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Deuteron formation mechanism

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Abstract. The microscopic mechanism of deuteron formation in high-energy collisions with nuclei is discussed. The formula for the deuteron emission cross section is derived in two ways: in the first the wavefunction description is used, while in the second we apply the density matrix formalism.

1. Introduction

According to the widely accepted picture, most deuterons produced in high-energy collisions with nuclei are formed by final-state interactions of nucleons with small relative momenta. In spite of an extensive literature on this problem (see, e.g., the review [1]) some aspects of the microscopic mechanism leading to deuteron formation have not been worked out. There is confusion related to the fact that the elastic interaction of two on-shell nucleons cannot lead to deuteron production because of conservation of four-momenta. In the pioneering paper by Butler and Pearson [2] it was assumed that the nucleons interact with an external optical potential of the nucleus, which makes deuteron formation possible. However, this solution does not agree with the experimental data [1] because of the momentum-dependent coefficient present in the formula for the deuteron production cross section (see below). In the phenomenological coalescence model [3] the nucleons are assumed to form a deuteron if their relative momentum is smaller than the critical value p_0 (coalescence radius), which is a free parameter of the model. The problem presented above is not taken into account at all.

The important point that has not been exposed in the literature is the following. The nucleons produced in high-energy collisions with nuclei are emitted from the finite space–time region Ω. So, their four-momenta are not precisely determined due to the uncertainty principle. Because the time interval of deuteron formation and the deuteron radius are close to the respective parameters of Ω, the uncertainties of energy ΔE and momentum Δp are comparable with the values of ΔE and Δp caused by the off-shell effect in a deuteron. Therefore the uncertainty of nucleon four-momenta allows deuteron formation, and no additional third body is needed†.

In fact the above picture of deuteron formation follows from the calculations by Sato and Yazaki [4], and from the formalism developed by Remler and coauthors [5], though it was not explicitly expressed there.

† An essential role of the uncertainty principle in bound-state formation has been pointed out to me by L L Nemenov.

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In § 2 the formula for the deuteron formation cross section is derived in terms of wavefunctions. In § 3 we apply the more appropriate density matrix formalism used earlier in references [4, 5]. In § 4 we discuss the results.

2. The wavefunction description

A fast projectile interacts with a nucleus and nucleons are emitted from the interaction region Ω. How many deuterons are there among the neutron–proton pairs? To answer this question one has to project the nucleon-pair wavefunction onto the deuteron wavefunction. The matrix element of interest is expressed as

$$\mathcal{M} = \lim_{t\to\infty} \int \mathrm{d}^3\boldsymbol{r}_1\, \mathrm{d}^3\boldsymbol{r}_2\, \psi_\mathrm{D}^*(\boldsymbol{r}_1, \boldsymbol{r}_2, t)\psi(\boldsymbol{r}_1, \boldsymbol{r}_2, t) \tag{1}$$

where $\psi(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$ and $\psi_\mathrm{D}(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$ are the time-dependent wavefunctions of the nucleon pair and the deuteron, respectively. Spin indices are suppressed. We take the limit $t\to\infty$ since we are interested in the deuteron fraction in the outgoing asymptotic nucleon-pair wavefunction. The problem of the time evolution of the matrix element from equation (1) has been studied by Remler and coauthors [5]†. In this paper we, in fact, omit this problem by means of the following simplified argument. We assume that the neutron–proton pair can be treated as an isolated system after the time $t=0$ when the pair leaves the interaction zone Ω. This is, of course, an idealisation of the real situation. This idealisation, however, justifies the wavefunction description used in this section. The neutron–proton potential, which is responsible for the time evolution of the pair wavefunction, is time independent. Therefore the pair wavefunction can be expanded in a series of energy eigenfunctions with time-independent coefficients. Because the deuteron wavefunction is an energy eigenfunction the scalar product in equation (1) is time independent for $t \geqslant 0$, and the limit $t\to\infty$ is trivial.

