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Generalizing Φ-measure of event-by-event fluctuations in high-energy heavy-ion collisions

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Abstract

The Φ-measure of event-by-event fluctuations in high-energy heavy-ion collisions corresponds to the second moment of the fluctuating quantity distribution of interest. It is shown that the measure based on the third moment preserves the properties of Φ but those related to the higher moments do not. In particular, only the second and third moment measures are intensive as thermodynamic quantities. The Φ_2- and Φ_3-measure of $p_\perp$-fluctuations are computed for the hadron gas in equilibrium and the results are analyzed in context of the experimental data. © 1999 Published by Elsevier Science B.V. All rights reserved.

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Large acceptance detectors allow one for a detailed analysis of individual collisions of heavy-ions at high-energies. Due to hundreds or even thousands of particles produced in these collisions, variety of statistical methods can be applied. There are several interesting proposals [1–5] to use the fluctuation measurements as a potential source of information on the collision dynamics. However, one faces a problem how to disentangle the 'dynamical' fluctuations from the 'trivial' geometrical ones due to the impact parameter variation. The latter fluctuations are large and dominate the fluctuations of all extensive event characteristics such as multiplicity or transverse energy. Using the fluctuation (or correlation) measure Φ, which was introduced in our paper [1], resolves the problem in a specific way. By construction, Φ is exactly the same for nucleon-nucleon (N–N) and nucleus-nucleus (A–A) collisions if the A–A collision is a simple superposition of N–N interactions. Consequently, Φ is independent of the centrality of A–A collision in such a case. On the other hand, Φ equals zero when the inter-particle correlations are entirely absent. The Φ-measure can be applied to the fluctuations of kinematical quantities such as the event energy or transverse momentum and to the fluctuations of event chemical composition [6,7].

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The NA49 Collaboration plans to study the chemical fluctuations in a near future [8] while the data on the transverse momentum fluctuations have been already published [9,10]. The value of $\Phi_{p_\perp}$ in the central Pb–Pb collisions at 158 GeV per nucleon has appeared to be smaller than expected. It has been also claimed [10] that the correlations, which are of short range in the momentum space, are responsible for the nonzero positive value of $\Phi_{p_\perp}$ being observed. The result has been widely discussed [11–16]. In particular, our calculations of $\Phi_{p_\perp}$ in the equilibrium hadron gas show [14] that the positive value of $\Phi_{p_\perp}$ appears due to the boson statistics of pions. When the hadronic system at freeze-out is identified with the pion gas, the calculated $\Phi_{p_\perp}$ slightly overestimates the experimental value [10] but, as discussed here, the inclusion of the pions which come from the resonance decays removes the discrepancy.

The Φ-measure corresponds to the second moment of the fluctuating quantity, say the event transverse momentum. Recently, it has been suggested [17] to use the higher moments in an analogous way. However, the authors of [17] have not realized that the fluctuation measures based on the higher moments, except that of the third one, do not possess a key property of Φ which has been mentioned above. Namely, $\Phi_{\rm NN} = \Phi_{\rm AA}$ if the A–A collision is a simple superposition of N–N interactions. When treated as thermodynamic quantities, the second and third moment measures are intensive while the higher moment ones are not. The aim of this note is to substantiate the comment and to discuss usefulness of the third moment measure. We focus our attention on the $p_\perp$-fluctuations which have been already studied experimentally [9,10].

Let us first introduce the Φ-measure. One defines the single-particle variable $z \overset{\rm def}{=} x - \bar{x}$ with the overline denoting averaging over a single particle inclusive distribution. The event variable Z, which is a multiparticle analog of z, is defined as $Z \overset{\rm def}{=} \sum_{i=1}^{N}(x_i - \bar{x})$, where the summation runs over particles from a given event. By construction, $\langle Z \rangle = 0$, where $\langle \dots \rangle$ represents averaging over events. Finally, the Φ-measure is defined in the following way

$$\Phi \overset{\rm def}{=} \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}} \,. \tag{1}$$

There is an obvious generalization of the definition (1) suggested in [17]. Namely,

$$\Phi_n \overset{\rm def}{=} \left(\frac{\langle Z^n \rangle}{\langle N \rangle} \right)^{1/n} - \left(\overline{z^n}\right)^{1/n} . \tag{2}$$

The fact that $\Phi_n = 0$, when no inter-particle correlations are present, is evident [1,17]. We are now going to show that Φ_3 as Φ_2, in contrast to Φ_n with $n > 3$, possesses another nontrivial property which is so useful in the data analysis. Namely, Φ_2 and Φ_3 are *independent* of the source number distribution if the particles originate from several identical sources. Then, Φ_2 and Φ_3 are independent of the impact parameter if the A–A collision is a superposition of N–N interactions. Let us prove this property.

