\sum_{i=1}^{N} \cos(2(\phi_i - \psi_{\rm E}))\right)^2 \\
&= \frac{1}{\langle R_2 \rangle^2}\left[\left(\frac{1}{2}+\frac{1}{2}v_4\,\left(2\langle R_2^2\rangle - 1\right)\right)\left\langle \frac{1}{N}\right\rangle + v_2^2\langle R_2^2\rangle\left\langle \frac{N-1}{N}\right\rangle\right].
\end{aligned} \tag{6}$$

It has been found in Au–Au collisions at RHIC [3] that $\langle v_4 \rangle \ll \langle v_2 \rangle$ while $\langle v_2 \rangle$ reaches the value of about 0.07 for rather peripheral collisions. Taking these numbers into account, we estimate the fluctuations of v_2 as

$$\mathsf{Var}(v_2) = \frac{1}{2\langle R_2\rangle^2\langle N\rangle} + \langle v_2\rangle^2\,\frac{\langle R_2^2\rangle - \langle R_2\rangle^2}{\langle R_2\rangle^2}\,, \tag{7}$$

where we have also assumed that $\langle N\rangle \gg 1$ and that the multiplicity fluctuations are small.

The second term in r.h.s of Eq. (7) depends on the number of particles M used to determine the impact parameter direction. Since M is usually rather large, we assume in accord with [3] that R_2 does not much deviate from unity. Then, as argued in [24, 25], we have

$$\langle R_2\rangle = \langle\cos(2(\psi_{\rm R}-\psi_{\rm E}))\rangle \approx 1-\langle(\psi_{\rm R}-\psi_{\rm E})^2\rangle \approx 1-\frac{a}{2\langle M\rangle}\,, \tag{8}$$

where the parameter a depends on the type of weights which are applied. An actual value of a is irrelevant for our considerations. Using the arguments which lead us to the result (8), one finds

$$\langle R_2^2\rangle = \langle\cos^2(2(\psi_{\rm R}-\psi_{\rm E}))\rangle \approx 1-2\langle(\psi_{\rm R}-\psi_{\rm E})^2\rangle \approx 1-\frac{a}{\langle M\rangle}\,.$$

Thus, $\langle R_2^2\rangle - \langle R_2\rangle^2 \sim \langle M\rangle^{-2}$. Since the number of particles used to determine the reaction plane is larger or at least similar to that which is involved in finding v_2, we conclude that the second term in r.h.s of Eq. (7) can be neglected. Thus, we finally estimate the statistical noise as

$$\delta v_2 = \frac{1}{\langle R_2\rangle\sqrt{2\langle N\rangle}}\,. \tag{9}$$

As an extra check, we have performed a Monte Carlo simulation of fake events with N particles generated according to $P(\phi)\sim(1{+}2v_2\cos(2(\phi{-}\psi_{\rm R})))$. The obtained variation of v_2 is, of course, in full agreement with the expression given above.

In the subsequent sections, we discuss physical phenomena which lead to the fluctuations of v_2 different than those described by Eq. (9), *i.e.* originating from true event-by-event fluctuations of flow.

4. Impact parameter and multiplicity fluctuations

As already noted, the observed elliptic flow is naturally described in the hydrodynamic model [7–12]. Therefore, are going to discuss here how large are the fluctuations of v_2 within the hydrodynamics. One should distinguish the fluctuations due to the varying impact parameter and those due to the thermodynamic fluctuations at fixed collision geometry. We start with the former ones.

As well known, $\langle v_2\rangle$ strongly depends on the collision impact parameter b. In the case of Au–Au collisions at RHIC at $\sqrt{s_{NN}} = 130$ GeV, the dependence has been parameterized [3] as

$$\langle v_2\rangle = a_1 b + a_2 b^2 + a_3^3 + a_4 b^4 + a_5 b^5 + a_6 b^6\,, \tag{10}$$

where b is measured in fm and $a_1 = -3.94\times10^{-4}$, $a_2 = 2.1\times10^{-3}$, $a_3 = -7.06\times10^{-5}$, $a_4 = -3.2\times10^{-5}$, $a_5 = 3.58\times10^{-6}$, and $a_6 = -1.17\times10^{-7}$. The parameterization assumes that $\langle v_2\rangle$ vanishes for $b = 0$ and for $b > b_{\rm max} = 14.7$ fm.