We calculate the matrix element for the time $t=0$ when the nucleons are emitted from the interaction region. Then the parameters of the pair wavefunction are related to the space–time characteristics of Ω. We use the non-relativistic approximation, where the centre-of-mass motion of two particles can be separated from the relative motion. The relativistic generalisation is discussed at the end of the section. The wavefunctions from equation (1) can be expressed in the form (the time variable is further suppressed)

$$\begin{aligned} &\psi_\mathrm{D}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{\sqrt{V}} \exp(\mathrm{i}\boldsymbol{P}\boldsymbol{R})\varphi_\mathrm{D}(\boldsymbol{r}) \\ &\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \Phi_{\boldsymbol{K}}(\boldsymbol{R})\varphi_{\boldsymbol{k}}(\boldsymbol{r}) \\ &\boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \end{aligned} \tag{2}$$

where $\varphi_\mathrm{D}(\boldsymbol{r})$ is the deuteron wavefunction of the relative motion and $\Phi_{\boldsymbol{K}}(\boldsymbol{R})$, $\varphi_{\boldsymbol{k}}(\boldsymbol{r})$ are the wave packets of the centre-of-mass motion and the relative motion of the nucleon pair. $\boldsymbol{K}$ and $\boldsymbol{k}$ denote the respective average momenta. V occurs because of the plane-wave normalisation of the centre-of-mass wavefunction of the deuteron. Substituting (2) in (1) one finds

$$\mathcal{M} = \frac{1}{\sqrt{V}}\, \tilde{\Phi}(\boldsymbol{P}-\boldsymbol{K}) \int \mathrm{d}^3\boldsymbol{r}\, \varphi_\mathrm{D}^*(\boldsymbol{r})\varphi(\boldsymbol{r}) \tag{3}$$

† Remler *et al* [5] have, in fact, studied the time evolution of the matrix element involving the density matrices rather than wavefunctions.

where

$$\tilde{\Phi}(\boldsymbol{P}-\boldsymbol{K})=\int \mathrm{d}^3\boldsymbol{R}\,\exp(\mathrm{i}\boldsymbol{P}\boldsymbol{R})\Phi_{\boldsymbol{K}}(\boldsymbol{R}).$$

The cross section for deuteron production is (see appendix)

$$\mathrm{d}\sigma=\left(\int \mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2|\mathcal{M}|^2\,\frac{\mathrm{d}\sigma^{\mathrm{np}}}{\mathrm{d}^3\boldsymbol{p}_1\mathrm{d}^3\boldsymbol{p}_2}\right)\frac{V\,\mathrm{d}^3\boldsymbol{P}}{(2\pi)^3} \tag{4}$$

where $\mathrm{d}\sigma^{\mathrm{np}}/\mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2$ is the inclusive cross section for the emission of a neutron and proton with momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, respectively. $V\,\mathrm{d}^3\boldsymbol{P}/(2\pi)^3$ is the deuteron phase-space element, and

$$\boldsymbol{K}=\boldsymbol{p}_1+\boldsymbol{p}_2 \qquad \boldsymbol{k}=\tfrac{1}{2}(\boldsymbol{p}_1-\boldsymbol{p}_2).$$

The cross section $\mathrm{d}\sigma^{\mathrm{np}}/\mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2$ describes neutron–proton pairs including deuterons. Because the n–p pair is treated as an isolated system the cross section does not include pairs from heavier fragments like ^{3}H, ^{3}He etc. If $\mathrm{d}\sigma^{\mathrm{np}}/\mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2$ depends weakly on $\boldsymbol{k}$ and $\boldsymbol{K}$ when compared with $|\mathcal{M}|^2$, the cross section taken at $\boldsymbol{K}=\boldsymbol{P}$ and $\boldsymbol{k}=0$ can be placed in front of the integral (4). The validity of this assumption is discussed briefly below. Keeping in mind the normalisation condition of the wavefunction $\tilde{\Phi}(\boldsymbol{P})$ one easily finds