$P_1(X)$ is the normalized distribution of $X \overset{\rm def}{=} \sum_{i=1}^{N} x_i$, when the particles come from the single source. If we have k sources distributed according to p_k, the X-distribution reads

$$P(X) = \sum_{k=1}^{\infty} p_k \int dX_1 \dots dX_k \, P_1(X_1) \dots P_1(X_k) \times \delta(X - (X_1 + \dots + X_k)) .$$

The moments of $P(X)$ are

$$\langle X^n \rangle \overset{\rm def}{=} \int dX\, X^n P(X) = (-i)^n \frac{d^n}{dQ^n} \mathscr{F}(Q) \bigg|_{Q=0} , \tag{3}$$

where the generating function $\mathscr{F}$ equals

$$\mathscr{F}(Q) \overset{\rm def}{=} \int dX\, e^{iQX} P(X) = \sum_{k=1}^{\infty} p_k [\mathscr{F}_1(Q)]^k$$

with

$$\mathscr{F}_1(Q) \overset{\rm def}{=} \int dX\, e^{iQX} P_1(X) .$$

Using Eq. (3), one computes the first five moments of $P(X)$ as

$$\langle X \rangle = \langle k \rangle \langle X \rangle_1 ,$$

$$\langle X^2 \rangle = \langle k \rangle \langle X^2 \rangle_1 + \langle k(k-1) \rangle \langle X \rangle_1^2 ,$$

$$\langle X^3 \rangle = \langle k \rangle \langle X^3 \rangle_1 + 3\langle k(k-1) \rangle \langle X^2 \rangle_1 \langle X \rangle_1 + \langle k(k-1)(k-2) \rangle \langle X \rangle_1^3 ,$$

$$\langle X^4 \rangle = \langle k \rangle \langle X^4 \rangle_1 + 4\langle k(k-1) \rangle \langle X^3 \rangle_1 \langle X \rangle_1 + 3\langle k(k-1) \rangle \langle X^2 \rangle_1^2 + 3\langle k(k-1)(k-2) \rangle \langle X^2 \rangle_1 \langle X \rangle_1^2 + \langle k(k-1)(k-2)(k-3) \rangle \langle X \rangle_1^4 ,$$

$$\langle X^5 \rangle = \langle k \rangle \langle X^5 \rangle_1 + 5\langle k(k-1) \rangle \langle X^4 \rangle_1 \langle X \rangle_1 + 10\langle k(k-1) \rangle \langle X^3 \rangle_1 \langle X^2 \rangle_1 + 7\langle k(k-1)(k-2) \rangle \langle X^3 \rangle_1 \langle X \rangle_1^2 + 9\langle k(k-1)(k-2) \rangle \langle X^2 \rangle_1^2 \langle X \rangle_1 + 7\langle k(k-1)(k-2)(k-3) \rangle \times \langle X^2 \rangle_1 \langle X \rangle_1^3 + \langle k(k-1)(k-2)(k-3)(k-4) \rangle \times \langle X \rangle_1^5 ,$$

where

$$\langle X^n \rangle_1 \overset{\text{def}}{=} \int dX X^n P_1(X) \quad \text{and} \quad \langle k^n \rangle \overset{\text{def}}{=} \sum_{k=1}^{\infty} k^n p_k .$$

Applying these formulas to the variable Z and taking into account that by definition $\langle Z \rangle = \langle Z \rangle_1 = 0$, we get

$$\langle Z^2 \rangle = \langle k \rangle \langle Z^2 \rangle_1 , \quad \langle Z^3 \rangle = \langle k \rangle \langle Z^3 \rangle_1 ,$$

$$\langle Z^4 \rangle = \langle k \rangle \langle Z^4 \rangle_1 + 3\langle k(k-1) \rangle \langle Z^2 \rangle_1^2 ,$$

$$\langle Z^5 \rangle = \langle k \rangle \langle Z^5 \rangle_1 + 10\langle k(k-1) \rangle \langle Z^3 \rangle_1 \langle Z^2 \rangle_1 .$$

Since $\langle N \rangle = \langle k \rangle \langle N \rangle_1$, one finds that

