ON THE PION AND KAON PRODUCTION IN NUCLEUS–NUCLEUS COLLISIONS NEAR THE ABSOLUTE THRESHOLD

Stanisław MRÓWCZYŃSKI [1]
High-Energy Department, Institute for Nuclear Studies, ul. Hoża 69, PL-00-681 Warsaw, Poland

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A simple phase-space model of particle production in nucleus–nucleus collisions near the absolute threshold is developed. The model parameters are determined by comparison with pion production experimental data, and then the model is used to estimate the kaon production cross section.

It is well known that particle production in nucleus–nucleus collisions is possible at initial energies per nucleon significantly smaller than the threshold energy for particle production in interactions between two free nucleons. These are the so-called subthreshold processes, see e.g. ref. [1]. Recently, the development of the experimental technique has allowed studies of the production of pions in nuclear collisions at bombarding energies greater than the absolute threshold energy only by some tens of MeV [2]. The work *absolute* means that the total kinetic energy of the colliding nuclei has to be converted (in the center-of-mass frame) into pion mass to satisfy the energy–momentum conservation.

The experimental studies of subthreshold kaon production have also started [3], and it is the aim of this note to give a crude estimation of the production cross section near the absolute threshold. I develop a very simple phase-space model, similar to that of Shyam and Knoll [4], which I apply to pion production to determine the model parameters. Then the model is used to discuss the kaon production processes. It occurs that in contrast to the pion case the kaon production cross section near the absolute threshold is most probably too small to be measurable. The kaon production proceeds at a higher momentum transfer to a nucleus than that of the pion production. Although both momentum transfers are of the same order the probability that a nucleus absorbs the momentum needed to produce a kaon is smaller by many orders of magnitude than that of the pion case.

In further considerations I discuss the subthreshold π^+ and K^+ production in collisions of two ^{12}C nuclei. The pion production reactions of the lowest energy threshold read

$$^{12}C+{}^{12}C \to {}^{24}Na+\pi^+ , \qquad (1)$$

$$^{12}C+{}^{12}C \to {}^{12}C+{}^{12}B+\pi^+ . \qquad (2)$$

It is expected that the initial energy dependence of the processes (1) and (2) near the threshold is governed by the phase-space behaviour. Keeping in mind that all particles involved in (1) and (2) are nonrelativistic (near the threshold), one easily calculates two- and three-particle phase-space volumes, which increase with initial energies as $(E-E_0)^{1/2}$ for the process (1) and as $(E-E_0)^2$ for (2). E denotes the total kinetic energy of colliding nuclei in the center-of-mass frame and E_0 is the threshold energy. Assuming that the matrix element of the process (1) and (2) is constant as a function of the initial energy, one parametrizes the respective cross sections, for the energies greater than the threshold energy as

$$\sigma^{\pi}_{CC}(E)=\beta E^{-1/2}(E-m_\pi)^{1/2} , \qquad (3)$$

$$\sigma^{\pi}_{CC}(E)=\alpha^{\pi}_{CC}E^{-1/2}(E-m_\pi)^2 . \qquad (4)$$

For simplicity I have assumed that the binding ener-

[1] Present address: Institut für Theoretische Physik, Universität Regensburg, Postfach 397, D-8400 Regensburg, Fed. Rep. Germany.

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gies per nucleon of ^{12}C, ^{12}B and ^{24}Na are equal to one another, and then the threshold energy equals the pion mass. The factor $E^{-1/2}$ appeared because of the relative velocity of colliding nuclei which is present in the cross section definition; β and α^{π}_{CC} are the initial-energy-independent parameters.

Let me confront the formulas (3) and (4) with the experimental data [2]. Normalizing both parametrizations at the same energy ($E=360$ MeV) one finds, see fig. 1, that the parametrization related to the pionic fusion reaction (1) is in disagreement with the data. One the other hand, eq. (4) with $\alpha^{\pi}_{CC}\approx 1.2\times 10^{-2}$ mb GeV$^{-3/2}$ can describe the data up to the energy of about 0.4 GeV. Therefore I assume that the process (2) gives the dominant contribution to the pion production near the absolute threshold. The process (1) can contribute only at the lowest energies which are now experimentally unavailable.

