STREAM INSTABILITIES OF THE QUARK–GLUON PLASMA

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The instabilities occurring when two streams of plasma collide are studied by means of the kinetic-theory approach to the quark–gluon plasma oscillations. The relevance of the phenomenon for the plasma from ultrarelativistic heavy-ion collisions is discussed.

The perspectives to generate the QCD plasma in ultrarelativistic heavy-ion collisions stimulate extensive studies of the possible signals which will allow us to conclude that the state of deconfined quarks and gluons is indeed produced in these collisions. For the general background see ref. [1]. In this paper I consider the phenomena well-known from the electrodynamic (electron–ion) plasma, which can be helpful in distinguishing the quark–gluon plasma. Specifically, I discuss instabilities i.e. specific oscillations, the amplitudes of which increase, usually exponentially, in time. The existence of instabilities is probably the most characteristic feature of the electromagnetic plasma. The development of them makes the motion of plasma particles essentially collective and it leads to a specific plasma behaviour far different from that of normal fluids or gases. The first steps towards studying the quark–gluon plasma instabilities have been taken. The existence of the so-called pinch instability has been suggested in ref. [2], and the filamentation instability, which is also discussed here, has been briefly considered in ref. [3].

The instabilities occur when the state of the system deviates from global thermodynamical equilibrium. According to the terminology from the electron–ion plasma [4], the instabilities caused by the system inhomogenity are called macroscopic, while those, which appear when the momentum distribution of the plasma particles differs from the equilibrium one, are known as the microscopic instabilities. In this paper I discuss only instabilities of the second type. They seem relevant for the QCD plasma from nuclear collisions since it is expected that the momentum distribution along the beam axis is much broader than that of the perpendicular momentum.

The essential point for the relevance of an instability is the time scale of its development. To get insight in the problem, I consider the idealized situation of two homogenous colliding streams of colorless plasma. This particular situation could be studied by means of the *chromohydrodynamic* equations [5], which describe hydrodynamic evolution of the plasma interacting with the chromodynamic field. I use here, however, the much more powerful kinetic theory approach described in detail in ref. [6], where the quark–gluon oscillations around the global equilibrium have been discussed.

The approach [6] is based on the gauge covariant transport theory [7] of the QCD plasma with the SU(N) gauge group. The kinetic equations of the quark and gluon distribution functions are linearized around the state where the color four-current and consequently the chromodynamic field vanish. The linearized equations are solved (with the chromodynamic field treated as external) and the chromoelectric permeability tensor is found.

To study the oscillations of the anisotropic plasma, one has to modify some of the results of ref. [6], where the plasma has been assumed isotropic. An isotropic plasma does not interact with the chromomagnetic mean field [8]. Following ref. [6] and us-

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ing the methods of the electron–ion plasma, see e.g. refs. [4,9,10], one easily finds the chromoelectric permeability tensor of the anisotropic collisionless plasma

$$\epsilon^{\alpha\beta}(k)=\delta^{\alpha\beta}+\frac{g^2}{2\omega}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{v^\alpha}{\omega-\boldsymbol{kv}+i0^+}\times\frac{\partial(n(p)+\tilde{n}(p)+2Nn_g(p))}{\partial p^\gamma}\times\left[\left(1-\frac{\boldsymbol{kv}}{\omega}\right)\delta^{\gamma\beta}+\frac{k^\gamma v^\beta}{\omega}\right], \quad (1)$$

where α, β, γ label the space axes; $k=(\omega, \boldsymbol{k})$ is the wave four-vector; g is the coupling constant ($\alpha_s=g^2/4\pi$); $p=(E, \boldsymbol{p})$ is the plasma particle four-momentum; $\boldsymbol{v}=\boldsymbol{p}/E$ is the particle velocity; $n(p)$, $\tilde{n}(p)$ and $n_g(p)$ are the dynamical equilibrium distribution functions of quarks, antiquarks and gluons, respectively. The units are used, where $c=\hbar=k=1$. The chromoelectric permeability tensor has been derived under the assumption that the plasma in the dynamical equilibrium is colorless, i.e. the color four-current is zero, or, in other words, the dynamical equilibrium distribution functions are unit matrices in color space. As shown in ref. [6], the chromoelectric tensor (1), which emerges from the transport equations linearized around the colorless equilibrium state, has no color indices.

In the case of the linear approach to the QCD plasma oscillations [6], the dispersion relations are defined as in the electromagnetic plasma [4], i.e. by the equation

$$\det|\boldsymbol{k}^2\delta^{\alpha\beta}-k^\alpha k^\beta-\omega^2\epsilon^{\alpha\beta}(k)|=0 . \quad (2)$$

The solutions of eq. (2) with the frequency which is pure real correspond to stable oscillation modes (the amplitude is constant in time), those with negative imaginary part of the frequency represent the damped modes (the amplitude exponentially decreases in time), and finally the solutions with positive imaginary frequency describe the unstable modes, the amplitude of which increases in time.