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{\langle Z^2 \rangle_1}{\langle N \rangle_1} , \quad \frac{\langle Z^3 \rangle}{\langle N \rangle} = \frac{\langle Z^3 \rangle_1}{\langle N \rangle_1} ,$$

but analogous formulas do not hold for $\langle Z^4 \rangle$ and $\langle Z^5 \rangle$. Instead,

$$\frac{\langle Z^4 \rangle}{\langle N \rangle} = \frac{\langle Z^4 \rangle_1}{\langle N \rangle_1} + 3\frac{\langle k(k-1) \rangle}{\langle k \rangle} \frac{\langle Z^2 \rangle_1^2}{\langle N \rangle_1} ,$$

$$\frac{\langle Z^5 \rangle}{\langle N \rangle} = \frac{\langle Z^5 \rangle_1}{\langle N \rangle_1} + 10\frac{\langle k(k-1) \rangle}{\langle k \rangle} \frac{\langle Z^3 \rangle_1 \langle Z^2 \rangle_1}{\langle N \rangle_1} .$$

Therefore, Φ_2 and Φ_3 are independent of the source number distribution while Φ_4, Φ_5 and, obviously, Φ_n with $n > 5$ do depend on p_k. The inclusive distribution, which determines $\overline{z^n}$, is, of course, independent of the source distribution. The above results also show that Φ_2 and Φ_3 are intensive quantities, i.e. they are independent of the system size, while Φ_n with $n > 3$ are not. Indeed, when the source number is fixed, $\langle k^l \rangle = k^l$ and one observes that only Φ_2 and Φ_3 do not dependent on k. Let us note here that the independence of k is, in principle, a weaker requirement than the independence of p_k.

The Φ_2-measure is sensitive to the fluctuations or correlations of various origin. For example, it acquires a finite value, which is positive for bosons and negative for fermions, due to the quantum statistics [14]. The correlation between the particle multiplicity and their kinematical characteristics also influences Φ_2 [1]. The energy-momentum conservation and presence of the collective motion introduces additional inter-particle correlations. Thus, one concludes that the nonvanishing value of Φ_2 signals the existence of the correlations in the system but it does not explain their origin. In such a situation, Φ_3 seems to be very useful. Indeed, simultaneous measurements of Φ_2 and Φ_3 might help to identify the fluctuations which dominate in the system. For this purpose one should theoretically estimate contributions of various correlations to Φ_2 and Φ_3

In our paper [14] we have discussed how to compute Φ_2 in the ideal quantum gas. Now, we are going to extend these calculations to the case of Φ_3. For comparison, we also present here the earlier published [14] results on Φ_2. At first, the energy fluctuations are considered. Therefore, the single particle variable x is identified with the particle energy E. Then, one immediately finds that

$$\overline{z^n} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} (E - \overline{E})^n \frac{1}{\lambda^{-1} e^{\beta E} \pm 1} , \tag{4}$$

where the single particle average energy is

$$\bar{E} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{E}{\lambda^{-1} e^{\beta E} \pm 1} ,$$

while the particle density ρ equals

$$\rho = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\lambda^{-1} e^{\beta E} \pm 1} ; \tag{5}$$

$\beta \equiv T^{-1}$ is the inverse temperature; $\lambda \equiv e^{\beta\mu}$ denotes the fugacity and μ the chemical potential; $E \equiv \sqrt{m^2 + \boldsymbol{p}^2}$ with m being the particle mass and $\boldsymbol{p}$ its momentum; the upper sign is for fermions while the lower one for bosons.

Since $Z = U - N\bar{E}$, where U is the system energy, $\langle Z^2 \rangle$ and $\langle Z^3 \rangle$ are computed as

$$\langle Z^2 \rangle = \frac{1}{\Xi} \left[\frac{\partial^2}{\partial \beta^2} + 2\bar{E}\lambda \frac{\partial^2}{\partial\beta\,\partial\lambda} + \bar{E}^2 \left(\lambda \frac{\partial}{\partial \lambda} \right)^2 \right] \Xi(V,T,\lambda) ,$$

$$\langle Z^3 \rangle = -\frac{1}{\Xi} \left[\frac{\partial^3}{\partial \beta^3} + 3\bar{E} \frac{\partial^2}{\partial\beta^2} \lambda \frac{\partial}{\partial\lambda} + 3\bar{E}^2 \frac{\partial}{\partial\beta} \left(\lambda \frac{\partial}{\partial \lambda} \right)^2 + \bar{E}^3 \left(\lambda \frac{\partial}{\partial \lambda} \right)^3 \right] \Xi(V,T,\lambda) , \tag{6}$$

where $\Xi(V,T,\lambda)$ is the grand canonical partition function [18] defined as

$$\Xi(V,T,\lambda) = \sum_N \sum_\alpha \lambda^N e^{-\beta U_\alpha} ,$$

with V denoting the system volume and the index α numerating the system quantum states [2]. As well known [18], the grand canonical partition function of the quantum ideal gas equals

$$\ln \Xi(V,T,\lambda) = \pm g V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm \lambda\, e^{-\beta E}\right] , \tag{7}$$

with g being the number of the particle internal degrees of freedom. After a rather lengthy calculation, one finds

