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Color filamentation in ultrarelativistic heavy-ion collisions

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Abstract

We study color fluctuations in the quark-gluon plasma produced at the early stage of nucleus-nucleus collision at RHIC or LHC. The fluctuating color current, which flows along the beam, can be very *large* due to the strong anisotropy of the parton momentum distribution. A specific fluctuation, which splits the parton system into the current filaments parallel to the beam direction, is argued to grow exponentially. The physical mechanism responsible for the phenomenon, which is known as a filamentation instability, is discussed.

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In the near future the nucleus-nucleus collisions will be studied experimentally at the accelerators of a new generation: Relativistic Heavy-Ion Collider (RHIC) at Brookhaven and Large Hadron Collider (LHC) at CERN. The collision energy will be larger by one or two orders of magnitude than that one of the currently operating machines. A copious production of partons, mainly gluons, due to hard and semihard processes is expected in the heavy-ion collisions at this new energy domain [1]. Thus, one deals with the many-parton system at the early stage of the collision. The system is on average locally colorless but random fluctuations can break the neutrality. Since the system is initially far from equilibrium specific color fluctuations can exponentially grow in time and then noticeably influence the system evolution. While the very existence of such instabilities, similar to those which are known from the electron-ion plasma, see e.g. [2], is fairly obvious and was commented upon long time ago [3], it is far less trivial to find those instabilities which are relevant for the parton system produced in ultrarelativistic heavy-ion collisions.

A system of two interpenetrating beams of nucleons [4] or partons [5,6] was argued to be unstable with respect to the so-called filamentation or Weibel instability [7]. Unfortunately, such a system appears rather unrealistic from the experimental point of view. However, we have recently argued [8] that the filamentation can occur under weaker conditions which are very probable for heavy-ion collisions at RHIC and LHC. Instead of the two streams of partons, it appears sufficient to assume a strongly anisotropic momentum distribution. We have then found the modes exponentially growing in time. Specifically, we have analysed the dispersion equation which for the anisotropic plasma is

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$$\det|k^2\delta^{nm} - k^n k^n - \omega^2 \epsilon^{nm}(\omega, \mathbf{k})| = 0 ,$$
$$n, m = x, y, z , \quad (1)$$

where $k \equiv (\omega, \mathbf{k})$ is the four-wave vector and ϵ^{nm} is the chromodielectric tensor derived [6,9] within the linear response approach of the QCD transport theory [9,10] as

$$\epsilon^{nm}(\omega, \mathbf{k}) = \delta^{nm} + \frac{g^2}{2\omega} \int \frac{d^3p}{(2\pi)^3} \frac{v^n}{\omega - \mathbf{k}\mathbf{v} + i0^+} \times \frac{\partial f(\mathbf{p})}{\partial p^l} \left[\left(1 - \frac{\mathbf{k}\mathbf{v}}{\omega}\right)\delta^{lm} + \frac{k^l v^m}{\omega} \right] , \quad (2)$$

where g is the QCD coupling constant, $p = (E_p, \mathbf{p})$ is the parton four-momentum with $E_p = |\mathbf{p}|$ being the energy of the massless quark or gluon; $\mathbf{v} \equiv \mathbf{p}/E_p$ denotes the parton velocity and $f(\mathbf{p})$ represents the effective parton distribution function (see below). The chromodielectric tensor (2) is a unit matrix in the color space i.e. $\epsilon^{nm}_{ij} = \delta^{ij}\epsilon^{nm}$ with $i, j = 1, 2, 3$ being the color indices in the fundamental representation. In spite of its similarity to the analogous QED expression, the tensor (2) takes into account the non-Abelian effects. Namely, due to the gluon-gluon coupling there is the gluon contribution to the distribution function from Eq. (2). It has been also shown [11] that the semiclassical QCD transport theory effectively incorporates the resummation over the so-called hard thermal loops. Thus, the formula (2) is more reliable than it looks at first glance.

When $f(\mathbf{p})$ is strongly anisotropic there are solutions of the dispersion Eq. (1) representing the unstable modes [8] which split the system into the filaments parallel to the beam. The aim of this note is to discuss how these modes are initiated. We show that the fluctuations, which act as seeds of the filamentation, are *large*, much larger than in the equilibrium plasma. Since the system of interest is far from equilibrium, the fluctuations are not determined by the chromodielectric permeability tensor (2). The fluctuation-dissipation theorem does not hold in such a case. Thus, we derive the color current correlation function which provides the fluctuation spectrum.