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{P}}=A\,\frac{\mathrm{d}\sigma^{\mathrm{np}}}{\mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2} \tag{5}$$

with $\boldsymbol{p}_1=\boldsymbol{p}_2=\boldsymbol{P}/2$ and

$$A=\tfrac{3}{4}\int \mathrm{d}^3\boldsymbol{k}\left|\int \mathrm{d}^3\boldsymbol{r}\varphi_{\mathrm{D}}^*(\boldsymbol{r})\varphi(\boldsymbol{r})\right|^2. \tag{6}$$

The coefficient $\frac{3}{4}$ comes from averaging over the polarisations of the nucleon pair and summing over the deuteron polarisations. In other words, it has been assumed that the nucleons emitted from the interaction region Ω are unpolarised. The wavefunctions from equation (6) are spin independent.

The result of Butler and Pearson [2] is similar to that of equation (5) but the factor A depends on the deuteron momentum as $\boldsymbol{P}^{-2}$.

If one assumes that

$$\frac{\mathrm{d}\sigma^{\mathrm{np}}}{\mathrm{d}^3\boldsymbol{p}_1\,\mathrm{d}^3\boldsymbol{p}_2}=\frac{1}{\sigma^0}\frac{\mathrm{d}\sigma^{\mathrm{n}}}{\mathrm{d}^3\boldsymbol{p}_1}\frac{\mathrm{d}\sigma^{\mathrm{p}}}{\mathrm{d}^3\boldsymbol{p}_2}$$

where σ^0 is the total inelastic cross section, the formula (5) coincides with that of the coalescence model [1, 3].

Let us discuss the practical meaning of formulae (5) and (6). The essence of this result is the proportionality of the cross section for deuteron production to the cross section for emission of an n–p pair with respective momenta of the nucleons. It should be stressed that the value of the coefficient A is, in fact, unpredictable, since the wavefunction $\varphi_{\boldsymbol{k}}(\boldsymbol{r})$ is unknown. To determine this function the complete scattering problem should be solved, which is far beyond our capabilities. This function, in particular, describes primordial deuteron production, which is the result of dynamical correlations among neutrons and protons. For example, such correlations can occur when the neutron–proton pair interacts coherently with a projectile. If one neglects correlations the wavefunction $\varphi_{\boldsymbol{k}}(\boldsymbol{r})$ is determined by the size of the interaction region Ω. Then we can parametrise this function, for a time $t=0$ when the nucleons leave the interaction zone Ω, in the following gaussian form:

$$\varphi_k(\boldsymbol{r})=\frac{1}{\pi^{3/4}\mathscr{R}^{3/2}}\exp(-\mathrm{i}\boldsymbol{kr})\exp(-\boldsymbol{r}^2/2\mathscr{R}^2).$$

Choosing the analogous parametrisation for the deuteron wavefunction $\varphi_D(\boldsymbol{r})$ we get

$$A=6\pi^{3/2}/(\mathscr{R}^2+\mathscr{R}_D^2)^{3/2}$$

The parameters $\mathscr{R}_D$ and $\mathscr{R}$ should be associated with the size of the deuteron and the interaction region from which the nucleons are emitted.

In deriving formula (5) we have assumed that the cross section of the n–p pair emission depends weakly on the nucleon momenta when compared with the matrix element squared $|\mathscr{M}|^2$. The validity of this assumption can be easily verified for the n–p pair cross section expressed through the one-particle cross sections. Then for the gaussian form of all functions of interest one gets the condition $\mathscr{R}^{-2}\ll mT_0$, where m is the nucleon mass and T_0 is the slope parameter (often called the effective temperature) of the proton spectra. This parameter depends strongly on the emission angle and consequently it is easier to fulfil the above condition for the deuterons emitted forwards than those emitted backwards. Because the cross section $\mathrm{d}\sigma^{np}/\mathrm{d}^3\boldsymbol{p}_1\mathrm{d}^3\boldsymbol{p}_2$ contains the short-range (in momentum space) correlations, the assumption that this cross section depends weakly on the nucleon relative momentum is, in any case, incorrect. As briefly discussed in § 4 the formula (6) is modified if one takes into account the short-range neutron–proton correlations.