Let me now consider two colliding streams of the quark–gluon plasma. The streams are assumed infinite in space and homogenous. The densities of both streams in their rest frames are equal to one another. It is also assumed that the thermal energy of the plasma particle is much smaller than the particle energy related to the stream collective motion. Then, the distribution function of quarks reads

$$n(p)=(2\pi)^3\rho[\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q})+\delta^{(3)}(\boldsymbol{p}+\boldsymbol{q})] , \quad (3)$$

where N_ρ has to be interpreted [for the SU(N) gauge group] as the quark density of the stream in the reference frame, where the stream velocities are opposite. The form of the distribution functions of antiquarks and gluons is analogous, however the gluon density is $(N^2-1)\rho_g$.

Substituting the distribution functions (3) in (1) and assuming that the vector $\boldsymbol{q}$ is parallel to the z-axis one gets

$$\epsilon^{xx}(k)=\epsilon^{yy}(k)=1-\frac{\omega_0^2}{\omega^2},$$

$$\epsilon^{xy}(k)=\epsilon^{yx}(k)=0 ,$$

$$\epsilon^{xz}(k)=\epsilon^{zx}(k)=-\frac{\omega_0^2}{\omega^2}\frac{k_xk_zu^2}{\omega^2-k_z^2u^2},$$

$$\epsilon^{yz}(k)=\epsilon^{zy}(k)=-\frac{\omega_0^2}{\omega^2}\frac{k_yk_zu^2}{\omega^2-k_z^2u^2},$$

$$\epsilon^{zz}(k)=1-\frac{\omega_0^2}{\omega^2}-\frac{\omega_0^2}{\omega^2}u^2\frac{\omega^2+k_z^2u^2}{(\omega^2-k_z^2u^2)^2}(k_x^2+k_y^2)-\frac{\omega_0^2}{\omega^2}\frac{m^2}{E^2}k_z^2u^2\frac{3\omega^2-k_z^2u^2}{(\omega^2-k_z^2u^2)^2}, \quad (4)$$

where $\omega_0^2=g^2(\rho+\tilde{\rho}+2N\rho_g)/E$ is the plasma frequency, $E=\sqrt{m^2+\boldsymbol{q}^2}$ is the particle energy in the stream and m is the particle thermal energy, which is assumed to be identical for quarks and gluons. (If one considers the streams of massive particles with temperatures much smaller than the particle mass, m is just the particle mass.) Finally, $u=|\boldsymbol{q}|/E$ is the stream velocity.

Treating the plasma in the stream as a baryonless ideal gas of massless quarks and gluons, the parton thermal energy and the densities can be expressed through the gas temperature T as

$$m=3T,$$

and

$$\rho^0+\tilde{\rho}^0+2N\rho_g^0=\frac{(3N_f+4N)\zeta(3)}{\pi^2}T^3,$$

where N_f is the number of quark flavours; the index

0 labels the densities in the stream rest frame. Then, the plasma frequency reads

$$\omega_0^2 = g^2 \frac{(3N_f + 4N)\zeta(3)}{3\pi^2} T^2 . \qquad (5)$$

Substituting the chromoelectric tensor (4) in (2) one finds the dispersion relations. Since we are interested in the relativistic streams, $u=1$ in further considerations. To simplify the analysis, let me consider two specific cases.

(1) *Oscillations along the beam axis.* Only the z-component of the wave vector is nonzero [$\boldsymbol{k}=(0, 0, k)$]. Then, eq. (2) is of the form

$$(k^2 - \omega^2 \epsilon^{xx})(k^2 - \omega^2 \epsilon^{yy})\epsilon^{zz} = 0 .$$

There are two solutions ($k^2 - \omega^2\epsilon^{xx} = k^2 - \omega^2\epsilon^{yy} = 0$), related to the transverse modes (the chromoelectric field is perpendicular to the wave vector),

$$\omega^2 = \omega_0^2 + k^2 , \qquad (6)$$

which are stable, and there is one solution corresponding to the longitudinal mode $\epsilon^{zz}=0$. For ultrarelativistic streams ($E \gg m$) the fourth term of ϵ^{zz} from (4) can be neglected (except $\omega^2 = k^2$) and the dispersion relation reads

$$\omega^2 = \omega_0^2 . \qquad (7)$$

Therefore, the longitudinal mode, as the transverse ones, is stable. It is interesting to note that the analogous longitudinal mode for nonrelativistic cold streams ($m=E$) is unstable [4].

