

Energy loss of a high-energy parton in the quark–gluon plasma

Stanisław Mrówczyński¹

High-Energy Department, Soltan Institute for Nuclear Studies, ul. Hoża 69, PL-00 681 Warsaw, Poland

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The energy loss of a high-energy parton (quark or gluon) traversing the equilibrium quark–gluon plasma is discussed. The collisions with the thermalized plasma partons and the plasma polarization effects are considered in detail. The calculations and the final results are confronted with those of previous papers.

Hadron jets produced in ultrarelativistic nucleus–nucleus collisions have been suggested to carry information about possible generation of quark–gluon plasma (QGP) in these collisions [1,2]. In this context the energy loss of a high-energy parton traversing the QGP is of crucial importance. The first estimate [1] of this quantity has taken into account the energy loss due to elastic collisions with plasma partons. Later on the role of collective plasma modes, or equivalently – the plasma polarization effects, has been discussed [3]. The aim of this paper is to reconsider the problem and to give a completely analytical solution which exploits general properties of the plasma chromodielectric tensor. Based on this approach we clarify some points, which, in our opinion, have been misinterpreted in ref. [3], and provide numerical estimates of the parton energy loss^{#1}.

The problem of energy loss of a test particle traversing the plasma is, in fact, very complex, see e.g. ref. [5], even in the case of an equilibrium plasma considered here. It is usually solved in the following way [5]. The test particle interactions in the plasma are split into two classes: those with *high*-momentum transfer corresponding to the collisions with plasma particles and those with *low*-momentum transfer dominated by the interactions with plasma collective modes. The latter processes, which can be treated in a classical way, are often called the processes of plasma polarization. In principle, one can study the particle energy loss in the approach which simultaneously treats the *low*- and *high*-momentum transfer processes. However, such an approach is rather complicated and we will consider here these processes separately. When the test particle energy (E) is comparable to the plasma temperature (T) one should also take into account the interaction with field fluctuations [5]. However, we assume $E \gg T$ throughout and neglect this. Let us start with the energy loss due to the plasma polarization effects.

The classical (nonquantum) expression for parton energy loss per unit time reads

$$\left(\frac{dE}{dt}\right)_p = \int d^3x j^a(x) E^a(x), \quad (1)$$

where E^a , $a=1, \dots, 8$ is the chromoelectric field induced in the plasma by the test particle current j^a . Assuming that the field E^a is *weak*, it can be calculated by means of the (abelian) Maxwell equations. After eliminating the chromomagnetic field one finds the following equation in the momentum space:

¹ E-mail address: MROW@PLEARN.bitnet.

^{#1} After completion of this paper the author was informed that conceptually similar but technically different calculations have been performed by Braaten and Thoma [4]. They have dealt with the QED plasma; however, their considerations, more elaborated than those presented here, can be also applied to the QCD case.

$$\left[\epsilon_{ij}(k) - \frac{\mathbf{k}^2}{\omega^2} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \right] E_j^a(k) = \frac{1}{i\omega} j_i^a(k), \tag{2}$$

where $i, j = 1, 2, 3$ are the space indices, $k = (\omega, \mathbf{k})$ is the wave four-vector and $\epsilon_{ij}(k)$ is the plasma chromodielectric tensor. As discussed in e.g. ref. [6], it is a color independent quantity, which for an isotropic medium can be decomposed as

$$\epsilon_{ij}(k) = \epsilon_L(k) \frac{k_i k_j}{\mathbf{k}^2} + \epsilon_T(k) \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right),$$

with $\epsilon_{L,T}(k)$ the longitudinal and transversal chromodielectric functions. With such a form of the dielectric tensor one can easily invert the matrix in the left-hand side of eq. (2).

The color current of the classical parton is

$$j^a(x) = q^a v \delta^{(3)}(\mathbf{x} - \mathbf{v}t), \tag{3}$$

where v is the parton velocity, which is supposed to be constant; the quantity q^a is the color charge of the test parton and $q^a q^a = g^2 C_F$ when the test parton is a quark and $q^a q^a = g^2 C_A$ for a gluon; C_F, C_A are the so-called Casimir invariants, which for the SU(3) group equal $\frac{4}{3}$ and 3, respectively; g is the QCD coupling constant, $\alpha_s = g^2/4\pi$.

