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The Ideal Gases of Tachyons (*).

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Summary. — The formalism of statistical mechanics of particles slower than light has been considered from the point of view of the application of this formalism for the description of tachyons. Properties of ideal gases of tachyons have been discussed in detail. After finding general formulae for quantum, Bose and Fermi gases the classical limit has been considered. It has been shown that Bose-Einstein condensation occurs. The tachyon gas of bosons violates the third principle of thermodynamics. Degenerated Fermi gas has been considered and in this case the entropy vanishes at zero temperature. Difficulties of formulating covariant statistical mechanics have been discussed.

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1. – Introduction.

In this paper we consider properties of the ideal gases of quantum and classical tachyons—particles faster than light [1-3].

There are two aspects of studying these hypothetical particles. In order

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(1) For a review of tachyons see E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 (1974), see also [2,3].

(2) *Proceedings of Session « Tachyons, Monopoles and Related Topics », Erice, 1976*, edited by E. RECAMI (North-Holland Publishing Company, Amsterdam, 1978).

(3) P. CALDIROLA and E. RECAMI: *Causality and Tachyons in Relativity*, in *Italian Studies in the Philosophy and Science*, edited by M. L. DALLA CHIARA (D. Reidel Publishing Company, Boston, Mass., 1980), p. 249-298.

to propose the experiment that could give an answer to the question on the existence of tachyons, theoretical investigations are needed. It should be stressed that a theoretical support of the majority of experiments (4) which have been carried out is very weak (1). Thus their negative result cannot be counted as conclusive. The problem is that there is no satisfactory model of interacting tachyons, thus there are not any indications of how to construct a detector of tachyons (*). On the other hand, studying tachyons is interesting from a pure theoretical point of view. There are many problems such as, *e.g.*, the applicability (level of universality) of the scheme of the theory of particles slower than light—bradyons for the description of tachyons. In the case of statistical mechanics being considered there are some concrete questions. For example, is the third principle of thermodynamics satisfied by tachyon gases? Does the Bose-Einstein gas of tachyons condensate at low temperature? Or a more general question: is it possible to formulate covariant statistical mechanics of tachyons by the simple extension formalism proposed for bradyons (5)?

In this paper we give answers to the above questions.

2. – Tachyon, what is it?

The free tachyon is described by the spacelike four-momentum, p^μ, lying on the single-sheeted hyperboloid

$$p^\mu p_\mu = -m^2 ,$$

where m is the real tachyon rest mass. From the above statement it follows that the energy of tachyon E_p is connected with momentum $\boldsymbol{p}$ by the formula

$$E_p = \sqrt{\boldsymbol{p}^2 - m^2} .$$

Because $|\boldsymbol{p}| \equiv p \geqslant m$, the energy is always real. We use the units where $c = h = k = 1$, $\hbar = 1/2\pi$. Our metric is $(+, -, -, -)$. The sign of a zero component of a spacelike vector can be changed by Lorentz transformation. Thus the tachyon, which has a positive energy and travels forward in time in one frame, can, having a negative energy, travel backward in time in another moving frame. This makes it possible to construct a causal loop (1-3). Both paradoxial facts, negative energy and motion backward in time can be excluded from the

(4) For extensive list of experiments see a bibliography by V. F. PEREPELITSA: Preprints, ITEP-100, ITEP-165, (Moscow, 1980).

(*) For some considerations concerning the tachyon detector see A. O. BARUT, G. D. MACCARONE and E. RECAMI: *Nuovo Cimento A*, **71**, 509 (1982).

(5) B. TOUSCHEK: *Nuovo Cimento B*, **58**, 295 (1968).

theory by applying the reinterpretation principle (6), according to which a negative-energy tachyon travelling backward in time has to be regarded as a positive-energy antitachyon travelling forward in time. The above postulate solves causal paradoxes (1-3).