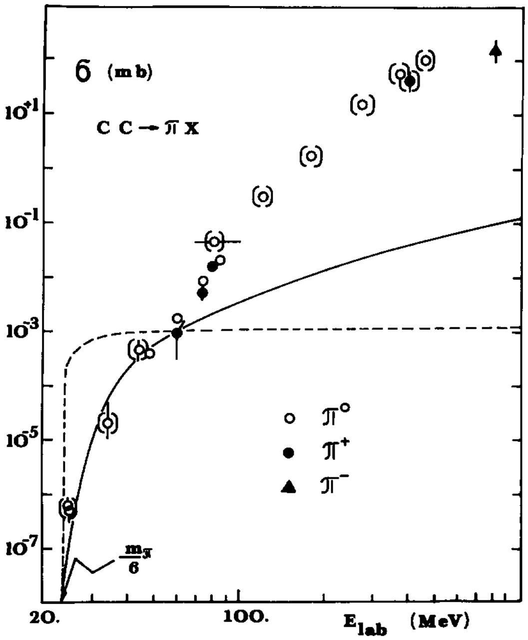

Fig. 1. The inclusive cross section of pion production in C–C collisions versus initial kinetic energy per nucleon in the lab frame. (In the non-relativistic region $E_{lab}=\frac{1}{6}E$.) The data are taken from ref. [2]. The points in parentheses correspond to cross sections measured in other than C–C combinations of target and projectile. These points have been rescaled according to the experimentally found cross section dependence ($A_p^{2/3}A_t^{2/3}$) on projectile and target mass numbers. The dashed line corresponds to eq. (3) and the solid line is the prediction of eq. (4). Both parametrizations are normalized at $E_{lab}=\frac{1}{6}E=60$ MeV.

It is easy to understand why the formula (4) fails at higher energies. When the initial energy increases from the pion production threshold, the allowed number of particles (nucleons and nuclear fragments) in final states also increases. One can show, see e.g. ref. [4], that the phase-space volume of the process with N non-relativistic particles in the final state increases with initial energy as $(E-E_0)^{(N-1)/2}$. Therefore the increase with initial energy is steeper when the number of particles is higher. To improve the model one has to include, as is done in ref. [4], the processes with the number of particles in the final state higher than three. The importance of these processes increases with intial energy.

Let me parametrize the cross section of the process

$$p+p\to p+n+\pi^+ \qquad (5)$$

as previously, i.e., as

$$\sigma^{\pi}_{pp}(E)=\alpha^{\pi}_{pp}E^{-1/2}(E-m_\pi)^2\,. \qquad (6)$$

The comparison of eq. (6) with data, see fig. 2, provides the value of $\alpha^{\pi}_{pp}\approx 240$ mb GeV$^{-3/2}$. In this way one finds that $\alpha^{\pi}_{CC}/\alpha^{\pi}_{pp}\approx 5\times 10^{-5}$. This ratio shows that it is much less probable to produce a pion in nucleus–nucleus interactions than to produce it in nucleon–nucleon collisions having the same portion of available (kinetic) energy. As we will see later the value of $\alpha^{\pi}_{CC}/\alpha^{\pi}_{pp}$ is so small due to the nuclear form-factor effect.

Let me now consider the kaon production. The processes analogous to (2) and (5), respectively, read

$$^{12}C+{}^{12}C\to{}^{12}C+{}^{12}_{\Lambda}B+K^+\,, \qquad (7a)$$

$$^{12}C+{}^{12}C\to{}^{12}_{\Lambda}C+{}^{12}B+K^+ \qquad (7b)$$

and

$$p+p\to p+\Lambda+K^+\,, \qquad (8)$$

where $^{12}_{\Lambda}C$ and $^{12}_{\Lambda}B$ are hypernuclei.

I parametrize the cross section of the sum of processes (7) as

$$\sigma^{K}_{CC}(E)=2\alpha^{K}_{CC}E^{-1/2}[E-(m_\Lambda-m_p)-m_K]^2 \qquad (9)$$

and the process (8) in the same way, i.e, as

$$\sigma^{K}_{pp}(E)=\alpha^{K}_{pp}E^{-1/2}[E-(m_\Lambda-m_p)-m_K]^2\,. \qquad (10)$$