The distribution functions of quarks $Q_{ij}(t, \mathbf{x}, \mathbf{p})$, antiquarks $\bar{Q}_{ij}(t, \mathbf{x}, \mathbf{p})$, and gluons $G_{ab}(t, \mathbf{x}, \mathbf{p})$ with $i, j = 1, 2, 3$ and $a, b = 1, 2, ..., 8$ are [9,10] matrices in the color space; t and $\mathbf{x}$ denote the time and position. The color current expressed through these functions reads [9,10]

$$j^\mu_a(t, \mathbf{x}) = \frac{1}{2} g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \times \left(\tau^a_{ji}\left(Q_{ij}(t, \mathbf{x}, \mathbf{p}) - \bar{Q}_{ij}(t, \mathbf{x}, \mathbf{p})\right) + i f^{abc} G_{bc}(t, \mathbf{x}, \mathbf{p}) \right) ,$$

where τ^a is the $SU(3)$ group generator and f^{abc} the respective structure constant. We assume that the quark-gluon plasma is on average locally colorless, homogeneous, and stationary. Thus, the distribution functions averaged over ensemble are of the form

$$\langle Q_{ij}(t, \mathbf{x}, \mathbf{p}) \rangle = \delta^{ij} n(\mathbf{p}) ,$$
$$\langle \bar{Q}_{ij}(t, \mathbf{x}, \mathbf{p}) \rangle = \delta^{ij} \bar{n}(\mathbf{p}) ,$$
$$\langle G_{ab}(t, \mathbf{x}, \mathbf{p}) \rangle = \delta^{ab} n_g(\mathbf{p}) ,$$

which give the zero average color current.

We study the fluctuations of the color current generalizing a well-known approach to the fluctuating electric current [2]. For a system of noninteracting quarks and gluons we have derived (in the classical limit) the following expression of the current correlation tensor

$$M^{\mu\nu}_{ab}(t, \mathbf{x}) \stackrel{\text{def}}{=} \langle j^\mu_a(t_1, \mathbf{x}_1) j^\nu_b(t_2, \mathbf{x}_2) \rangle = \frac{1}{8} g^2 \delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p}) \, \delta^{(3)}(\mathbf{x} - \mathbf{v}t) , \quad (3)$$

where the effective parton distribution function $f(\mathbf{p})$ equals $n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 6n_g(\mathbf{p})$ and $(t, \mathbf{x}) \equiv (t_2 - t_1, \mathbf{x}_2 - \mathbf{x}_1)$. Due to the average space-time homogeneity the correlation tensor depends only on the difference $(t_2 - t_1, \mathbf{x}_2 - \mathbf{x}_1)$. The physical meaning of the formula (3) is transparent. The space-time points $(t_1, \mathbf{x}_1)$ and $(t_2, \mathbf{x}_2)$ are correlated in the system of noninteracting particles if the particles fly from $(t_1, \mathbf{x}_1)$ to $(t_2, \mathbf{x}_2)$. For this reason the delta $\delta^{(3)}(\mathbf{x} - \mathbf{v}t)$ is present in the formula (3). The momentum integral of the distribution function simply represents the summation over particles. The fluctuation spectrum is found as a Fourier transform of the tensor (3) i.e.

$$M^{\mu\nu}_{ab}(\omega,\boldsymbol{k}) = \frac{1}{8} g^2 \delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\boldsymbol{p}) \times 2\pi\delta(\omega - \boldsymbol{kv}) . \quad (4)$$

When the system is in equilibrium the fluctuations are given, according to the fluctuation-dissipation theorem, by the respective response function. For $f(\boldsymbol{p})$ being the classical equilibrium distribution function one indeed finds from Eqs. (2), (4) the standard fluctuation-dissipation relation [2] valid in the g^2-order. For example,

$$M^{00}_{ab}(\omega,\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2\pi} \frac{T}{\omega} \,\mathrm{Im}\, \epsilon^L_{ab}(\omega,\boldsymbol{k}) ,$$

where T is the temperature and ϵ^L_{ab} represents the longitudinal part of the chromodielectric tensor with the color indices in the adjoint representation.

We model the parton momentum distribution at the early stage of ultrarelativistic heavy-ion collision in two forms:

$$f(\boldsymbol{p}) = \frac{1}{2Y}\Theta(Y-y)\Theta(Y+y)\, h(p_\perp)\, \frac{1}{p_\perp \,\mathrm{ch} y} , \quad (5)$$

and

$$f(\boldsymbol{p}) = \frac{1}{2\mathcal{P}}\Theta(\mathcal{P} - p_\parallel)\Theta(\mathcal{P} + p_\parallel)\, h(p_\perp) , \quad (6)$$

where y, $p_\parallel$, and $p_\perp$ denote the parton rapidity, the longitudinal and transverse momenta, respectively. The parton momentum distribution (5) corresponds to the rapidity distribution which is flat in the interval $(-Y, Y)$. The distribution (6) is flat for the longitudinal momentum $-\mathcal{P} < p_\parallel < \mathcal{P}$. We do not specify the transverse momentum distribution $h(p_\perp)$, which is assumed to be of the same shape for quarks and gluons, because it is sufficient for our considerations to demand that the distributions (5), (6), are strongly elongated along the z-axis i.e. $e^Y \gg 1$ and $\mathcal{P} \gg \langle p_\perp \rangle$.