In some cases it is useful to introduce the inverse velocity of a tachyon, $\eta = v^{-1}$. With this variable the energy and momentum of a tachyon are expressed as follows (7):

$$E = \frac{m\eta}{\sqrt{1-\eta^2}}, \qquad P = \frac{m}{\sqrt{1-\eta^2}}.$$

At low energy, $\eta \ll 1$ (« nonrelativistic limit »), we get

$$E = m(\eta + O(\eta^3)), \qquad p = m(1 + \tfrac{1}{2}\eta^2 + O(\eta^4)). \tag{1}$$

3. – Some basic notions of statistical mechanics of tachyons.

In our considerations we follow the well-known Huang's book (8). We start from a microcanonical ensemble and the postulate of « *a priori* equal probabilities » of different microscopic states which lead to the same macroscopic one. We do not see any reason for changing this fundamental postulate in the case of tachyons. Without any differences, we define the entropy of a system and the temperature of a subsystem through the entropy. In this way one finds a canonical ensemble and a partition function of N particles Q_N. For the classical system we have

$$Q_N(V, T) = \int \frac{\mathrm{d}^3p_1 \dots \mathrm{d}^3p_N \mathrm{d}^3q_1 \dots \mathrm{d}^3q_N}{N!} \exp[-\beta H(\boldsymbol{p}_1, \dots, \boldsymbol{p}_N, \boldsymbol{q}_1, \dots, \boldsymbol{q}_N)]$$

with V the volume of the system, $\beta^{-1} = T$ the temperature and $H(\boldsymbol{p}_1 \dots \boldsymbol{p}_N, \boldsymbol{q}_1 \dots \boldsymbol{q}_N)$ the Hamiltonian of N particles depending on their momenta, $\boldsymbol{p}_i$, and positions, $\boldsymbol{q}_i$: The integration is performed over momenta greater than the rest masses of tachyons.

For quantum systems one finds

$$Q_N(V, T) = \sum_n \exp[-\beta\varepsilon_n],$$

where ε_n is the energy of the system in state n.

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(6) O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. J. Phys.*, **30**, 718 (1982).

(7) A. F. ANTIPPA: *Nuovo Cimento A*, **10**, 389 (1972).

(8) K. HUANG: *Statistical Mechanics* (John Wiley and Sons, Inc., New York, N. Y., 1963).

It is possible to find the partition function of the ideal gas of classical tachyons (9). In this paper, however, the classical gas is considered as a limiting case of quantum gases. Thus, we introduce a grand canonical partition function, Ξ:

$$\Xi(z, V, T) = \sum_{N=1}^{\infty} z^N Q_N(V, T)$$

The fugacity z, which is related to the chemical potential μ by the equality

$$z = \exp[\beta\mu],$$

can be eliminated due to the equation

$$N = z\frac{\partial}{\partial z}\ln\Xi, \tag{2}$$

where N has to be treated as an average number of tachyons in the system. The connection with thermodynamics is expressed through the formulae

$$pV = T\ln\Xi, \qquad U = -\frac{\partial}{\partial\beta}\ln\Xi, \tag{3}$$

where p is the pressure and U the internal energy of the gas.

4. – The ideal gases. General formulae.

Assuming that the number of particles in the same state is 0 or 1 for fermion tachyons and 0, 1, 2 ... for boson tachyons, one finds (8) a grand partition function for ideal Fermi-Dirac (upper sign) and Bose-Einstein (lower sign) gases:

$$\Xi(z, V, T) = \exp\left[\pm g\sum_{p}\ln\left[1 \pm Z\exp\left[-\beta E_p\right]\right]\right],$$

g is the number of internal degrees of freedom of a particle. Using formulae (2) and (3), we get

$$\begin{cases} \dfrac{pV}{T} = \pm g\sum_{p}\ln\left[1 \pm z\exp\left[-\beta E_p\right]\right], \\ N = g\sum_{p}\left[z^{-1}\exp\left[\beta E_p\right] \pm 1\right]^{-1}, \\ U = g\sum_{p}E_p\left[z^{-1}\exp\left[\beta E_p\right] \pm 1\right]^{-1}. \end{cases} \tag{4}$$

(9) S. MRÓWCZYŃSKI: Preprint, E2-83-476 (Dubna, 1983); *Lett. Nuovo Cimento*, **38**, 247 (1983).