The v_2 fluctuations due to the varying impact parameter can be estimated by the formula

$$\delta v_2 = \frac{d\langle v_2\rangle}{d\,b}\,\delta b\;.$$

The impact parameter fluctuations are, in principle, measurable through the observation of multiplicity of participating nucleons N_p which in turn is controlled by the collision trigger conditions. Then, δb can be recalculated into δN_p. Adopting the linear dependence

$$N_p = 2Z\left(1 - \frac{b}{b_{\rm max}}\right)\,,$$

where $Z = 79$ is the number of protons in a gold nucleus, one gets the formula

$$\delta v_2 = \left(a_1 + 2\,a_2 b + 3\,a_3^2 + 4\,a_4 b^3 + 5\,a_5 b^4 + 6\,a_6 b^5\right)\,\frac{b_{\rm max}}{2Z}\,\delta N_p\,. \qquad (11)$$

We note that for $b\approx 10$ fm where the flow is maximal the v_2 fluctuations due to the impact parameter variation vanish because the derivative $d\langle v_2\rangle/db$ is then zero. For $b = 5$ fm where $\langle v_2\rangle\approx 0.03$, Eq. (11) gives $\delta v_2\approx 8\times10^{-4}\,\delta N_p$ which should be compared to the statistical noise (9). For $b = 5$ fm, $\delta N_p = 30$ and $\langle N\rangle = 500$, the magnitude of the v_2 fluctuations caused by the impact parameter variation is approximately equal to that of statistical noise. Thus, not only the statistical noise but also the centrality fluctuations must be subtracted from the measured v_2 fluctuations to observe the dynamical fluctuations of interest.

Let us now consider the fluctuations of v_2 due to the variation of thermodynamic parameters. The most important are presumably the multiplicity fluctuations. Here, we present some general formulas, considering an example of non-statistical fluctuations of multiplicity in the next section. (We follow a similar analysis [21] of the mean $p_{\rm T}$ event-by-event fluctuations due to radial flow fluctuations.) We assume here that the multiplicity of produced particles is not directly used to determine the collision centrality. In such a case the predicted v_2 fluctuations could be significantly reduced.

The fluctuations of v_2 can be estimated as

$$\delta v_2 = \frac{d\langle v_2\rangle}{d\langle N\rangle}\,\delta N\,,$$

which can be rewritten in the form

$$\frac{\delta v_2}{\langle v_2\rangle} = \frac{\delta N}{\langle N\rangle} P_h\,, \tag{12}$$

where the index P_h (the effective power) is

$$P_h \equiv \frac{d\ln\langle v_2\rangle}{d\ln\langle N\rangle} = \frac{\langle N\rangle}{\langle v_2\rangle}\,\frac{d\langle v_2\rangle}{d\langle N\rangle}\,,$$

with h denoting a hadron species used to determine v_2. We note that STAR collaboration has already reported the data on v_2 for π, K, K_s, p, Λ [2]. The first *stochastic* factor in Eq. (12) is the relative multiplicity fluctuation which drives the fluctuations of v_2, while the second *dynamical* factor P_h shows how a change in entropy transfers into v_2. P_h is obviously different for various secondary hadron species which can be used to test the idea further.

Assuming the poissonian character of multiplicity fluctuations, Eq. (12) can be rewritten as

$$\delta v_2 = \frac{\langle v_2\rangle}{\sqrt{N}}\,P_h\,. \tag{13}$$

The value of the index P_h can be estimated within the hydrodynamics. The calculations presented in [11] for EoS LH8 show that changing dN/dy from 200 to 400 leads to the increase of v_2 for pions from 0.028 to about 0.04 in a good agreement with NA49 and STAR data, see Fig. 24 in [11]. Reading a logarithmic slope from that figure, we find $P_\pi \approx 0.4$, which will be used below[3].

Comparing Eqs. (9) and (13), one finds that the ratio of the hydrodynamic fluctuations to the statistical noise is $\sqrt{2}\,\langle R_2\rangle\langle v_2\rangle\,P_h$ which for $\langle R_2\rangle = 0.6$, $\langle v_2\rangle = 0.07$ [3] and $P_h = 0.4$ is 0.02. The effect is indeed rather small. However, as discussed in Sec. 3, the magnitude of the statistical noise can be well controlled, and consequently, the hydrodynamic fluctuations seem to be detectable.