The QCD-based computations, see e.g. [1], show that the rapidity distribution of partons produced at the early stage of heavy-ion collisions is essentially gaussian with the width of about one to two units. When the distribution (5) simulates the gaussian one, Y does not measure the size of the 'plateau' but rather the range over which the partons are spread. If one takes the gaussian distribution of the variance σ and the distribution (5) of the same variance, then $Y = \sqrt{3}\,\sigma$.

The rapidity range covered by the distribution (5) is very large – the estimated value of Y is 2.5 for RHIC and 5.0 for LHC [12]. One may wonder how the partons of vastly different rapidities, which interact with each other very weakly, can contribute to the fluctuation initiating the instability. It is important to observe that the fluctuations grow due to the *collective* interaction of many partons and not due to the individual parton-parton collisions. As we discuss in detail below, the fluctuation causes the current which generates the chromomagnetic field. The field in turn slightly deflects the partons and the current grows. Thus, the individual parton-parton interactions do not matter very much.

The parton system described by the distribution functions (5), (6) is assumed to be homogeneous and stationary. Applicability of this assumption is very limited because there is a correlation between the parton longitudinal momentum and its position, i.e. partons with very different momenta will find themselves in different regions of space shortly after the collision. However, one should remember that we consider the parton system at a very early stage of the collision, soon after the Lorentz contracted ultrarelativistic nuclei traverse each other. At this stage partons are most copiously produced but do not have enough time to escape from each other. Thus, the assumption of homogeneity holds for the space-time domain of the longitudinal size, say, 2–3 fm and life time 2–3 fm/c. As shown in our paper [8], this time is long enough for the instability to occur.

After these comments let us calculate the correlation tensor (4) for the distribution functions (5), (6). Due to the symmetry $f(\boldsymbol{p}) = f(-\boldsymbol{p})$ of these distributions, the tensor $M^{\mu\nu}$ is diagonal i.e. $M^{\mu\nu} = 0$ for $\mu \neq \nu$. Since the average parton longitudinal momentum is much bigger than the transverse one, it obviously follows from Eq. (4) that the largest fluctuating current appears along the z-axis. Therefore, we discuss the M^{zz} component of the correlation tensor. $M^{zz}(\omega,\boldsymbol{k})$ depends on the $\boldsymbol{k}$-vector orientation and there are two generic cases: $\boldsymbol{k} = (k_x, 0, 0)$ and $\boldsymbol{k} = (0, 0, k_z)$. The inspection of Eq. (4) shows that the fluctuations with $\boldsymbol{k} = (k_x, 0, 0)$ are much larger than those with $\boldsymbol{k} = (0, 0, k_z)$. Thus, let us consider $M^{zz}(\omega, k_x)$. Substituting the distributions (5), (6) into (4) one finds after azimuthal integration that $M^{zz}_{ab}(\omega, k_x)$ reaches the maximal values for $\omega^2 \ll k_x^2$. So, we compute M^{zz}_{ab} at

$\omega = 0$. Keeping in mind that $e^Y \gg 1$ and $\mathcal{P} \gg \langle p_\perp \rangle$ we get the following approximate expressions for the flat y- and $p_\parallel$-distributions:

$$M_{ab}^{zz}(\omega = 0, k_x) = \frac{1}{8} g^2 \delta^{ab} \frac{e^Y}{Y} \frac{\langle \rho \rangle}{|k_x|}, \quad (7)$$

$$M_{ab}^{zz}(\omega = 0, k_x) = \frac{1}{8} g^2 \delta^{ab} \frac{\mathcal{P}}{\langle p_\perp \rangle} \frac{\langle \rho \rangle}{|k_x|}, \quad (8)$$

where $\langle \rho \rangle$ is the effective parton density given as

$$\langle \rho \rangle \equiv \int \frac{d^3p}{(2\pi)^3} f(\mathbf{p}) = \frac{1}{4\pi^2} \int_0^\infty dp_\perp p_\perp \, h(p_\perp)$$
$$= \frac{1}{3} \langle \rho \rangle_{q\bar{q}} + \frac{3}{4} \langle \rho \rangle_g ,$$

with $\langle \rho \rangle_{q\bar{q}}$ denoting the average density of quarks and antiquarks, and $\langle \rho \rangle_g$ that of gluons. For the flat $p_\parallel$-case we have also used the approximate equality

$$\int_0^\infty dp_\perp \, h(p_\perp) \cong \frac{1}{\langle p_\perp \rangle} \int_0^\infty dp_\perp p_\perp \, h(p_\perp)$$

to get the expression (8). It is instructive to compare the results (7), (8) with the analogous one for the equilibrium plasma which is

$$M_{ab}^{zz}(\omega = 0, k_x) = \frac{\pi}{16} g^2 \delta^{ab} \frac{\langle \rho \rangle}{|k_x|} .$$

