Antideuteron production and the size of the interaction zone ☆

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The description of (anti)deuteron production in high-energy collisions in the framework of the coalescence model provides a method to measure the size of the interaction zone. This method is compared with that based on pion interferometry, and then both are applied to pp collisions at $\sqrt{s}=53$ GeV. The data on antideuterons from Υ decays are also discussed.

The space–time characteristics of the interaction zone of high-energy collisions are traditionally studied by means of the pion interferometry method (for a review see ref. [1]). The aim of this note is to discuss another method (*the coalescence-model method*), which is often applied to high-energy collisions with nuclei, where the information about the size of the interaction region is obtained by comparing the production rates of nucleons and of deuterons [2–4]. Observation of (anti)deuterons in high-energy pp collisions [5,6] and e^+e^- annihilations [7] allows to apply the coalescence-model method also in these cases.

It has been realized in the sixties that the coalescence of nucleons due to final state interaction is responsible for the deuteron production [8] (an analogous observation concerning antideuterons was made immediately after their discovery [9], see also refs. [5,10]); however, only more recent studies [2–4] have established the relation of the coalescence model parameters with the space–time characteristics of the interaction zone.

The (anti)nucleons produced in a high-energy collision are emitted from a finite space–time region. Their momenta are not precisely determined because of the uncertainty principle. Therefore, two (anti)-nucleons which are close to each other in phase space can form an (anti)deuteron (no additional third body is needed). Assuming the factorization of the matrix element into parts corresponding to the (anti)-nucleon-pair production and the final state interaction one finds, see refs. [2–4], the Lorentz invariant cross section for (anti)deuteron production expressed through the Lorentz invariant cross section of (anti)nucleon-pair production as

$$E\frac{d\sigma^{\bar{d}}}{d^3\boldsymbol{P}}=\frac{2}{m}\mathscr{A}E_1E_2\frac{d\sigma^{\bar{n}\bar{p}}}{d^3\boldsymbol{p}_1\,d^3\boldsymbol{p}_2}\,, \tag{1}$$

with $E_1=E_2=\frac{1}{2}E$ and $\boldsymbol{p}_1=\boldsymbol{p}_2=\frac{1}{2}\boldsymbol{P}$; m is the nucleon mass and $\mathscr{A}$ is the following overlap integral [#1]:

$$\mathscr{A}=\tfrac{3}{4}(2\pi)^3\int d^3r\,|\phi_d(\boldsymbol{r})|^2\mathscr{D}(\boldsymbol{r})\,, \tag{2}$$

where $\phi_d(\boldsymbol{r})$ is the deuteron wave function and $\mathscr{D}(\boldsymbol{r})$ is the relative distance distribution of the (anti)nucleon pair in the interaction zone at the instant of the pair emission. The coefficient $\frac{3}{4}$ comes from the averaging over the polarizations of the (anti)nucleon pair and summing over the (anti)deuteron polarizations. The emitted (anti)nucleons are assumed unpolarized. Eq. (1) additionally assumes that the (anti)nucleon-pair production cross section weakly depends on the (anti)nucleon relative momentum when compared with the momentum dependence of the (anti)deuteron wave function. (The relation be-

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[#1] In the first version of this paper a coefficient $(2\pi)^3$ was missing in eq. (2), and consequently the quantitative results presented there were partially incorrect.

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tween the free nucleon and deuteron production cross sections appears to have first been obtained by Migdal [11] who studied the reactions $NN \to NN\pi$ and $NN \to d\pi$.)