I have assumed that the binding energy of Λ in a car-

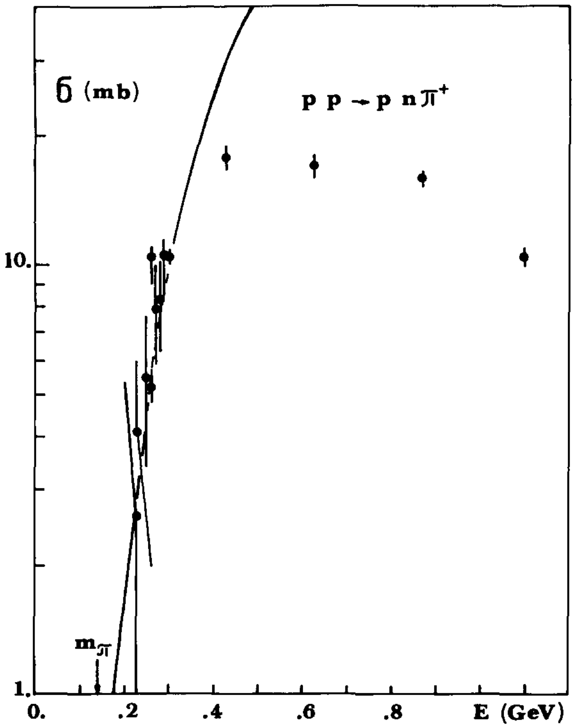

Fig. 2. The cross section of the reaction $pp \to pn\pi^+$ versus initial kinetic energy in the CM frame. The data are taken from ref. [5]. The solid line is the prediction of eq. (6) normalized at $E=0.3$ GeV.

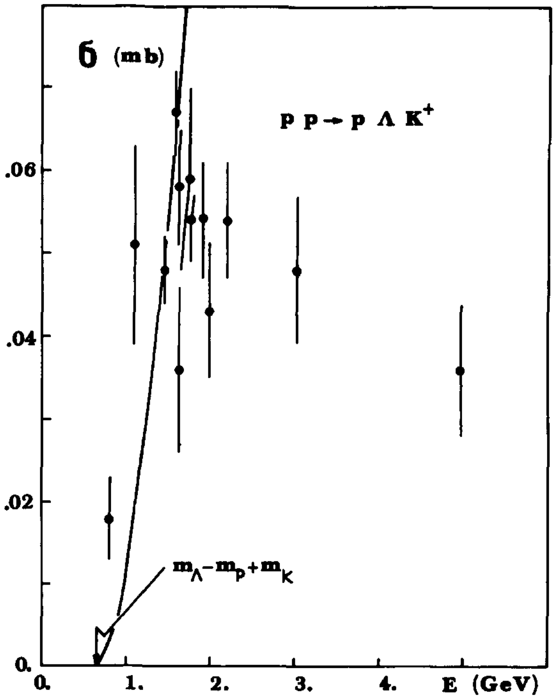

Fig. 3. The cross section of the reaction $pp \to p\Lambda K^+$ versus initial kinetic energy in the CM frame. The data are taken from ref. [5]. The solid line is the prediction of eq. (10) normalized at $E=1.5$ GeV.

bon hypernucleus equals the binding energy of a nucleon in a carbon nucleus.

Comparing formula (10) with the experimental data, see fig. 3, I have found that $\alpha^{K}_{pp} \approx 0.1$ mb $\mathrm{GeV}^{-3/2}$.

How to determine the value of the parameter α^{K}_{CC}?

The simplest assumption, which gives the very optimistic value of $\alpha^{K}_{CC} = 5\times10^{-6}$ mb $\mathrm{GeV}^{-3/2}$, is

$$\frac{\alpha^{\pi}_{CC}}{\alpha^{\pi}_{pp}} = \frac{\alpha^{K}_{CC}}{\alpha^{K}_{pp}} . \tag{11}$$

This assumption, however, is unrealistic since the processes (7) proceeds at a momentum transfer which is significantly greater than that of the process (2). The momentum transfer to a carbon nucleus in the center-of-mass frame at the threshold initial energy is $q_\pi = 1.25$ GeV in the case of the pion production, and it is $q_K = 2.74$ GeV for the kaon case. The question arises in which way such large momenta can be absorbed by a nucleus?

I consider two limiting cases. First, when the whole momentum transfer is absorbed by a single nucleon. Then the probability that a nucleus survives in a bound state (it is demanded by energy–momentum conservation) receiving the momentum transfer q is proportional to $|F(q)|^2$, where $F(q)$ is the nuclear form factor.