5. Fluctuations induced by cluster formation

In general, cluster production induce event-by-event fluctuations of local multiplicity or dN/dy larger than pure statistical noise, simply because the number of clusters is smaller than the number of particles. The estimates,

[3] P. Kolb was kind enough to provide the results for pions and nucleons from his version of hydrodynamics, and the indices turned out to be three times smaller, $P_\pi \approx 0.12, P_p \approx 0.13$. These number, however, are affected by some artifacts of the freeze-out approximations used, especially for low (SPS) energies. As a result, there is a non-monotonous dependence of v_2 *versus* dN/dy with a minimum, reducing the index. We note that a compilation of the AGS-SPS-RHIC data show a monotonous rise, as found in [11].

which we will give, follow ordinary statistical arguments, assuming that the cluster production happens in statistically independent way. This is justified by the observation that they all appear at different locations in the transverse plane, and also that they depend on specific hadronic and vacuum configurations at the moment of the collision. The original presentation of the idea that clusters should lead to observable event-by-event fluctuations of flows has been made by one of us at the 2001 CERN workshop [29].

Experimentally, existence of strong clustering of produced secondaries in p–p collisions has been known since 2-body correlation function was measured at ISR long time ago. To our knowledge, however, precise properties of such clusters have been never really explained or well quantified. As a relatively recent example of the cluster study, we refer to Fermilab experiment [33] where high multiplicity $\bar{p}$–p collisions have been analyzed with the conclusion that the average charged multiplicity per cluster is about 4. A single isolated cluster is produced in the so-called Pomeron–Pomeron process. Another example is provided by a recent analysis of the old UA8 data [34], showing production of clusters of 3–5 GeV mass. It was further shown that such clusters, with mass up to 5 GeV, decay isotropically in their rest frame[4]. Understanding of such clusters is very important to clarify a long-standing problem of "soft Pomeron" dynamics.

As a theoretical motivation, we suggest topological cluster formation in heavy-ion collisions. Inhomogeneous structure of the QCD vacuum, with relatively dilute gas of instantons, results in also dilute set of topological clusters arising at the collision early stage when the system is promptly excited, from virtual to real classical fields. For more discussion of these ideas, specific formulas and original references, see recent paper by one of us [21] and some subsequent works [27,28] where the cluster production and decay into gluons and quarks is discussed. For the purpose of this paper, it is enough to know that such a cluster, a QCD sphaleron, is like a heavy resonance which is expected to decay into about 3 gluons and 6 quarks and antiquarks. In p–p those should hadronize into specific final states, while in heavy ion case these partons are absorbed by the fireball and simply increase the local entropy density. This should cause event-by-event fluctuations of radial flow [21] as well as of elliptic flow we discuss in this note.

Let us now quantify particle number fluctuations caused by the cluster formation. As previously, N denotes particle multiplicity in a given $p_{\rm T}$ and y window and $N_{\rm cl}$ is the number of hadrons which can be attributed to the cluster decays. With N_0 we denote the hadron multiplicity from all sources

[4] Unfortunately, the UA2 detector, which was used to collect the data, was just a simple calorimeter, and we do not know anything about the structure of these clusters or even mean multiplicities. RHIC detectors and especially STAR can do a lot of clarification in p–p mode, provided proper triggers are implemented.

different than the topological clusters. Then, the relative fluctuation of the particle multiplicity can be written as

$$\frac{\delta N}{\langle N\rangle}=\frac{\langle N_0\rangle}{\langle N\rangle}\frac{\delta N_0}{\langle N_0\rangle}+\frac{\langle N_{\rm cl}\rangle}{\langle N\rangle}\frac{\delta N_{\rm cl}}{\langle N_{\rm cl}\rangle}\,.$$

Now, we assume that the fluctuations of both the cluster number $n_{\rm cl}$ and N_0 are poissonian, *i.e.*

$$\frac{\delta n_{\rm cl}}{\langle n_{\rm cl}\rangle}=\frac{1}{\sqrt{\langle n_{\rm cl}\rangle}}\,,\qquad\qquad \frac{\delta N_0}{\langle N_0\rangle}=\frac{1}{\sqrt{\langle N_0\rangle}}\,,$$

and that every cluster provides k hadrons ($N_{\rm cl}=k\,n_{\rm cl}$). Then, the relative fluctuation of the hadron number is

$$\frac{\delta N}{\langle N\rangle}=\frac{1}{\sqrt{\langle N\rangle}}\left(\sqrt{1-f}+\sqrt{fk}\right)\,, \tag{14}$$

where $f\equiv\langle N_{\rm cl}\rangle/\langle N\rangle$ denotes the fraction of final state hadrons produced due to the cluster formation.