As has been argued, the uncertainty of nucleon four-momenta plays an essential role in deuteron formation. Therefore the nucleons have to be described by means of wave packets. Let us observe that if one uses a plane wave to describe the relative motion of the neutron and proton the fictitious normalisation volume V will remain in the final formula. It has a simple physical interpretation. The probability that the neutron and proton, which are not initially localised (because of the plane wave), form a deuteron is inversely proportional to the box volume V in which the particles are confined.

On the other hand, it is not important for the problem of deuteron formation whether the centre of mass of the nucleon pair is localised or not. One can check that the formulae (5) and (6) will remain unchanged if one uses the plane wave to describe the motion of the centre of mass of the nucleon pair.

The relativistic generalisation of formulae (5) and (6) is not trivial because of the well known difficulties in the proper relativistic treatment of bound states. We will now use simple arguments to modify (5) and (6) to include relativistic effects, but our reasoning is far from rigorous.

In general the factorisation of the two-particle wavefunction into parts describing the relative motion and the centre-of-mass motion is invalid in a relativistic approach, although, if the relative motion is non-relativistic in the centre-of-mass frame, this factorisation is approximately correct as long as we are not interested in the processes of high momentum transfer. For such processes the high momentum tails of the wavefunctions are of principal importance and the form of the deuteron wavefunction given in equation (2) is incorrect. Because the momentum transfer in the process of deuteron formation is of the order of the inverse deuteron radius, and this momentum transfer is small, one can use the deuteron wavefunction in the form (2). Then the only relativistic effect is the Lorentz contraction of the wavefunctions of relative motion of the nucleon pair and the deuteron. The simple calculation shows that the coefficient A should be multiplied by the Lorentz factor γ related to the deuteron motion. Using the Lorentz-invariant cross sections one can write down the relativistic analogue of the formula (5):

$$E\frac{d\sigma}{d^3\boldsymbol{P}}=\frac{2}{m}AE_1E_2\frac{d\sigma^{np}}{d^3\boldsymbol{p}_1d^3\boldsymbol{p}_2} \tag{7}$$

with $E_1=E_2=E/2$ and $\boldsymbol{p}_1=\boldsymbol{p}_2=\boldsymbol{P}/2$, which is the well known result of the coalescence model [1].

3. The density matrix formalism

The neutron–proton pair is part of a very complex system created in a high-energy collision with a nucleus. Therefore the density matrix formalism used in the papers by Sato and Yazaki [4] and by Remler and coauthors [5] is more appropriate than the wavefunction description used in the previous section.

The probability of finding a deuteron among the n–p pairs is

$$W=\lim_{t\to\infty}\int d^3\boldsymbol{r}_1d^3\boldsymbol{r}_1'd^3\boldsymbol{r}_2d^3\boldsymbol{r}_2'\rho(\boldsymbol{r}_1,\boldsymbol{r}_1';\boldsymbol{r}_2,\boldsymbol{r}_2';t)\psi_D^*(\boldsymbol{r}_1',\boldsymbol{r}_2',t)\psi_D(\boldsymbol{r}_1,\boldsymbol{r}_2,t) \tag{8}$$

where $\rho(\boldsymbol{r}_1,\boldsymbol{r}_1';\boldsymbol{r}_2,\boldsymbol{r}_2';t)$ is the two-particle density matrix. As in the previous section the problem of the time dependence of the matrix element from equation (8), studied in [5], is simplified here by the assumption that after the time $t=0$ of the emission of nucleons from the interaction zone Ω, the density matrix describes the free expansion of nucleons, deuterons, tritons, etc, not interacting with one another. Then the matrix element present in (8) is time independent, and we calculate it for $t=0$. The time dependence will now be suppressed. Expressing the deuteron wavefunction in the form (2) and using the centre-of-mass variables $\boldsymbol{R}$ and $\boldsymbol{r}$, the formula (8) can be rewritten as

$$W=\frac{1}{V}\int d^3\boldsymbol{r}d^3\boldsymbol{r}'d^3\boldsymbol{R}d^3\boldsymbol{R}'\rho(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')\exp[i\boldsymbol{P}(\boldsymbol{R}-\boldsymbol{R}')]\varphi_D^*(\boldsymbol{r}')\varphi_D(\boldsymbol{r}). \tag{9}$$