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} (E - \bar{E})^2 \frac{\lambda^{-1} e^{\beta E}}{(\lambda^{-1} e^{\beta E} \pm 1)^2} , \tag{8}$$

and

$$\frac{\langle Z^3 \rangle}{\langle N \rangle} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} (E - \bar{E})^3 \times \frac{\lambda^{-1} e^{\beta E} (\lambda^{-1} e^{\beta E} \mp 1)}{(\lambda^{-1} e^{\beta E} \pm 1)^3} . \tag{9}$$

As expected, Φ_2 and Φ_3, which are given by the formulas (4), (8), (9), are intensive thermodynamic quantities, i.e. they are independent of the system volume. We also note that Φ_2 and Φ_3 are independent of g. One observes that the sign of Φ_2 is definite i.e. $\Phi_2 < 0$ for fermions, $\Phi_2 > 0$ for bosons and $\Phi_2 = 0$ in the classical limit ($\lambda^{-1} \gg 1$) [14]. The sign of Φ_3 is not definite but Φ_3 still vanishes for the classical particles.

When the particles are massless and their chemical potential vanish ($\lambda = 1$), the calculations can be performed analytically to the end. Then, Eqs. (4), (8) and (9) give

$$\Phi_2 \cong \begin{pmatrix} -0.07 \\ 0.40 \end{pmatrix} T , \quad \Phi_3 \cong \begin{pmatrix} 0.11 \\ -\infty \end{pmatrix} T ,$$

where the upper case is for fermions and the lower one for bosons. For $m = \mu = 0$ the bosonic Φ_3 appears to be (logarithmically) divergent due to the singular character of the function $(e^{\beta E} - 1)^{-3}$ at $E \to 0$.

[2] The formulas from [14] analogous to (6) and (7) are erroneously written but the final results are correct.

One immediately modifies Eqs. (4), (8) and (9) for the case of the transverse momentum. The respective equations read:

$$\overline{z^n} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^n \frac{1}{\lambda^{-1}e^{\beta E} \pm 1}, \quad (10)$$

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^2 \frac{\lambda^{-1}e^{\beta E}}{(\lambda^{-1}e^{\beta E} \pm 1)^2}, \quad (11)$$

$$\frac{\langle Z^3 \rangle}{\langle N \rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^3 \times \frac{\lambda^{-1}e^{\beta E}(\lambda^{-1}e^{\beta E} \mp 1)}{(\lambda^{-1}e^{\beta E} \pm 1)^3}, \quad (12)$$

where $p_\perp = p \sin\Theta$ with $p \equiv |\mathbf{p}|$ and Θ being the angle between $\mathbf{p}$ and the beam (z) axis, and

$$\bar{p}_\perp = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} \frac{p_\perp}{\lambda^{-1}e^{\beta E} \pm 1}.$$

In Figs. 1 and 2 we present with dashed lines the Φ_2- and Φ_3-measure of $p_\perp$-fluctuations in the ideal pion gas. The pions are, of course, massive ($m_\pi =$ 140 MeV), so Φ_2 and Φ_3 are found numerically from Eqs. (10)–(12). The calculations are performed for several values of the pion chemical potential. In the chemical equilibrium $\mu = 0$. As seen, Φ_2 is positive but Φ_3 is negative. At $T \cong 200$ MeV and $\mu = 70$ MeV, Φ_3 experiences a rapid growth. This happens because the first term from Eq. (2) changes the sign from positive to negative at $T \cong 200$ MeV.