In full analogy to the (anti)deuteron production one derives [4] the production cross section of two particles (e.g. pions), which takes into account the final state interaction between these particles. Specifically,

$$E_1 E_2 \frac{d\sigma^{\pi\pi}}{d^3p_1\, d^3p_2} = \mathscr{R}(p_1 - p_2) E_1 E_2 \frac{d\tilde{\sigma}^{\pi\pi}}{d^3p_1\, d^3p_2}, \quad (3)$$

with

$$\mathscr{R} = \int d^3r\, |\phi_{2\pi}(r)|^2 \mathscr{D}(r), \quad (4)$$

where $\phi_{2\pi}(r)$ is the two-particle wave function of relative motion. The expression following the function $\mathscr{R}$ on the right-hand-side of eq. (3) is the two-particle production cross section which neglects particle correlations due to the final state interactions. Eq. (4) has been derived under the assumption that the two particles are emitted from the source at the same moment of time (in their center-of-mass frame). (Sakharov [12] found a formula analogous to (3) studying the final state interaction in electron–positron pair production.)

Now, let us apply eqs. (1)–(4) to describe antideuteron production and two identical pion correlations in proton–proton collisions at $\sqrt{s} = 53$ GeV.

Unfortunately, the cross section of (anti)nucleon-pair production is not experimentally measured. It can be approximated as

$$E_1 E_2 \frac{d\sigma^{\bar{n}\bar{p}}}{d^3p_1\, d^3p_2} = \frac{1}{\sigma_{\rm inel}} E_1 \frac{d\sigma^{\bar{n}}}{d^3p_1} E_2 \frac{d\sigma^{\bar{p}}}{d^3p_2}, \quad (5)$$

where $\sigma_{\rm inel}$ is the total inelastic cross section which equals 35 mb [13]. The ansatz (5) assumes independent from each other production of two (anti)nucleons. In the case of nucleon emission in nucleus–nucleus interactions this assumption seems justified but it can be strongly violated when production of antinucleons in pp collisions is considered. Therefore, eq. (5) is expected to give rather an upper limit of the cross section of antinucleon-pair production.

The inclusive cross section of antiproton production measured at $\theta = 90°$ in the center-of-mass frame has been parametrized as [14]

$$E \frac{d\sigma^{\bar{p}}}{d^3p} = A \exp(-Bp_T), \quad (6)$$

where $A = 2.29$ mb GeV^{-2}, $B = 2.49$ GeV^{-1} and p_T is the antiproton perpendicular momentum. Furthermore, it is assumed that the cross sections of antiproton and antineutron production are equal to each other.

We use $\phi_d(r)$, $\phi_{2\pi}$ and $\mathscr{D}(r)$ in the following forms:

$$\phi_d(r) = \left(\frac{\alpha}{2\pi}\right)^{1/2} \frac{\exp(-\alpha r)}{r}, \quad (7a)$$

$$\phi_{2\pi}(r) = (\tfrac{1}{2})^{1/2} [\exp(iq\cdot r) + \exp(-iq\cdot r)], \quad (7b)$$

$$\mathscr{D}(r) = \frac{1}{\frac{4}{3}\pi R^3} \Theta(R - r), \quad (7c)$$

where $r \equiv |r|$ and $\alpha = 0.232$ fm^{-1} [15]. Eq. (7b) represents a symmetrized wave function of two noninteracting pions with relative momentum q.

Substituting the parametrizations (7) into eqs. (2), (4) one finds

$$\mathscr{A} = \tfrac{9}{2}\pi^2 \frac{1 - \exp(-2\alpha R)}{R^3}, \quad (8)$$

$$\mathscr{R}(q) = 1 + \lambda \frac{3}{(2qR)^3} [\sin(2qR) - 2qR\cos(2qR)], \quad (9)$$

where $q \equiv |q|$. As is traditionally done in the pion interferometry analysis [1] we have introduced ad hoc the parameter λ in eq. (9), which according to our derivation should be equal to 1. This parameter varying from 0 to 1 represents the degree of source chaoticity with $\lambda \to 1$ corresponding to the completely chaotic (independent) pion emission. The source lifetime parameter is not present in the correlation function (9) because it has been assumed that both pions of a pair are emitted from the source at the same moment of time. We have made this simplifying assumption to treat antideuteron production and pion correlations in exactly the same way. In fact, one could avoid this assumption [4] in both cases. However, the analysis, which is then much more complicated, cannot be performed because of the rather scarce antideuteron data.