The above series are convergent and N is positive when $0 \leqslant z \leqslant \infty$ for fermions and $0 \leqslant z \leqslant 1$ for bosons.

If $V \to \infty$, sums (4) can be changed into integrals, namely

$$\sum_{p} \to V\int d^3p\,.$$

For bosons we assume that $z < 1$. The case $z = 1$, Bose-Einstein condensation, is discussed in sect. **6**.

Thus

$$(5)\qquad \begin{cases} \dfrac{P}{T} = \pm\, g\displaystyle\int d^3p \ln\left[1 + z\exp\left[-\beta E_p\right]\right], \\ \dfrac{N}{V} = \quad g\displaystyle\int d^3p\left(z^{-1}\exp\left[\beta E_p\right] \pm 1\right)^{-1}, \\ \dfrac{U}{V} = \quad g\displaystyle\int d^3p\, E_p\left(z^{-1}\exp\left[\beta E_p\right] \pm 1\right)^{-1}. \end{cases}$$

Expanding the functions under integrals (5) in the series of powers of $z\exp[\beta E_p]$, one finds

$$(6)\qquad \begin{cases} \dfrac{P}{T} = 4\pi g m^2 T \displaystyle\sum_{n=1}^{\infty} \frac{(\mp 1)^{n+1}}{n^2}\, S_{02}(n\beta m) z^n\,, \\ \dfrac{N}{V} = 4\pi g m^2 T \displaystyle\sum_{n=1}^{\infty} \frac{(\mp 1)^{n+1}}{n}\, S_{02}(n\beta m) z^n\,, \\ \dfrac{U}{V} = 4\pi g m^2 T^2 \displaystyle\sum_{n=1}^{\infty} \frac{(\mp 1)^{n+1}}{n}\left[S_{02}(n\beta m) - n\beta m\, S'_{02}(n\beta m)\right] z^n\,. \end{cases}$$

We have used the equality ([10])

$$\int_1^{\infty} dy\, y^2 \exp\left[-x\sqrt{y^2-1}\right] = \int_0^{\infty} dt \cosh^2 t \exp\left[-x \sinh t\right] = \frac{1}{x}\, S_{02}(x)\,.$$

$S_{02}(x)$ is the so-called Lommel function ([10]). Differentiation is denoted by prime. $S'_{02}(x)$ can be expressed through another Lommel function according to the formula ([10])

$$S'_{\mu\nu}(x) = \frac{\nu}{x}\, S_{\mu\nu}(x) + (\mu - \nu - 1) S_{\mu-1,\nu+1}(x)\,.$$

([10]) I. S. GRADSHTEYN and I. M. RYZHIK: *Tables of Integrals, Series and Products*, edited by A. JAFFREY (Academic Press, New York, N. Y., 1965).

In order not to complicate the form of our results, we do not use, however, this formula.

For a gas of relativistic bradyons formulae (6) are the same, although the Lommel functions $S_{02}(x)$ have to be changed into the Macdonald functions $K_2(x)$ [11,12].

5. – Classical limit.

Considering eqs. (5), one observes that the fugacity z strongly decreases when the density, N/V, is going to zero and/or the temperature increases to infinity. Thus, for $N/V \to 0$ and/or $\beta \to 0$, one can approximate series (6) by the first terms

$$\frac{P}{V} = 4\pi g m^2 T S_{02}(\beta m) z\,,$$

$$\frac{N}{V} = 4\pi g m^2 T S_{02}(\beta m) z\,,$$

$$\frac{U}{V} = 4\pi g m^2 T^2 [S_{02}(\beta m) - \beta m S'_{02}(\beta m)] z\,.$$