The Wigner transformation of the density matrix provides the quantum-mechanical analogue of the distribution function, i.e. the probability density of finding an np pair with centre-of-mass momentum $\boldsymbol{K}$ and relative momentum $\boldsymbol{k}$, and centre-of-mass position $\boldsymbol{X}$ and relative distance $\boldsymbol{x}$. This probability density is expressed as

$$P(\boldsymbol{x},\boldsymbol{X},\boldsymbol{k},\boldsymbol{K})=\int d^3\boldsymbol{u}d^3\boldsymbol{U}\exp(i\boldsymbol{KU}+i\boldsymbol{ku})\rho(\boldsymbol{x}-\tfrac{1}{2}\boldsymbol{u},\boldsymbol{x}+\tfrac{1}{2}\boldsymbol{u};\boldsymbol{X}-\tfrac{1}{2}\boldsymbol{U},\boldsymbol{X}+\tfrac{1}{2}\boldsymbol{U}). \tag{10}$$

It is normalised as

$$\int d^3\boldsymbol{x}d^3\boldsymbol{X}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{d^3\boldsymbol{K}}{(2\pi)^3}\mathscr{P}(\boldsymbol{x},\boldsymbol{X},\boldsymbol{k},\boldsymbol{K})=1.$$

Assuming that $\mathscr{P}(\boldsymbol{x},\boldsymbol{X},\boldsymbol{k},\boldsymbol{K})$ can be expressed in the form

$$\mathscr{P}(\boldsymbol{x},\boldsymbol{X},\boldsymbol{k},\boldsymbol{K})=D(\boldsymbol{x},\boldsymbol{X})G(\boldsymbol{k},\boldsymbol{K})$$

and inverting the formula (10) one gets

$$\rho(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')=D\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2},\frac{\boldsymbol{R}+\boldsymbol{r}'}{2}\right)$$
$$\times\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{K}}{(2\pi)^3}\exp[-\mathrm{i}\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')-\mathrm{i}\boldsymbol{K}(\boldsymbol{R}-\boldsymbol{R}')]G(\boldsymbol{k},\boldsymbol{K}). \tag{11}$$

Substituting the expression (11) into (9) one finds

$$W=\frac{1}{V}\int\mathrm{d}^3\boldsymbol{r}\mathrm{d}^3\boldsymbol{r}'\mathrm{d}^3\boldsymbol{R}\mathrm{d}^3\boldsymbol{R}'\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{K}}{(2\pi)^3}\exp[\mathrm{i}(\boldsymbol{P}-\boldsymbol{K})(\boldsymbol{R}-\boldsymbol{R}')-\mathrm{i}\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')]$$
$$\times D\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2},\frac{\boldsymbol{R}+\boldsymbol{R}'}{2}\right)G(\boldsymbol{k},\boldsymbol{K})\varphi_{\mathrm{D}}^{*}(\boldsymbol{r}')\varphi_{\mathrm{D}}(\boldsymbol{r}). \tag{12}$$