Substituting the chromodielectric field found from eq. (2) and the Fourier transformed current (3) into eq. (1), one finds

$$\left(\frac{dE}{dt} \right)_p = -ig^2 C_{F,A} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\omega}{\mathbf{k}^2 \epsilon_L(k)} + \frac{v^2 - \omega^2/\mathbf{k}^2}{\omega [\epsilon_T(k) - \mathbf{k}^2/\omega^2]} \right), \tag{4}$$

where $\omega = \mathbf{k}v$. There is in eq. (4) a contribution from the test parton self-interaction, which, in fact, does not contribute to the parton energy loss. The simplest way to get ride of this effect is to subtract from the field generated in the plasma the field generated in the vacuum where $\epsilon_{L,T}(k) = 1$. Then, one gets

$$\left(\frac{dE}{dx} \right)_p = -ig^2 C_{F,A} \frac{1}{v} \int \frac{d^3k}{(2\pi)^3} \left[\frac{\omega}{\mathbf{k}^2} \left(\frac{1}{\epsilon_L(k)} - 1 \right) + \frac{v^2 - \omega^2/\mathbf{k}^2}{\omega} \left(\frac{1}{\epsilon_T(k) - \mathbf{k}^2/\omega^2} - \frac{1}{1 - \mathbf{k}^2/\omega^2} \right) \right], \tag{5}$$

where the energy loss per unit time has been changed into the energy loss per unit length; $v \equiv |v|$.

At this step Thoma and Gyulassy [3] substituted into eq. (5) the explicit form of the dielectric function taken from ref. [7], see also refs. [6,8-10], and performed a numerical integration. It appears, however, that this problem can be resolved in a simpler and more elegant analytical way.

Let us introduce the cylindrical coordinates (k_T, k_L) with k_L along the test parton velocity. Because $k_L = \omega/v$, eq. (5) can be rewritten as

$$\left(\frac{dE}{dx} \right)_p = g^2 C_{F,A} \frac{1}{v^2} \int_0^{k_0} dk_T k_T \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} \frac{\omega}{k_T^2 + \omega^2/v^2} \left[\frac{1}{\epsilon_L(k)} - 1 + k_T^2 v^2 \left(\frac{1}{\omega^2 \epsilon_T(k) - \mathbf{k}^2} - \frac{1}{\omega^2 - \mathbf{k}^2} \right) \right], \tag{6}$$

where $\mathbf{k}^2 = k_T^2 + \omega^2/v^2$. As will be seen below, the integral over k_T is logarithmically divergent, and consequently one has to introduce a cut-off k_0 . The appearance of this divergency is not surprising. The point is that the classical approach based on eq. (1) breaks for large k_T . When the wavelength of the electric field is comparable to the test parton de Broglie wavelength, the classical current expression (3) cannot be used. Further, the chromoelectric field given by eq. (1) is of *macroscopic* character, and consequently, it is meaningless to consider the field with a wavelength shorter than the average distance between plasma partons. We postpone the discussion of the parameter k_0 and proceed considering separately the contributions to the energy loss (6) due to the longitudinal and transversal fields. The longitudinal part reads

$$\left(\frac{dE_L}{dx}\right)_p = g^2 C_{F,A} \int_0^{k_0} \frac{dk_T k_T}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} \frac{\omega}{k_T^2 v^2 + \omega^2} \frac{1 - \epsilon_L(k)}{\epsilon_L(k)}. \tag{7}$$

We change the integration over ω , which extends from $-\infty$ to $+\infty$, into a contour integration in the complex ω -plane. The contour runs along the real axis and a *large* circle in the upper half-plane. The dielectric function $\epsilon_L(k)$ has zeros only in the lower half-plane of ω [11]. It means that the plasma oscillations around the global equilibrium state are damped. We assume that the antidamped modes found by several authors [12] are of nonphysical character. For the discussion of this controversial point see ref. [13]. Since all zeros of $\epsilon_L(k)$ are in the lower half-plane, the only pole of the function under the integral (7) appears in the upper half-plane at $ik_T v$, when $k^2 = k_T^2 + \omega^2/v^2 = 0$. Performing the integration along the contour, eq. (7) yields

$$\left(\frac{dE_L}{dx}\right)_p = g^2 C_{F,A} \int_0^{k_0} \frac{dk_T k_T}{2\pi} \frac{1 - \epsilon_L(\omega = ik_T v, \mathbf{k} = 0)}{2\epsilon_L(\omega = ik_T v, \mathbf{k} = 0)}. \tag{8}$$