Eliminating z from the above formulae, we get the equation of state

$$p \cdot V = N \cdot T$$

and the internal energy

$$U = NmT\left(1 - \beta m \frac{S'_{02}(\beta m)}{S_{02}(\beta m)}\right).$$

The specific heat, defined as follows:

$$c_v = \frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_v,$$

is expressed by the formula

$$c_v = 1 + (\beta m)^2 \frac{S''_{02}(\beta m)}{S_{02}(\beta m)} - \left[\beta m \frac{S'_{02}(\beta m)}{S_{02}(\beta m)}\right]^2.$$

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(11) S. BELENKIJ and L. D. LANDAU: *Usp. Fiz. Nauk*, **56**, 309 (1955); also in *Collected Papers of L. D. Landay*, edited by D. TER HAAR (Gordon and Breach, New York, N.Y., 1965).

(12) J. GOSSET, J. I. KAPUSTA and G. D. WESTFALL: *Phys. Rev. C*, **18**, 844 (1978).

In two extreme cases $x \gg 1$ and $1/x \gg 1$, $S_{02}(x)$ can be approximated as

$$S_{02}(x) = \begin{cases} \dfrac{2}{x^2} + O(\ln x) & \text{for } \dfrac{1}{x} \gg 1\,, \\ \dfrac{1}{x} + O\left(\dfrac{1}{x^2}\right) & \text{for } x \gg 1\,. \end{cases} \tag{7}$$

Let us compare the above expression with the analogous approximations for the Macdonald function:

$$K_2(x) = \begin{cases} \dfrac{2}{x^2} + O(x^2 \ln x) & \text{for } \dfrac{1}{x} \gg 1\,, \\ \sqrt{\dfrac{\pi}{2x}} \exp[-x]\left[1 + O\left(\dfrac{1}{x}\right)\right] & \text{for } x \gg 1\,. \end{cases}$$

Because $K_2(x)$ and $S_{02}(x)$ are approximate by the same function for $x \to 0$, the gases of bradyons and tachyons are very similar at a high-temperature limit. Namely, for both gases, we have

$$U \simeq 3 \cdot N \cdot T\,, \qquad c_v \simeq 3\,.$$

(The equation of state is the same at any temperature.) The above result is obvious if one recalls that a high-energy tachyon as well as a bradyon behaves as a luxon—massless particle.

More details on the ideal gas of classical tachyons can be found in our previous paper ([13]).

6. – Bose-Einstein condensation.

For bosons, sums (4) cannot be changed into integrals (5) when $z \simeq I$, because the terms related to $E_p = 0$ can give finite contributions to the series. Thus we write

$$N = gV\int d^3p(z^{-1} \exp[\beta E_p] - 1)^{-1} + \langle n_0 \rangle\,,$$

where

$$\langle n_0 \rangle = \frac{z}{1-z}\,.$$

The finite part of particles is in the lowest-energy state, Bose-Einstein con-

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([13]) S. CHANDRASEKHAR: *An Introduction to the Study of Stellar Structure* (Dover, New York, N.Y., 1939), Chapter X.

densation occurs when

$$\langle n_0 \rangle = N - gV\int d^3p(\exp[\beta E_p] - 1)^{-1} > 0\,. \tag{8}$$

Using (6), one finds

$$\langle n_0 \rangle = N - 4\pi g m^2 T V \sum_{n=1}^{\infty} \frac{S_{02}(n\beta m)}{n}\,. \tag{9}$$

As the sum in (9) is a monotonically decreasing function of β, one expects that condition (8) can be fulfilled at low temperature. Thus we approximate S_{02} by the asymptotic form (7). In this way, one gets

$$\langle n_0 \rangle = N - 4\pi g m T^2 V \zeta(2)$$

with $\zeta(s)$ the Riemann zeta-function ([10]):

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}\,, \qquad \zeta(2) = \frac{\pi^2}{6}\,, \qquad \zeta(3) = 1.202\,\ldots\,.$$