It is a far going idealization to treat a fireball at freeze-out as an ideal gas of pions. A substantial fraction of the final state pions come from the hadron resonances. These pions do not 'feel' the Bose-Einstein statistics at freeze-out and consequently the values of Φ_2 and Φ_3 should be significantly reduced. We estimate the role of resonances in the following way. The spectrum of pions, which originate from the resonance decays, is not dramatically different than that given by the equilibrium distribution [19]. Therefore, we treat the fireball at freeze-out as a mixture of 'quantum' pions – those called 'direct' – and the 'classical' pions which come from the resonance decays. Since the weighting functions in Eqs. (10)–(12) are all equal to $\lambda e^{-\beta E}$ in the classical limit, the formulas analogous to (10), (11), (12) are

$$\overline{z^n} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^n \times \left[\frac{1}{\lambda^{-1}e^{\beta E} - 1} + \lambda_r e^{-\beta E}\right], \quad (13)$$

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^2 \times \left[\frac{\lambda^{-1}e^{\beta E}}{(\lambda^{-1}e^{\beta E} - 1)^2} + \lambda_r e^{-\beta E}\right], \quad (14)$$

$$\frac{\langle Z^3 \rangle}{\langle N \rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^3 \times \left[\frac{\lambda^{-1}e^{\beta E}(\lambda^{-1}e^{\beta E} + 1)}{(\lambda^{-1}e^{\beta E} - 1)^3} + \lambda_r e^{-\beta E}\right], \quad (15)$$

with

$$\bar{p}_\perp = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} p_\perp \left[\frac{1}{\lambda^{-1}e^{\beta E} - 1} + \lambda_r e^{-\beta E}\right],$$

$$\rho = \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{\lambda^{-1}e^{\beta E} - 1} + \lambda_r e^{-\beta E}\right].$$

The parameter λ_r is chosen is such a way that the number of 'classical' pions equals to the number of pions from the resonance decays. Thus, λ_r is temperature dependent. In the actual calculations, we have taken into account the lightest resonances: $\rho(770)$ and $\omega(782)$ which give the dominant contribution. The life time of ρ, which is 1.3 fm$/c$, is not much longer than the time of the fireball decoupling and some pions from the ρ decays can still 'feel' the effect of Bose statistics. Therefore, the contribution of ρ to the 'classical' pions is presumably overestimated in our calculations. Since we neglect the heavier resonances and weakly decaying particles, which also contribute to the final state pions, the two effects partially compensate each other. In any case, our calculations show that the resonances do not change the values of Φ_2 and Φ_3 dramatically in the domain of temperatures of interest.

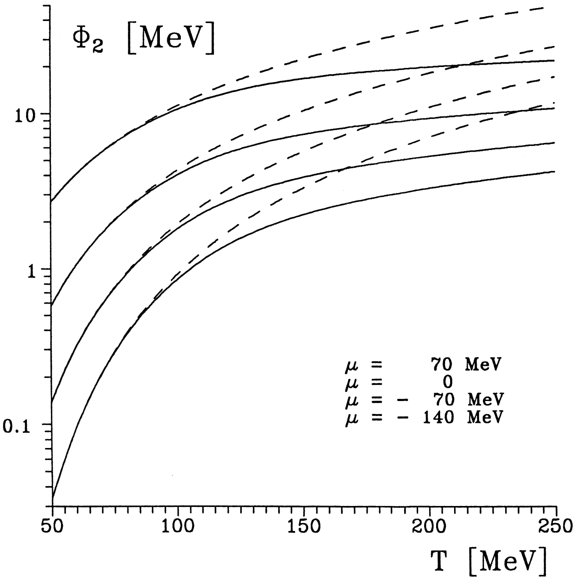

Fig. 1. Φ_2-measure of $p_\perp$-fluctuations in the hadron gas as a function of temperature for four values of the chemical potential. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed and solid lines correspond to $\mu = 70$ MeV, the lower ones to $\mu = 0$, etc.

In Figs. 1 and 2 the solid lines represents Φ_2- and Φ_3-measure which include the resonances. The chemical potentials of ρ and ω are assumed to be equal to that of pions. As seen, the role of the resonances is negligible at the temperatures below 100 MeV but above this temperature the resonances reduce the fluctuations noticeably. As already mentioned, Φ_2-measure of $p_\perp$-fluctuations has been experimentally measured in the central Pb–Pb collisions by the NA49 collaboration. The first result has been published as $\Phi_2 = 0.7 \pm 0.5$ MeV [9] but the value of Φ_2 is increased to 4.6 ± 1.5 MeV when the two-track resolution effect is properly taken into account [10]. If we identify the system freeze-out temperature with the slope parameter deduced from the pion transverse momentum distribution $T \cong 180$ MeV [20]. Then, the value of Φ_2, which is read out from Fig. 1 for $\mu = 0$, equals 15 MeV for no resonances and $\Phi_2 = 8.7$ MeV when the resonances are included. The temperature is significantly reduced if the transverse hydrodynamic expansion is taken into account. The freeze-out temperature obtained by means of the simultaneous analysis of the single particle spectra and the two-particle correlations is about 120 MeV [20]. Then, the value of Φ_2 for $\mu = 0$ equals 6.5 MeV for the case of no resonances and $\Phi_2 = 5.6$ MeV when the resonances are included. The latter number agrees perfectly well with the mentioned above experimental value. This strongly supports the claim [10] that the short range correlations due to the Bose-Einstein statistics of pions play a dominant role in the hadronic system produced in central heavy-ion collisions. However, it would be very interesting to check whether the experiment also confirms our prediction on Φ_3 which is presented in Fig. 2. As seen, $\Phi_3 = -12.3$ MeV for $T = 120$ MeV and $\mu = 0$ when the resonances are taken into account.