The explicit form of the dielectric function in the long wave limit ($\mathbf{k} \rightarrow 0$) is well known as

$$\epsilon_L(\omega, \mathbf{k} = 0) = 1 - \frac{\omega_0^2}{\omega^2}, \tag{9a}$$

where ω_0 is the so-called plasma frequency, which for the baryonless plasma with the $SU(3)$ gauge group is (see e.g. ref. [6])

$$\omega_0^2 = \frac{1}{18} g^2 (N_f + 6) T^2, \tag{10a}$$

with N_f being the number of quark flavors. Eq. (9a) assumes that the imaginary part of the dielectric function is infinitesimally small. Such an assumption is correct as long as the coupling constant is *small*, because the imaginary part of the dielectric function is of higher order in the perturbative expansion than the real part (see below). Since the value of α_s is not smaller than 0.1 at the plasma temperature of interest (200–400 MeV), it might be important to take into account the finite imaginary part of ϵ_L . Using the transport equations with the collision terms in the relaxation time approximation one easily finds the following form of the dielectric function in the long-wave limit (see e.g. ref. [6]):

$$\epsilon_L(\omega, \mathbf{k} = 0) = 1 - \frac{\omega_0^2}{\omega(\omega + i\nu)}, \tag{9b}$$

where the parameter ν has been estimated as [6,10]

$$\nu \cong \frac{g^2}{4\pi} T. \tag{10b}$$

Substituting the dielectric function (9b) into eq. (8) one finds

$$\left(\frac{dE_L}{dx}\right)_p = -g^2 C_{F,A} \frac{\omega_0^2}{4\pi} \int_0^{k_0} \frac{dk_T k_T}{k_T^2 + k_T \nu + \omega_0^2}.$$

Let us observe here that the absolute value of the energy loss reaches maximum when $\nu \rightarrow 0$. After integration over k_T we obtain the following result:

$$\left(\frac{dE_L}{dx}\right)_p = -\frac{2\pi}{3v^2} C_{F,A} \left(1 + \frac{1}{6} N_f\right) \alpha_s^2 T^2 \times \left[\ln \frac{\omega_0^2 + k_0 v + k_0^2}{\omega_0^2} - \frac{2\nu}{\sqrt{4\omega_0^2 - \nu^2}} \left(\arctg \frac{2k_0 + \nu}{\sqrt{4\omega_0^2 - \nu^2}} - \arctg \frac{\nu}{\sqrt{4\omega_0^2 - \nu^2}} \right) \right]. \tag{11}$$

Let us now discuss the energy loss due to the transversal electric field which is

$$\left(\frac{dE_T}{dx}\right)_p = g^2 C_{F,A} v^2 \int_0^{k_0} \frac{dk_T k_T^3}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} \frac{\omega}{k_T^2 v^2 + \omega^2} \left(\frac{1}{\omega^2 [\epsilon_T(k) - 1/v^2] - k_T^2} - \frac{1}{\omega^2 (1 - 1/v^2) - k_T^2} \right). \tag{12}$$

The situation here is more complicated, when compared to the longitudinal field case, because the functions $\omega^2 (1 - 1/v^2) - k_T^2$ and $\omega^2 [\epsilon_T(k) - 1/v^2] - k_T^2$ have, in contrast to $\epsilon_L(k)$, zeros in the upper half-plane of ω . The zero of interest of the first function is $i\gamma v k_T$ with $\gamma \equiv (1 - v^2)^{-1/2}$. The positions of zeros of the second function can be determined only numerically due to a complicated form of the dielectric function, see e.g. refs. [6,8]. Fortunately, there is an approximate way to calculate the integral in eq. (12).