Let me recall that the form factor is defined as

$$F(q) = \int d^3r \exp(i\mathbf{q}\cdot\mathbf{r})\, \phi(\mathbf{r})\phi^*(\mathbf{r}) ,$$

where $q \equiv |\mathbf{q}|$ and $\phi(\mathbf{r})$ is the ground state wavefunction of a nucleon in a nucleus. Since $\exp(i\mathbf{q}\cdot\mathbf{r})\phi(\mathbf{r})$ is the wavefunction of the nucleon to which the momentum $\mathbf{q}$ has been instantaneously transferred, the form factor is the projection of the *kicked* nucleon wavefunction on the ground state one, and $|F(q)|^2$ gives the probability of survival of the *kicked* nucleon in the ground state. Let me notice that the characteristic time of pion production equals the inverse pion mass and is 1.4 fm/c. Since this time is substantially longer than the characteristic time of internal nucleus motion, it is reasonable to treat the momentum transfer to a nucleus as instantaneous.

It should be also stressed that the numerical value of the form factor, where the conjugate wavefunction is the wavefunction of excited state instead of the ground state one, is not very different from the value of the form factor with the ground state function if the momentum transfer is much greater than the characteristic momentum of nucleon motion in a nucleus.

Using the probabilistic interpretation of the form factor one finds that cross section of the process (2) near the threshold is proportional to $|F(q_\pi)|^4$ (two nuclei in the final state). Taking the form factor [6]

$$F(q)=[1-\tfrac{2}{39}(qa)^2]\exp[-\tfrac{3}{26}(qa)^2] \qquad (12)$$

with $a=2.42$ fm, which corresponds to the harmonic well density distribution, one finds that $|F(q_\pi)|^4$ is of the order 10^{-43}. The smallness of this coefficient is in obvious contradiction with the existing experimental data on pion production. On the other hand it is hard to imagine that in nucleus–nucleus collisions, most probably central ones, all momentum is transferred to a single nucleon. So, let me consider the second, more optimistic, limiting case that the momentum transfer is uniformly distributed among all nucleons of the carbon nucleus. Then, the probability that the nucleus survives in the bound state receiving the momentum transfer q is proportional to $|F(\frac{1}{12}q)|^{24}$. The value of $|F(\frac{1}{12}q_\pi)|^{48}$ with the form factor (12) is of the order of 10^{-6}. Let us observe that it is not very far, as expected, from the previously determined value of $\alpha^\pi_{CC}/\alpha^\pi_{pp}$. Therefore I propose the following improvement of relation (11):

$$|F(\tfrac{1}{12}q_\pi)|^{-48}\frac{\alpha^\pi_{CC}}{\alpha^\pi_{pp}}=|F(\tfrac{1}{12}q_K)|^{-48}\frac{\alpha^K_{CC}}{\alpha^K_{pp}} \qquad (13)$$

which gives the value of α^K_{CC} equal to 6×10^{-30} mb GeV$^{-3/2}$. The kaon production cross sections (9) with this value of α^K_{CC} for the energy $E=1$ GeV, which corresponds to laboratory energy per nucleon equal to 170 MeV (58 MeV over threshold), is of the order of 10^{-30} mb. This value is far beyond experimental possibilities.

Let me note here that the nuclear form factors are measured with high accuracy at the momentum transfers of order $\frac{1}{12}q_K$, and that the parametrization (12) fits the data very well [6]. Therefore, in spite of a very high power of the form factor in eq. (13), the accuracy of the value of α^K_{CC} found from (13) is sufficient for our purposes. Even if one takes the ratio of $F(\frac{1}{12}q_\pi)/F(\frac{1}{12}q_K)$ greater than that which follows from eq. (12) by 20%, the estimated kaon production cross section at $E=1$ GeV is of the order of 10^{-24} mb and my conclusion remains unaffected.

It is expected that the kaon production cross section, as the pion cross section, strongly increases (probably exponentially) with intial energy. However, the model discussed here cannot be extrapolated to energies E greater than about 1 GeV for the reasons quoted in the context of the pion production.

I conclude as follows. In contrast to the pion case, the kaon production cross section near the absolute threshold (up to the laboratory energy per nucleon of some tens of MeVs above the threshold) is probably experimentally unavailable. It occurs because the kaon production process proceeds at a momentum transfer greater by a factor of 2.2 than that of the pion production process and therefore the nuclear form factor damps the cross section. It should be stressed that this conclusion is independent of the details of my phase-space model. Recent analysis [7] of the existing subthreshold K^- production data suggests the lack of collective effects and consequently it supports my arguments. In spite of my pessimistic conclusion it would be very interesting to perform experimental studies to put at least an upper limit on the kaon production cross section near the absolute threshold. Our understanding of nucleus–nucleus collisions at high momentum transfers would demand serious revision if the measured cross section substantially exceeded the estimation found in this note.

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