Let us close this paper with a technical remark. When the Φ-measure is applied to the real data or simulated events, it is rather inconvenient to use the formula (1) because then one has to process the data twice; in the first run one evaluates the inclusive average $\bar{x}$ and then computes the moments of Z and

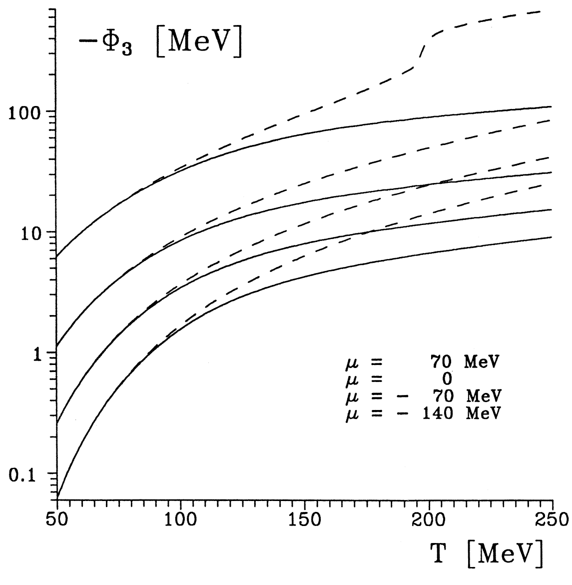

Fig. 2. Φ_3-measure of $p_\perp$-fluctuations in the hadron gas as a function of temperature for four values of the chemical potential. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed and solid lines correspond to $\mu = 70$ MeV, the lower ones to $\mu = 0$, etc.

z. To avoid the double data processing one can use the formula derived in [12] which is

$$\Phi_2 = \left(\frac{\langle X^2 \rangle}{\langle N \rangle} - \frac{2\langle X \rangle \langle XN \rangle}{\langle N \rangle^2} + \frac{\langle X \rangle^2 \langle N^2 \rangle}{\langle N \rangle^3} \right)^{1/2} - \left(\frac{\langle X_2 \rangle}{\langle N \rangle} - \frac{\langle X \rangle^2}{\langle N \rangle^2} \right)^{1/2}, \quad (16)$$

where the event variable X_2 is defined as $X_2 \stackrel{\text{def}}{=} \sum_{i=1}^{N} x_i^2$. The expression of Φ_3, which is analogous to (16), reads

$$\Phi_3 = \left(\frac{\langle X^3 \rangle}{\langle N \rangle} - \frac{3\langle X \rangle \langle X^2 N \rangle}{\langle N \rangle^2} + \frac{3\langle X \rangle^2 \langle XN^2 \rangle}{\langle N \rangle^3} - \frac{\langle X \rangle^3 \langle N^3 \rangle}{\langle N \rangle^4} \right)^{1/3} - \left(\frac{\langle X_3 \rangle}{\langle N \rangle} - \frac{3\langle X_2 \rangle \langle X \rangle}{\langle N \rangle^2} + \frac{2\langle X \rangle^3}{\langle N \rangle^3} \right)^{1/3},$$

with $X_3 \stackrel{\text{def}}{=} \sum_{i=1}^{N} x_i^3$.

We conclude our study as follows. The Φ_3-measure, which is based on the third moment of the fluctuating quantity distribution, preserves the advantageous properties of Φ_2 while the higher moment measures do not. Simultaneous usage of Φ_2 and Φ_3 may help in identifying the origin of correlations observed in the final state of heavy-ion collisions at high-energies. In particular, the measurement of Φ_3 of $p_\perp$-fluctuations can decisively confirm that the dominant correlations in the central collisions are those of the quantum statistics.

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