(2) *Oscillations perpendicular to the beam axis.* I choose the wave vector along the x-axis [$\boldsymbol{k}=(k, 0, 0)$]. For this case eq. (2) reads

$$\epsilon^{xx}(k^2 - \omega^2\epsilon^{yy})(k^2 - \omega^2\epsilon^{zz}) = 0 .$$

The dispersion relation of the longitudinal model coincides with (7) and the one of the transverse mode with the chromoelectric field along the y-axis has the form (6). Both modes are stable. For the transverse mode with the chromoelectric field along the z-axis, one finds two solutions of the equation $k^2 - \omega^2\epsilon^{zz} = 0$,

$$\omega_\pm^2 = \tfrac{1}{2}[\omega_0^2 + k^2 \pm \sqrt{(\omega_0^2 + k^2)^2 + 4\omega_0^2 k^2}] . \qquad (8)$$

One sees that $\omega_+^2 \geqslant 0$ and $\omega_-^2 \leqslant 0$. Therefore, the modes represented by ω_+ are stable. On the other hand, the frequency of the modes related to the ω_- solution is pure imaginary. The mode with the negative $\mathrm{Im}\,\omega$ is damped, and the one with the positive $\mathrm{Im}\,\omega$, which is called the filamentation mode [10], is unstable. In the further discussion I concentrate on the filamentation mode, the physical picture of which is the following. A density fluctuation of the initially homogenous streams occurs. When the density gradient is nonzero in the direction perpendicular to the beam, the fluctuation increases in time, and finally the colliding streams are split into filaments of transversal size equal to the half-wave-length of the initial fluctuation. The color currents are of the opposite sign in the neighbouring filaments.

Let me consider the characteristic time τ of development of the instability, which equals $1/\mathrm{Im}\,\omega_-$. One sees from eq. (8) that the absolute value of ω_-^2 increases with k^2. If $k^2 \gg \omega_0^2$ one finds

$$\omega_-^2 \cong -\omega_0^2(1 - \omega_0^2/k^2) .$$

Therefore the maximal negative value of ω_-^2 is $-\omega_0^2$. In this way one finds the minimal time of the instability development to be

$$\tau_{\min} = \omega_0^{-1} ,$$

which occurs for $k^2 \gg \omega_0^2$.

Let me estimate the numerical value of $\tau_{\min}$. Substituting $N_f=2$, $N=3$ and $g=1$ (the approach [6] does not demand the smallness of the coupling constant) in (5), one finds that $\tau_{\min}$ equals 1.2 fm/c for $T=200$ MeV and 0.5 fm/c for $T=500$ MeV. The value of $\tau_{\min}$ given in ref. [3] (0.1 fm/c) is, in my opinion, underestimated. It should be also stressed that the present reasoning provides a lower limit of the time of instability development, since the parton collisions and plasmon decays [6] have been neglected. Both phenomena provide damping mechanisms of the oscillations and increase the value of $\tau_{\min}$. One should also remember that I have considered a rather idealized problem with the distribution function (3). The calculations with more realistic functions are in progress.

The question arises whether the filamentation instability can occur in the quark–gluon plasma produced in heavy-ion collisions, and if it is possible to observe this process. The answer essentially depends on the conditions which are realized in the collisions, in particular, on the life-time of the deconfined state. The instability can develop if the plasma exists for a time significantly greater than $\tau_{\min}$. On the basis of hydrodynamic calculations (for a review see ref. [11]), one can hope that this is really the case. The problem of detecting the instability is more difficult. The proposition made in ref. [3] to observe hard photons emitted when the instability develops is, in my opinion, rather unrealistic, because of many photon sources, e.g. neutral pions, which occur in nuclear collisions at high energies.

It is commonly believed that the hadronization process is soft, i.e. that hadrons *remember* to some extent the momentum distribution of partons before the hadronization. Then, hadrons carry information about the collective motion of partons, which occurs due to the instability. Because the development of the instability most probably leads to the generation of the parton jet, I propose to measure the total energy versus total momentum of the hadrons in the hadron jet, and to compare it with the dispersion relation of the plasma oscillations. However, this problem needs further studies, which are now in progress.

Let me recapitulate the considerations. The system of two colliding streams of the quark–gluon plasma has been studied. It has been assumed that the parton thermal energy in the stream in much smaller than the parton energy related to the collective stream motion. The dispersion relations of the plasma waves have been discussed using the kinetic theory approach to the quark–gluon plasma oscillations [6]. Only one unstable mode, known as the filamentation instability, has been found. The relevance of the phenomenon for the plasma from nuclear collisions has been discussed.

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