Assuming that the cluster production alone is completely responsible for the p–p cross growth with the collision energy, one obtains the *upper bound*[5] on cluster production in Au–Au [35]. Adopting the scaling from p–p to Au–Au with the number of hard collisions, it was estimated in [35] that up to roughly 70 clusters per unit rapidity, $dn_{\rm cl}/dy\approx 70$, can be produced around $y=0$ in central Au–Au collisions at RHIC. This estimate, in turn, leads to an (upper limit) on cluster-related entropy of about *half* of the total value, and consequently of about half of the total multiplicity ($f\leq 0.5$). Keeping in mind that $dN/dy\approx 550$, one gets $k\approx 4$. Inserting these numbers into Eq. (14), we find that the formation and subsequent decays of clusters can (maximally) double the multiplicity fluctuations when compared to the poissonian fluctuations.

Using Eq. (12), one immediately translates the multiplicity fluctuations into the fluctuations of v_2. Since, the clusters can even double $\delta N/\langle N\rangle$, the same holds for $\delta v_2/\langle v_2\rangle$. Once we have concluded Sec. 4 that the hydrodynamic fluctuations seem to be measurable, we claim here that there is a chance to detect the fluctuation growth due to the cluster formation. However, it should stressed that the production and subsequent decay of the clusters can be observed directly studying the multiplicity fluctuations.

[5] This is an upper bound because nuclear modification of the structure functions is ignored. The realistic number is presumably factor 2 or so smaller.

6. Fluctuations induced by filamentation instability

When the momentum distribution of partons is strongly elongated in one direction, say along the z *i.e.* beam axis, the neutral system has a tendency to split into the filaments along z with the current flowing in the opposite directions in neighboring filaments. The reason is as follows: once the currents in the system occur, they generate the (chromo-)magnetic field, oscillating in the direction perpendicular to the beam axis, and the Lorentz force acts back on the charges which form the currents. It appears that the currents get focused and the current magnitude grows. This is the filamentation instability [30] which have been studied in the context of ultrarelativistic heavy-ion collisions in [18–20].

The breakdown of the azimuthal symmetry of the system due to the instability development gives a chance to observe it experimentally. It has been argued [18–20] that the instability growth leads to the energy transport along the wave vector which coincides with the Poynting vector of the generated chromodynamic field. Consequently, one expects significant a variation of the transverse energy as a function of the azimuthal angle.

Here we point another, presumably more realistic, possibility to detect the color filamentation. When the instability grows the trajectories of charge particles are focused in the centers of the filaments. Therefore, according to the Liouville theorem the distribution of the momentum perpendicular to the filaments, say along the x-axis, has to expand to conserve the phase space volume. The quantum mechanical counterpart of the argument relies on the uncertainty relation: once the particles are localized within the filaments their transverse momentum has to widen to the inverse filament thickness multiplied by $\hbar$. Below, we quantify this quantum mechanical reasoning.

It should be clearly stated that the collective motion caused by the instability development is not correlated with the reaction plane and it has nothing to do with the hydrodynamic flow. As such it would be called a"non-flow" effect. However, the filamentation generates a finite value of v_2 and it contributes to its fluctuations. So, let us consider the phenomenon in more detail.