One sees that *the current fluctuations in the anisotropic plasma are amplified by the large factor* which is e^Y/Y or $\mathcal{P}/\langle p_\perp \rangle$. With the estimated value of Y 2.5 for RHIC and 5.0 for LHC [12], the amplification factor e^Y/Y equals 4.9 and 29.7, respectively.

Following [13] let us now explain why the fluctuation, which contributes to $M_{ab}^{zz}(\omega = 0, k_x)$, grows in time. The form of the fluctuating current is

$$j_a(x) = j_a \, \hat{\mathbf{e}}_z \cos(k_x x) , \quad (9)$$

where $\hat{\mathbf{e}}_z$ is the unit vector in the z-direction. Thus, there are current filaments of the thickness $\pi/|k_x|$ with the current flowing in the opposite directions in the neighboring filaments. For the purpose of a qualitative argumentation presented here the chromodynamics is treated as an eightfold electrodynamics. Then,

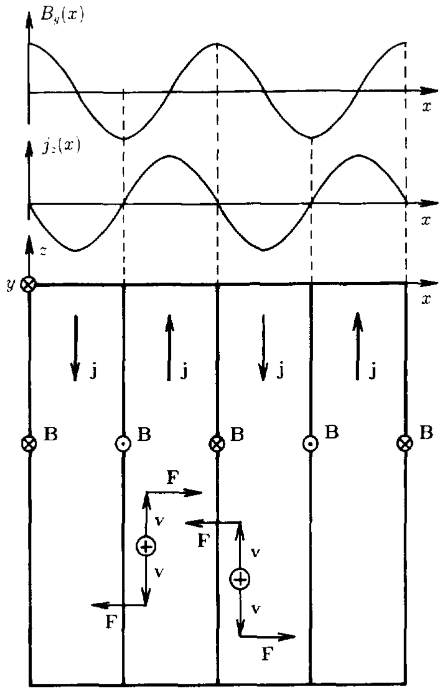

Fig. 1. The mechanism of filamentation. The phenomenon is, for simplicity, considered in terms of the electrodynamics. The fluctuating current generates the magnetic field acting on the positively charged particles which in turn contribute to the current (see text). ⊗ and ⊙ denote the parallel and, respectively, antiparallel orientation of the magnetic field with respect to the y-axis.

the magnetic field generated by the current (9) is given as

$$\mathbf{B}_a(x) = \frac{j_a}{k_x} \hat{\mathbf{e}}_y \sin(k_x x) ,$$

while the Lorentz force acting on the partons, which fly along the beam, equals

$$\mathbf{F}(x) = q_a \, \mathbf{v} \times \mathbf{B}_a(x) = -q_a \, v_z \frac{j_a}{k_x} \hat{\mathbf{e}}_x \sin(k_x x) ,$$

where q_a is the color charge. One observes, see Fig. 1, that the force distributes the partons in such a way that those, which positively contribute to the current in a given filament, are focused to the filament center while those, which negatively contribute, are moved to the neighboring one. Thus, the initial current is growing.

One asks whether the color instabilities are detectable in ultrarelativistic heavy-ion collisions. The answer seems to be positive because the occurrence of the filamentation breaks the azimuthal symmetry of the system and hopefully will be visible in the final state. The azimuthal orientation of the wave vector will change from one collision to another while the instability growth will lead to the energy transport along this vector (the Poynting vector points in this direction). Consequently, one expects significant variation of the transverse energy as a function of the azimuthal angle. This expectation is qualitatively different than that based on the parton cascade simulations [1], where the fluctuations are strongly damped due to the large number of uncorrelated partons. Due to the collective character of the filamentation instability the azimuthal symmetry will be presumably broken by a flow of large number of particles with relatively small transverse momenta. The jets produced in hard parton-parton interactions also break the azimuthal symmetry. However, the symmetry is broken in this case due to a few particles with large transverse momentum. The problem obviously needs further studies but the event-by-event analysis of the nuclear collision seems to give a chance to observe the color instabilities in the experiments planed at RHIC and LHC.

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