When $|\omega| \rightarrow \infty$ the two terms in eq. (12) cancel each other because $\epsilon_T \rightarrow 1$ in this limit. Therefore, the main contribution to the integral (12) can come from the *small- ω* region. Let us also observe that the two functions under the integral (12) are *large* when $\omega^2 - k^2$ and $\omega^2 \epsilon_T(k) - k^2$ are close to zero. Therefore, we consider the contribution to the integral (12) from the region where ω and k are both *small*. In this region the dielectric function weakly depends on k and can be approximated by $\epsilon_T(\omega, k=0)$. As is well known,

$$\epsilon_T(\omega, k=0) = \epsilon_L(\omega, k=0) \cong 1 - \frac{\omega_0^2}{\omega^2}. \tag{13}$$

With the dielectric function (13) one finds the zeros of $\omega^2 [\epsilon_T(k) - 1/v^2] - k_T^2$ as $\omega_{\pm} = \pm i\gamma v \sqrt{\omega_0^2 + k_T^2}$. Knowing all zeros in the upper half-plane of the function from eq. (12), the integral over ω can be performed. An elementary calculation shows that this integral equals exactly zero. Therefore, we conclude that the transversal field provides a negligible contribution to the energy loss. It corresponds to a well-known result [5] on the absence of Cherenkov radiation in an equilibrium electron-ion plasma.

In their numerical calculations Thoma and Gyulassy [3] identified the parameter k_0 with a test parton initial momentum and called the result “full expression for the energy loss”. As discussed under eq. (6), the classical formula (1) fails for *large* wave vectors of the chromodielectric field. Therefore, one gets a full expression of energy loss combining eq. (11) with that one of energy loss due to test parton collisions with thermalized plasma partons. This problem has been studied by Bjorken [1] and below we repeat his considerations with several minor improvements.

The energy loss per unit length of a parton due to collisions with plasma quarks and gluons is

$$\left(\frac{dE}{dx}\right)_c = \sum_i \int \frac{d^3k}{(2\pi)^3} n_i(k) [\text{flux factor}] \int dt \frac{d\sigma^i}{dt} \nu, \tag{14}$$

where the summation runs over quarks, antiquarks and gluons; $n_i(k)$ is the Bose-Einstein, or Fermi-Dirac distribution of the plasma partons, $\nu = E - E'$ is the energy transfer from the fast test parton to the thermalized one and $d\sigma^i/dt$ is the parton-parton cross section, which in the limit $s \gg t$ (s, t are the standard Mandelstam variables) reads

$$\frac{d\sigma^i}{dt} = C_i \frac{2\pi\alpha_s^2}{t^2},$$

where C_i equals $\frac{4}{9}$, 1 and $\frac{9}{4}$ for q-q, q-g and g-g collisions, respectively. Following Bjorken one introduces the effective parton distribution $n = \frac{2}{3} n_q + \frac{3}{2} n_g$ and the effective cross section $d\sigma/dt = C_{F,A} \pi \alpha_s^2 / t^2$ with C_F and C_A

equal, as previously, $\frac{4}{3}$ for a test quark and 3 for a test gluon.

When all partons are massless, there is, in the limit $E, E' \gg k$, several kinematical simplifications: $s = 2kE(1 - \cos \theta)$, where θ is the angle between momenta of colliding partons and k denotes $|\mathbf{k}|$, (not the wave four-vector as previously), [flux factor] = $1 - \cos \theta$ and $|t| = s\nu/E$. Thus, eq. (14) can be manipulated to

$$\left(\frac{dE}{dx}\right)_c = -C_{F,A} \pi \alpha_s^2 \int \frac{d^3k}{(2\pi)^3} \frac{n(k)}{2k} \ln \frac{\nu_{\max}}{\nu_{\min}}, \tag{15}$$

where the ν_{\max} and ν_{\min} are maximal and minimal energy transfers. When the partons collide in vacuum the minimal energy transfer is zero. It happens when the collision proceeds at infinite impact parameter. In the plasma, however, the interaction is screened at distances exceeding the Debye radius, and consequently there appears a finite minimal energy transfer. We choose the minimal momentum transfer $(\sqrt{-t})_{\min}$ equal k_0 , which has appeared previously as a maximal wave vector of the classical chromodielectric field. Therefore,

$$\nu_{\min} = \frac{Ek_0^2}{s} = \frac{k_0^2}{2k(1 - \cos \theta)},$$

and the lower limit of the integration over k is taken as k_0 . Bjorken [1] assumed that $\nu_{\max} = \frac{1}{2}E$. Now we integrate over \mathbf{k} in eq. (15) substituting an average momentum $\langle k \rangle$ equal