Finally we find that the Bose-Einstein condensation occurs at temperatures lower than the critical one, T_c, or/and at densities higher than the critical density ϱ_c:

$$T_c = \left(\frac{3N}{2\pi^3 g m V}\right)^{\frac{1}{2}}, \qquad \varrho_c = \frac{2}{3}\pi^3 g m T^2\,.$$

Now we focus our attention on the properties of the gas when a condensed phase exists, $z = 1$. One finds that

$$\frac{\langle n_0 \rangle}{N} = 1 - \left(\frac{T}{T_c}\right)^2.$$

Thus at $T = 0$ there is the condensed phase only. Formulae (6) with a low-temperature form of S_{02} (7) look like

$$\frac{P}{T} = 4\pi g m T^2 \zeta(3)\,, \qquad \frac{N}{V} = 4\pi g m T^2 \zeta(2)\,, \qquad \frac{U}{V} = 8\pi g m T^3 \zeta(3)\,.$$

The equation of state is the following:

$$pV = \frac{\zeta(3)}{\zeta(2)} NT\,.$$

The internal energy and the specific heat are expressed by the formulae

$$U = 2\frac{\zeta(3)}{\zeta(2)}NT\,, \qquad c_v = 2\frac{\zeta(3)}{\zeta(2)} = 1.461\,.$$

Since the entropy, S, of a system is defined by

$$S = N\int_0^T \frac{c_v}{T}\,\mathrm{d}T\,, \tag{10}$$

the entropy of the system is logarithmically divergent at $T = 0$. Thus *the third principle of thermodynamics is not satisfied by the Bose gas of tachyons.*

Entropy at zero temperature is related to the degeneration of the ground state of the system. The zero-energy tachyon carries a momentum equal to its mass. Thus the ground state of one tachyon is strongly degenerated. Consequently, the ground state of N bosons is stronger degenerated and the entropy at $T = 0$ differs from zero.

7. – Degenerated Fermi-Dirac gas.

When $\beta m \gg 1$ and $\beta\mu \gg 1$, it is seen that only the energies of tachyons up to the value of chemical potential, μ, significantly contribute to integrals (5). This means that the particles occupy the lowest-energy levels. Such a gas is called degenerated.

Let us consider the Fermi energy, μ_F, which, by definition, equals the chemical potential at zero temperature. The integral determining the density (5) at $T = 0$ looks like

$$\frac{N}{V} = 4\pi g\int_m^{p_F} \mathrm{d}p\,p^2 = \frac{4}{3}\pi g[p_F^3 - m^3]\,, \tag{11}$$

where

$$p_F = (\varphi_F^2 + m^2)^{\frac{1}{2}}\,.$$

Thus

$$\mu_F = \left[\left(\frac{3N}{4\pi g V} + m^2\right)^{\frac{2}{3}} - m^2\right]^{\frac{1}{2}}.$$

To discuss the properties of the degenerated tachyon gas, we consider two extreme cases: the «nonrelativistic» limit $m/\mu_F \gg 1$ and the «ultrarelativistic» limit $m/\mu_F \ll 1$.

A) « *Nonrelativistic* » *case.* Substituting (1) in (5) and taking into account the first nonvanishing order of η, we get

$$
(12)\qquad
\begin{cases}
p \;= \dfrac{4}{3}\pi g m^4 \left[\displaystyle\int_0^\infty \mathrm{d}\eta (z^{-1}\exp[\beta m\eta] + 1)^{-1} - \dfrac{\mu}{m} \right], \\[2ex]
\dfrac{N}{V} = 4\pi g m^3 \displaystyle\int_0^\infty \mathrm{d}\eta\, \eta (z^{-1}\exp[\beta m\eta] + 1)^{-1}, \\[2ex]
\dfrac{U}{V} = 4\pi g m^4 \displaystyle\int_0^\infty \mathrm{d}\eta\, \eta^2 (z^{-1}\exp[\beta m\eta] + 1)^{-1}.
\end{cases}
$$

To obtain the expression for pressure, we have firstly integrated partially integral (5). Since $\beta\mu \gg 1$, we have approximated $z+1$ by z.