Formula (12) is significantly simplified if one assumes that the function $G(\boldsymbol{k},\boldsymbol{K})$ depends weakly on $\boldsymbol{k}$ and $\boldsymbol{K}$ when compared with the Fourier transform of the product $\varphi_{\mathrm{D}}D$. Then the function $G(\boldsymbol{k},\boldsymbol{K})$ taken at $\boldsymbol{k}=0$ and $\boldsymbol{K}=\boldsymbol{P}$ can be placed in front of the integral (12). Now we can perform the integrations over $\boldsymbol{k}$ and $\boldsymbol{K}$ which provide the delta functions $(2\pi)^3\delta^{(3)}(\boldsymbol{r}'-\boldsymbol{r})$ and $(2\pi)^3\delta^{(3)}(\boldsymbol{R}'-\boldsymbol{R})$ respectively. After the trivial integrations over $\boldsymbol{r}'$ and $\boldsymbol{R}'$ we get

$$W=\frac{1}{V}G(\boldsymbol{k}=0,\boldsymbol{K}=\boldsymbol{P})\int\mathrm{d}^3\boldsymbol{r}\mathrm{d}^3\boldsymbol{R}|\varphi_{\mathrm{D}}(\boldsymbol{r})|^2D(\boldsymbol{r},\boldsymbol{R}).$$

Since $\int\mathrm{d}^3\boldsymbol{R}D(\boldsymbol{r},\boldsymbol{R})\overset{\mathrm{def}}{=}D(\boldsymbol{r})$ is the probability density of the relative distance of the neutron and proton of the pair, we finally find

$$W=\frac{1}{V}G(\boldsymbol{k}=0,\boldsymbol{K}=\boldsymbol{P})\int\mathrm{d}^3\boldsymbol{r}|\varphi_{\mathrm{D}}(\boldsymbol{r})|^2D(\boldsymbol{r}). \tag{13}$$

Expressing $G(\boldsymbol{k},\boldsymbol{K})$ through the cross section of nucleon pair emission one gets from equation (13) the formula for the deutron production cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{P}}=C\frac{\mathrm{d}\sigma^{\mathrm{np}}}{\mathrm{d}^3(\boldsymbol{P}/2)\mathrm{d}^3(\boldsymbol{P}/2)} \tag{14}$$

with

$$C=\tfrac{3}{4}(2\pi)^3\int\mathrm{d}^3\boldsymbol{r}|\varphi_{\mathrm{D}}(\boldsymbol{r})|^2D(\boldsymbol{r}). \tag{15}$$

As previously, the coefficient $\frac{3}{4}$ has been introduced because of averaging over nucleon-pair polarisations and summing over the final spin states of the deuteron. In the respective formula from reference [4] there is the additional coefficient 2^3, which arises from the momentum per nucleon used in the paper [4]. The np-pair cross section from equation (14) differs from that used in equation (4). The cross section here describes all neutron–proton pairs including deuterons and heavier fragments. On the other hand the cross section given by the formula (14) describes only deuterons and not the n–p pairs in heavier fragments.

Let us discuss the approximation used that $G(\boldsymbol{k},\boldsymbol{K})$ depends weakly on $\boldsymbol{k}$ and $\boldsymbol{K}$. Treating the function $G(\boldsymbol{k},\boldsymbol{K})$ as a constant in equation (11) one finds

$$\rho(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')=D(\boldsymbol{r},\boldsymbol{R})G(\boldsymbol{k},\boldsymbol{K})\delta^{(3)}(\boldsymbol{r}'-\boldsymbol{r})\delta^{(3)}(\boldsymbol{R}'-\boldsymbol{R}).$$

So, the approximation that leads to equations (14) and (15) is equivalent to the statement that the density matrix of np pairs is diagonal.

The physical interpretation of equation (15) is transparent. The deuteron formation cross section is proportional to the overlap of the deuteron wavefunction modulus squared and the probability distribution of the distance between the neutron and proton in Ω. One should remember that the distribution $D(\boldsymbol{r})$ contains, apart from information on the size of the emitting source, the dynamical correlations among the neutrons and protons. Therefore, by parametrising the function $D(\boldsymbol{r})$ in, say, gaussian form and comparing the formulae (14) and (15) with the experimental data, one finds the 'effective' radius of the interaction zone.

Using the arguments given in the previous section we get the relativistic generalisation of equation (14), which is identical to equation (8) but with C instead of A.