Substituting eqs. (5)–(7) into eq. (1) we get a formula for antideuteron production cross section in which R plays the role of a free parameter. In fig. 1 this formula is confronted with the experimental data [5,6]. To show the sensitivity of the method we present in fig. 1 the cross section calculated with three values of R. One sees that eq. (1) predicts the slope of cross section as a function of transverse momentum in full agreement with the data.

In fig. 2 we compare the correlation function (9) with the data [16]. In fact, the two-pion correlations are studied in ref. [16] as functions of transverse and longitudinal relative momenta (q_T, q_L) measured with respect to total pair momentum. However, no significant difference is seen between the dependence on q_L and q_T. Therefore, we assume that $\mathcal{R}(q)=\mathcal{R}(q_T)$ and we identify q_T with q in fig. 2. We have chosen the value of $\lambda=0.5$ (cf. ref. [1]), and, as previously, we show the correlation function for three values of R.

In fact, the parameter R cannot be identified with the interaction-zone radius since R corresponds to the relative distance of two particles. However, if one treats the parametrization (7c) as an approximation to a gaussian distribution (with the same average radius squared), the source radius, defined as the square root of average radius squared, equals $\sqrt{\frac{6}{5}}R$, where

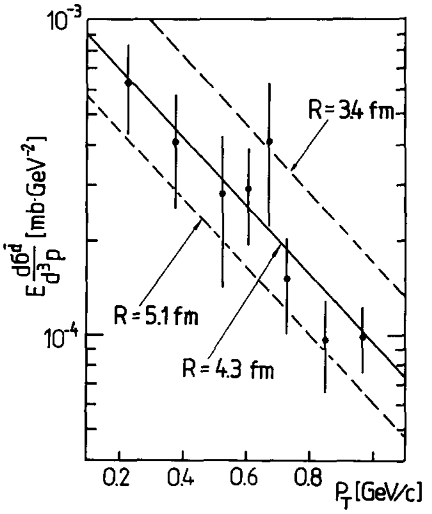

Fig. 1. Lorentz invariant cross section of the antideuteron production in pp collisions at $\sqrt{s}=53$ GeV versus perpendicular momentum. The data are taken from refs. [4,5].

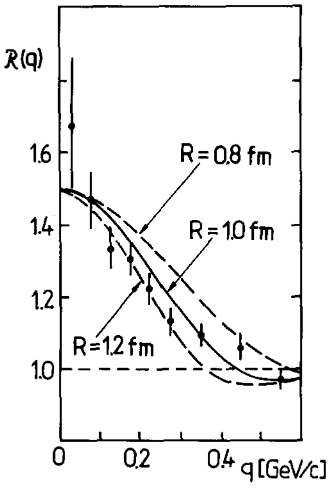

Fig. 2. The correlation function of positive pions from pp collisions at $\sqrt{s}=53$ GeV versus relative momentum. The data are taken from ref. [16].

we assumed that the particle positions in the source are independent of each other.

One observes that our value of the source radius found from the pion interferometry, which agrees with that given in ref. [16], is smaller by a factor 4 than the value obtained from the antideuteron data. In our opinion, at least part of this difference is due to the ansatz (5), which probably overestimates the cross section of antinucleon-pair production. If it is indeed the case, the *real* value of R is smaller than 4.3 fm. Therefore, one has to know the cross section of antinucleon-pair production to make the coalescence-model method a precise tool to measure the size of the interaction zone.