For computing integrals (12), we use the Sommerfeld lemma ([13]) according to which

$$
(13)\qquad
\begin{cases}
\displaystyle\int_0^\infty \mathrm{d}u (z^{-1}\exp[u+1])^{-1} \frac{\partial\varphi}{\partial u} = \varphi(u_0) + 2\sum_{k=1}^\infty C_{2k} \frac{\mathrm{d}^{2k}}{\mathrm{d}u^{2k}} \varphi(u)\bigg|_{u=u_0} + O(1/2), \\[2ex]
u_0 = \ln z, \\[2ex]
c_n = \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^n} = (1-2^{1-n})\,\zeta(n), \qquad c_2 = \frac{\pi^2}{12}.
\end{cases}
$$

$\varphi(u)$ in the regular function and $\varphi(0) = 0$.

In the case under consideration series (13) contains one term only. Thus

$$
(14)\qquad
\begin{cases}
p \;= \dfrac{4}{3}\pi g m^4 \left[\dfrac{\ln z}{\beta m} - \dfrac{\mu}{m} \right] = 0, \\[2ex]
\dfrac{N}{V} = 2\pi g m \mu^2 \left[1 + \dfrac{\pi^2}{3}\dfrac{T^2}{\mu^2} \right], \\[2ex]
\dfrac{U}{V} = \dfrac{4}{3}\pi g m \mu^3 \left[1 + \pi^2 \dfrac{T^2}{\mu^2} \right].
\end{cases}
$$

We have omitted the terms $O(\frac{1}{2})$.

We are led to a very curious result: the pressure of the degenerated gas of fermion tachyons decreases to zero when $m/\mu_F \gg 1$. Let us underline that the pressure is never negative. This follows from the fact that for fermions the function under integral (5) is nonnegative.

Putting $T = 0$, one gets

$$\mu_F = \left(\frac{N}{2\pi g V m}\right)^{\frac{1}{2}}. \tag{15}$$

The above formula is an approximation of (11) for $m/\varphi_F \gg 1$. Comparing (15) and (14), we find

$$\mu = \mu_F\left[1 - \frac{\pi^2}{6}\frac{T^2}{\mu_F^2} + O\left(\frac{T^4}{\mu_F^4}\right)\right],$$

$$\frac{U}{V} = \frac{4}{3}\pi g m \mu_F^3\left[1 + \frac{\pi^2}{2}\frac{T^2}{\mu_F^2} + O\left(\frac{T^4}{\mu_F^4}\right)\right].$$

The specific heat is expressed by

$$c_v \simeq \tfrac{4}{3}\pi^3 m \mu_F V T .$$

Because c_v $(T = 0) = 0$, entropy (10) vanishes at zero temperature. Thus *the third principle of thermodynamics is satisfied by the Fermi gas of tachyons.* The degeneration of the ground state of the system of tachyons described previously is, in the case of fermions, suppressed by Pauli quenching.

B) « *Ultrarelativistic* » *case.* For $\mu_F/m \gg 1$ we can use an approximation

$$E_p = p + O\left(\frac{m^2}{p}\right).$$

In this limit the gas of tachyons is equivalent to that of massless fermions, *e.g.* neutrinos. Thus, without comments, we give only some formulae, which have been found in the same way as those in the previous subsection:

$$\mu_F = \left(\frac{3N}{4\pi g V}\right)^{\frac{1}{3}}, \qquad \mu = \mu_F\left[1 - \frac{\pi^3}{3}\frac{T^2}{\mu_F^2} + O\left(\frac{T^4}{\mu_F^4}\right)\right],$$

$$pV = \frac{1}{3}U,$$

$$\frac{U}{V} = \pi g \mu_F^4\left[1 + \frac{2}{3}\pi^2\frac{T^2}{\mu_F^2} + O\left(\frac{T^4}{\mu_F^4}\right)\right],$$

$$c_v \simeq \frac{4}{3}\pi^3 g \mu_F^2 V T .$$

We have found the well-known equation of state for the gas of massless particles. The pressure is sure to differ from zero even at $T = 0$ and the third principle of thermodynamics is satisfied.