$$\langle k \rangle = \frac{\int d^3k n(k)}{\int d^3k n(k)/k}$$

instead of k and $\cos \theta = 0$ under the logarithm. Assuming $T \gg k_0$ one finds

$$\left(\frac{dE}{dx}\right)_c = -C_{F,A} \pi \left(1 + \frac{1}{6}N_f\right) \alpha_s^2 T^2 \ln \frac{2\langle k \rangle E}{k_0^2}, \tag{16}$$

where $\langle k \rangle \cong 2T$. As previously the plasma is assumed baryonless. As will be shown below, $k_0 \geq T$ and then the right-hand side of eq. (16) should be multiplied by a factor approximately equal to

$$\frac{(12N_f + 16) \int_{k_0}^{\infty} dk k e^{-k/T}}{12N_f \int_0^{\infty} dk k (e^{k/T} + 1)^{-1} + 16 \int_0^{\infty} dk k (e^{k/T} - 1)^{-1}} = \frac{12(3N_f + 4)}{\pi^2(3N_f + 8)} e^{-k_0/T} (1 + k_0/T).$$

In this case the average momentum can be approximated as

$$\langle k \rangle = \frac{2T^2 + 2k_0T + k_0^2}{T + k_0}. \tag{17}$$

The parameter k_0 represents a maximal momentum transfer in interactions with collective modes, on the other hand, a minimal momentum transfer in collisions with thermalized partons. The natural choice of k_0 is the inverse screening (Debye) radius. Thus, $k_0 = \sqrt{3} \omega_0$ (see e.g. ref. [6]), and eq. (10) yields

$$k_0^2 = 4\pi \left(1 + \frac{1}{6}N_f\right) \alpha_s T^2. \tag{18}$$

Let us observe that for two quark flavors $k_0 > T$ as long as $\alpha_s > 0.06$.

Combining eqs. (11) and (16) (the latter with the correction factor) we can write down the complete expression for the energy loss of an ultrarelativistic parton in the equilibrium QGP. Namely,

$$\frac{dE}{dx} = -C_{F,A} \alpha_s k_0^2 \left[\frac{3(3N_f + 4)}{\pi^2(3N_f + 8)} e^{-k_0/T} \left(1 + \frac{k_0}{T}\right) \ln \frac{2\langle k \rangle E}{k_0^2} + A \right], \tag{19}$$

where k_0 and $\langle k \rangle$ are given by eqs. (18) and (17), respectively, and

$$A \equiv \frac{1}{6} \left[\ln(4 + \sqrt{3}x) - \frac{2x}{\sqrt{4-x^2}} \left(\operatorname{arctg} \frac{2\sqrt{3+x}}{\sqrt{4-x^2}} - \operatorname{arctg} \frac{x}{\sqrt{4-x^2}} \right) \right]$$

with

$$x \equiv \frac{\nu}{\omega_0} \cong \left(\frac{3\alpha_s}{4\pi(1 + \frac{1}{6}N_f)} \right)^{1/2}.$$

The authors of ref. [3] have made two steps, which are not quite correct in our opinion. They have identified the parameter k_0 with energy E , and then have treated the polarization energy loss as a complete energy loss. However, their numerical results are not very different from that given by eq. (19). For example, eq. (19) predicts $dE/dx \cong 0.19$ GeV/fm for a 20 GeV light quark at $\alpha_s = 0.2$, $N_f = 2$ and $T = 0.25$ GeV, while the respective number given in ref. [3] approximately equals 0.24 GeV/fm. Thus, the conclusion of refs. [1,3] remains unchanged: the energy loss of the parton in the equilibrium QGP is much smaller than the parton energy loss in vacuum, which appears due to confining forces. The latter can be identified with the string tension equal about 1 GeV/fm. The situation probably changes when one deals with a strongly turbulent plasma, which might be the case for the plasma produced in nuclear collisions. Then one expects a much larger energy loss [5].

Let us briefly summarize our considerations. The energy loss of a high-energy parton (quark or gluon) going through the baryonless quark-gluon plasma in equilibrium has been discussed. The elastic collisions with thermalized partons and the interactions with plasma collective modes have been considered in detail. In both cases completely analytical solutions have been found. It has been argued that the transversal modes provide a negligible contribution. Several points have been treated in a different way than in the previous study but the numerical results are rather similar and confirm the conclusion of a very small parton energy loss in the deconfined phase.

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