Probably, the main limitation of the coalescence-model method is the smallness of antideuteron production cross section. Let us briefly discuss this problem. Assuming isotropy of the antiproton production cross section in the center-of-mass frame of colliding protons, one gets a conservative estimate for the total antideuteron cross section by integrating the cross section (1) with the parametrization (5), (6) and $R=4.3$ fm over antinucleon momenta. Using the energy dependence of the cross section (6) given in ref. [14] (measured for $23<\sqrt{s}<63$ GeV) we can parametrize the antiproton and antideuteron cross sections as

$$\sigma^{\bar{p}} \cong 2.5(s/s_0)^{0.31}\ \text{mb}\,, \tag{10a}$$

$$\sigma^{\bar{d}} \cong 8(s/s_0)^{0.62} \times 10^{-4}\ \text{mb}\,, \tag{10b}$$

where $\sqrt{s_0}=53$ GeV. If we extrapolate the parametrizations (10) to $\sqrt{s}=800$ GeV one finds $\sigma^{\bar{p}} \cong 14$ mb and $\sigma^{\bar{d}} \cong 2.4\times10^{-2}$ mb. (Taking into account the increase of σ_{inel} by a factor 1.6 between $\sqrt{s}=53$ GeV and $\sqrt{s}=800$ GeV [17], the latter cross section is decreased by this factor.) Therefore, at collider and higher energies the antideuteron production cross section seems big enough to perform effective measurements.

Recently, antideuterons have been observed in e^+e^- annihilations at 10 GeV centre-of-mass energy [7]. These antideuterons have been identified as products of Υ (bottomium) decays. The measured antideuteron production cross section has been related to that of antiprotons by the formula commonly used in the studies of the deuteron emission in nucleus–nucleus collisions, see e.g. refs. [2,18]. Specifically,

$$\frac{1}{\sigma}\frac{d\sigma^{\bar{d}}}{d^3\boldsymbol{p}} = \tfrac{4}{3}\pi p_0^3 \gamma \left(\frac{1}{\sigma}\frac{d\sigma^{\bar{n}}}{d^3\boldsymbol{p}}\right)\left(\frac{1}{\sigma}\frac{d\sigma^{\bar{p}}}{d^3\boldsymbol{p}}\right), \tag{11}$$

where σ is the cross section for the Υ hadronic decays, $\boldsymbol{p}$ is the momentum per antinucleon, γ is the Lorentz factor, $\gamma=(1+\boldsymbol{p}^2/m^2)^{1/2}$, and p_0 is the radius of the sphere in the momentum space. If the difference of the antinucleon momenta lies within the sphere, the antinucleons are assumed to form the antideuteron. The antideuteron data have been successfully described by eq. (11) with $p_0=130$ MeV/c. The cross section of antineutron production has been assumed to be equal to the antiproton one.

Comparison of eq. (11) with eq. (1) [with the ansatz (5)] provides the relation

$$\mathscr{A} = (1/2^3)\tfrac{4}{3}\pi p_0^3\,. \tag{12}$$

The coefficient 2^3 has appeared here because of the momentum per antinucleon used in eq. (11). Now, one can relate the radii of the momentum-space sphere to those of the interaction zone by means of eq. (12). In particular, $p_0=130$ MeV/c provides $R=6.5$ fm. Again, as in the case of pp collisions, we have got a large (much larger than the radius of the bottomium) value of R. The reason might also be the same – eq. (11) assumes that the ansatz (5) holds. We find it not very reasonable to discuss a possible *physics* behind the large value of R unless the relation (5) is experimentally examined.

The interaction-zone-size analysis exploiting antideuteron data can also be applied to heavy-ion collisions at the presently highest available energies. When compared with deuterons and pions copiously produced in these collisions, antideuterons might be more interesting since they are *really* produced objects and carry information which is not so substantially obscured by resonances as it happens with pions. Recently, several authors discussed the importance of measuring antiprotons and antideuterons in heavy-ion collisions [19].

Let us recapitulate our considerations. The comparison of antideuteron and antiproton production rates provides the (*coalescence-model*) method to measure the size of interaction zone. This method is compared with that based on the pion-correlation studies, and then we apply the two methods to proton–proton collisions at $\sqrt{s}=53$ GeV. The data on antideuteron production in e^+e^- collisions are also discussed. In both cases we stress the need to measure the antinucleon-pair production cross section to make the coalescence-model method a precise tool to study the size of the interaction zone. Finally, we advocate the usefulness of the method at higher energies, where the antideuteron production cross section is substantially bigger, and in heavy-ion collisions.

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