Let the wave vector of the filamentation mode (k) be oriented along the x-axis. Then, the single particle wave function describing the transverse degrees of freedom is the form

$$\psi(x,y) \sim \exp\left[-\frac{x^2+y^2}{4R^2}\right]\cos(kx+\alpha)\,, \tag{15}$$

where R is the system transverse radius. To simplify further analysis we put the phase α equal to zero. Then, as we will see, odd harmonics of the azimuthal distribution vanish due to the mirror symmetry of the wave function (15). Performing the Fourier transform of (15), one gets the momentum distribution as

$$P(p_x,p_y) = \left|\widetilde{\psi}(p_x,p_y)\right|^2 \sim \mathrm{e}^{-2R^2\left(p_x^2+p_y^2\right)} \times \left[\mathrm{e}^{4R^2p_xk} + \mathrm{e}^{-4R^2p_xk} + 2\right]. \tag{16}$$

Since $p_x = p\cos\phi$ and $p_y = p\sin\phi$, the distribution (16) provides the azimuthal distribution of the form

$$P(\phi) = \int_0^\infty dp\, p\, P(p\cos\phi, p\sin\phi) \sim \int_0^\infty dp\, p\, \mathrm{e}^{-2R^2p^2}\left[\mathrm{e}^{4R^2pk\cos\phi} + \mathrm{e}^{-4R^2pk\cos\phi} + 2\right]. \tag{17}$$

Neglecting the Jacobian p in Eq. (17), the integral over momentum can be performed analytically and the result reads

$$P(\phi) \sim \left[\mathrm{e}^{2R^2k^2\cos^2\phi} + 1\right]. \tag{18}$$

The distribution (18) gets a particularly simple form when the filament thickness is much smaller than the system size. Then, $R\,k \gg 1$ and

$$P(\phi) = \frac{1}{2(\pi-1)}\left[1 - \delta\left(\phi - \frac{\pi}{2}\right) - \delta\left(\phi - \frac{3\pi}{2}\right)\right]. \tag{19}$$

Using the distribution (19), one finds

$$v_2 = \frac{1}{\pi - 1}. \tag{20}$$

The value of v_2 is rather large. A realistic value of v_2 is presumably significantly smaller because not all particles produced in a given event would participate in a collective motion caused the instability development. The collective motion should be also convoluted with the thermal one. Thus, the effect of filamentation must be diluted. An appearance of the instability in the system is not a deterministic but a random process. Therefore, we expect that there are collisions with and without the instability. Consequently, v_2 varies between zero and maximal value (20). Thus, we expect large fluctuations of v_2.

It should be also stressed here that there are specific distinctive features of the collective flow and the flow fluctuations due to the filamentation. First of all we note that in contrast to hydrodynamically generated v_2, the flow caused by the instability development does not vanish at zero impact parameter. Thus, one should look for filamentation in maximally central collisions. It is also expected that particles with small $p_{\rm T}$ are particularly sensitive the collective motion of interest.

7. Final remarks

The aim of this paper is to advocate usefulness of the flow fluctuation analysis in revealing of the early stage dynamics of heavy-ion collisions. We have shown that even rather exotic phenomena can be studied in this way. Those presented should be, of course, treated only as examples motivating the measurements, and simple order-of-magnitude estimates.

Since v_2 is experimentally determined on the event-by-event basis anyway, the proposed fluctuation measurement presumably does not require much additional efforts. However, an accuracy of the measurements should be improved. Studying the flow fluctuations as a function of particle multiplicity one can check whether δv_2 scales like $1/\sqrt{\langle N\rangle}$, which is a characteristic feature of statistical noise. If not we deal with nontrivial dynamical fluctuations. However, before such a conclusion is achieved one has to properly subtract the fluctuations due to the impact parameter variation. We note that these fluctuations can be constrained by the collision trigger condition and that the fluctuations due to impact parameter vanish around the maximal flow where $d\langle v_2\rangle/db = 0$.

To disentangle various fluctuation sources, the data should be analyzed in a broad interval of impact parameters and varying acceptance windows. Fortunately, the mechanisms of interest contribute differently to δv_2. In particular, the cluster effect is expected to be the largest for most peripheral events, while that of the filamentation for the most central ones. It would be also desirable to study elliptic flow fluctuations simultaneously with fluctuations of particle multiplicity and other collision characteristics.

At the end, we mention one more supplementary method [31] to study the azimuthal fluctuations which seems to be particularly useful to detect non-flow correlations. The method, which uses the so-called Φ-measure of fluctuations [32], does not require the reaction plane reconstruction and can be rather easily applied to experimental data. The measure is sensitive to various sources of dynamical correlations and the integrated information provided by Φ can be combined with that offered by the Fourier analysis [24, 25]. Since all Fourier harmonics contribute to Φ one can check whether the measured harmonics saturate the observed value of Φ.

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