4. Discussion and conclusions

It is interesting to compare the results found on the basis of the wavefunction description and those from § 3.

Let us rewrite the integral (6) in the form

$$A=\tfrac{3}{4}\int d^3\boldsymbol{k}\,d^3\boldsymbol{r}\,d^3\boldsymbol{r}'\varphi_{\boldsymbol{k}}(\boldsymbol{r})\varphi_{\boldsymbol{k}}^*(\boldsymbol{r}')\varphi_{\mathrm{D}}^*(\boldsymbol{r})\varphi_{\mathrm{D}}(\boldsymbol{r}'). \tag{16}$$

Now we can try to perform the integration with respect to $\boldsymbol{k}$. Because the wave packets $\varphi_{\boldsymbol{k}}(\boldsymbol{r})$ are not the momentum eigenfunctions this integral does not equal $[(2\pi)^3/V]\delta^{(3)}(\boldsymbol{r}'-\boldsymbol{r})$. However, it is reasonable to assume that

$$\int d^3\boldsymbol{k}\varphi_{\boldsymbol{k}}(\boldsymbol{r})\varphi_{\boldsymbol{k}}^*(\boldsymbol{r}')=(2\pi)^3\Delta(\boldsymbol{r})\delta^{(3)}(\boldsymbol{r}'-\boldsymbol{r}) \tag{17}$$

with

$$\int d^3\boldsymbol{r}\Delta(\boldsymbol{r})=1.$$

This corresponds to the statement that the density matrix is diagonal. If $\Delta(\boldsymbol{r})=D(\boldsymbol{r})$ we recover equation (15) by substituting (17) in (16).

In the case of deuteron formation the bound-state radius is close to that of the interaction region Ω from which nucleons are emitted. Let us now briefly consider the process of bound-state formation when the radius of the bound state is much smaller than that of Ω, and also the opposite case. As explained above, in the first case bound-state formation due to final-state interaction is strongly suppressed because the off-shell effect greatly exceeds the uncertainty of the particle four-momenta. On the other hand our results (5), (6) and (14), (15) are invalid in this case because the momentum dependence of the particle production cross section cannot be counted as weak, and consequently this cross section cannot be placed in front of the integral (4) or (12).

The formation of pionium (the Coulomb bound state of π^+ and π^-) in high-energy collisions provides an example of the case when the radius of the bound state is much greater than that of Ω. Then the formulae (6) and (15) (without spin factors) give

$$A=C=(2\pi)^3|\varphi_{\mathrm{B}}(\boldsymbol{r}=0)|^2.$$

It is seen that in this case the coefficients A and C depend only on the bound-state wavefunction $\varphi_B(\boldsymbol{r})$. The same result has recently been found by Nemenov [6] on the basis of considerations somewhat different to ours.

The cross section $d\sigma^{np}/d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2$ is not available experimentally, and it is usually expressed through the one-particle nucleon cross sections. Then this cross section is weakly dependent on the relative nucleon momentum, as has been assumed in our considerations. However, $d\sigma^{np}/d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2$ contains the short-range n–p correlations and it should be written in the form

$$\frac{d\sigma^{np}}{d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2}=\frac{1}{\sigma_0}\frac{d\sigma^{n}}{d^3\boldsymbol{p}_1}\frac{d\sigma^{p}}{d^3\boldsymbol{p}_2}(1+\tilde{z}(\boldsymbol{p}_1-\boldsymbol{p}_2)).$$

The momentum dependence of the correlation function $\tilde{z}(\boldsymbol{p})$ is not weaker than that of the deuteron wavefunction, and equations (6) and (15) should be modified as

$$A\to A+\tfrac{3}{4}\int d^3\boldsymbol{k}\,\tilde{z}(\boldsymbol{k})\left|\int d^3\boldsymbol{r}\varphi_D^*(\boldsymbol{r})\varphi_{\boldsymbol{k}}(\boldsymbol{r})\right|^2$$

$$C\to C+\tfrac{3}{4}(2\pi)^3\int d^3\boldsymbol{r}d^3\boldsymbol{r}'z(\boldsymbol{r}-\boldsymbol{r}')\varphi_D^*(\boldsymbol{r}')\varphi_D(\boldsymbol{r})D\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2}\right)$$

where $\tilde{z}(\boldsymbol{k})$ is the Fourier transform of the function $z(\boldsymbol{r})$.