8. – Difficulties of formulation of covariant statistical mechanics of tachyons.

An invariant partition function, playing a central role in the covariant statistical mechanics of bradyons ([5]), for a classical ideal gas looks like

$$(16)\qquad \begin{cases} Q_N = \dfrac{1}{N!}\displaystyle\int d^4P \exp\left[-\beta^\mu P_\mu\right]\sigma(P^\mu)\,, \\ \sigma(P^\mu) = \displaystyle\int \prod_{i=1}^{N} \left(V_\mu p_i^\mu \delta(p_{i\mu}p_i^\mu - m^2)\,\Theta(p_i^0)\,d^4p_i\right)\delta^{(4)}\left(P^\mu - \sum_{i=1}^{N} p_i^\mu\right). \end{cases}$$

P^μ is the four-momentum of the system and β^μ is the four-vector which is $(1/T, 0, 0, 0)$ in the rest frame of the system. Analogously V^μ in the rest frame is equal to $(V, 0, 0, 0)$. The expression under multiplication mark is an analogue of the noninvariant phase-space element $V d^3 p_i$.

For bradyons the distinction between those with negative and these with positive energies is Lorentz invariant because positive- and negative-energy bradyons lie on separated sheets of a hyperboloid. Thus the function $\theta(p_i^0)$ which « chooses » bradyons with positive energy is Lorentz scalar. For tachyons which lie on a single-sheeted hyperboloid it is not the case. The sign of a zero component of a four-momentum can be changed by Lorentz transformation. Thus the theta-function violates Lorentz invariance. Integral (16) would be invariant without the θ-function. However, in this case, the integral is divergent due to the contribution of negative energies of tachyons. A detailed discussion of the phase-space integrals of tachyons can be found in ref. ([14]).

The problem discussed above is an example of an apparent conflict between the covariance requirements and acceptability to theory tachyons with positive energy only. The formulation of the covariant statistical mechanics of tachyons probably needs fundamental modifications of the formalism of bradyons.

9. – Conclusions.

Let us recapitulate our results. We have considered the formalism of statistical mechanics of bradyons restricted to the systems at rest and we have acknowledged that this formalism is applicable to tachyons. Afterwards we have discussed the properties of the ideal gases. We have not found any anomalies besides two results which are significantly different from those of bradyons. Namely the entropy of the Bose gas does not vanish at zero temperature and the pressure of the degenerated Fermi gas decreases to zero.

([14]) S. Mrówszyński: *Nuovo Cimento A*, **78**, 415 (1983).

We are faced with rather fundamental difficulties in an attempt to construct the covariant statistical mechanics of tachyons. The formalism of bradyons occurs to be unapplicable to superlight particles. Unfortunately this negative result casts a shadow on our previous considerations because the substantial properties of tachyons manifest themselves at Lorentz boosts (*).

(*) It is possible that the author's scepticism is not totally justified since the situation in relativistic statistical mechanics for bradyons is not quite clear, see, *e.g.* D. TER HAAR and H. WERGELAND: *Phys. Rep. C*, **1**, 31 (1971).

● RIASSUNTO (*)

Si è considerato il formalismo della meccanica statistica di particelle piú lente della luce dal punto di vista dell'applicazione di questo formalismo alla descrizione dei tachioni. Le proprietà dei gas ideali di tachioni sono state discusse in dettaglio. Dopo aver trovato formule generali per gas quantici, di Bose e di Fermi, si è considerato il limite classico. Si è mostrato che avviene condensazione di Bose-Einstein. I gas tachionico di bosoni viola il terzo principio della termodinamica. Si è considerato un gas di Fermi degenerato ed, in questo caso, l'entropia si annulla a temperatura zero. Si sono discusse le difficoltà di formulazione della meccanica statistica covariante.

(*) *Traduzione a cura della Redazione.*

Резюме не получено.