As noted by Sato and Yazaki [4] the formulae (14) and (15) can be used to determine the size of the interaction region from which the nucleons are emitted. However, one has to calculate the correlation function $\tilde{z}(\boldsymbol{p})$ to make this procedure fully quantitative. This problem will be discussed in our next paper where, in particular, we apply the formulae derived here to antideuterons produced in proton–proton collisions.

We conclude the considerations presented in this paper as follows. The uncertainty in the four-momenta of the neutron and proton emitted from the interaction region Ω allows deuteron formation without an additional third body. The formula for the deuteron emission cross section can be derived in the wavefunction language or by means of the density matrix formalism. In the first case it is important to use wave packets instead of plane waves to describe the nucleon motion. An interesting feature of the density matrix calculations is the fact the coalescence formula for the deuteron emission cross section relates to the diagonal density matrix of neutron–proton pairs.

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Appendix

We present here the derivation of equation (4).

The amplitude for deuteron production (in Dirac notation) is

$$\mathscr{F}_X=\langle D, X|S|i\rangle$$

where $\langle \mathrm{i}|$ is the initial state and $\langle \mathrm{D}, \mathrm{X}|$ is a state with a deuteron. We introduce the complete set of states $\langle \mathrm{np}, \mathrm{Y}|$, where np denotes a neutron–proton pair. Then

$$\mathscr{F}_{\mathrm{X}} = \sum_{\mathrm{np}, \mathrm{Y}} \langle \mathrm{D}, \mathrm{X}|\mathrm{np}, \mathrm{Y}\rangle \langle \mathrm{np}, \mathrm{Y}|\mathrm{S}|\mathrm{i}\rangle. \tag{A.1}$$

Now we perform the decomposition

$$\langle \mathrm{D}, \mathrm{X}| = \langle \mathrm{D}|\,\langle \mathrm{X}| \qquad \langle \mathrm{np}, \mathrm{Y}| = \langle \mathrm{np}|\,\langle \mathrm{Y}|, \tag{A.2}$$

which demands the independence of the np pair from other particles in the state. Such independence is approximately realised in multiparticle states such as those in high-energy collisions with nuclei. Substituting (A.2) into (A.1) and performing the summation over Y quantum numbers we find

$$\mathscr{F}_{\mathrm{X}} = \sum_{\mathrm{np}} \langle \mathrm{D}|\mathrm{np}\rangle \mathscr{A}^{\mathrm{X}}_{\mathrm{np}}$$

where

$$\mathscr{A}^{\mathrm{X}}_{\mathrm{np}} = \langle \mathrm{np}, \mathrm{X}|\mathrm{S}|\mathrm{i}\rangle.$$

If one assumes that the amplitudes of different np characteristics do not interfere with one another (which is the case for multiparticle final states) we get

$$|\mathscr{F}_{\mathrm{X}}|^2 = \sum_{\mathrm{np}} |\langle \mathrm{D}|\mathrm{np}\rangle|^2 |\mathscr{A}^{\mathrm{X}}_{\mathrm{np}}|^2.$$

One easily finds the inclusive cross section for deuteron production, which is proportional to $\Sigma_{\mathrm{X}}|\mathscr{F}_{\mathrm{X}}|^2$ since $\Sigma_{\mathrm{X}}|\mathscr{A}^{\mathrm{X}}_{\mathrm{np}}|^2$ can be expressed through $\mathrm{d}\sigma^{\mathrm{np}}/\mathrm{d}^3\boldsymbol{p}_1\mathrm{d}^3\boldsymbol{p}_2$.

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