# Heavy-quark potential in the quark-gluon plasma with cut-off low-momentum modes ☆

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Received 6 March 1990; revised manuscript received 21 May 1990

We discuss the potential acting between a heavy (static) quark and antiquark placed in the quark-gluon plasma using a plasma model, where the low-momentum parts of the gluon (and quark) spectra are cut off due to a nonperturbative mechanism. The Debye (screening) mass is found on the basis of a kinetic approach, and the potential is compared with Monte Carlo lattice data. In the case of the SU(2) gauge group, where the cut-off parameter has been fixed by fitting Monte Carlo data for the energy density and pressure, a very good agreement is found. The model seems to describe the SU(3) Monte Carlo data as well. However, the scarcity of data in this case does not allow us to draw very definite conclusions. At the end, we briefly discuss plasma oscillations with cut-off low-momentum modes.

Monte Carlo lattice calculations show a specific behaviour of the thermodynamical functions of the quark-gluon plasma. Namely, the energy density approaches a value close to that of the ideal gas of partons already for temperatures even slightly exceeding the deconfinement one, while the pressure strongly deviates from the ideal gas one. In the discussion of this problem [1-3], a very simple *non-perturbative* model of the quark-gluon plasma has been recently suggested [2,3] (see also refs. [4,5]), which we call for convenience the (*momentum*) *cut-off model*.

The high-momentum partons are assumed to be weakly interacting, and consequently can be treated in a *perturbative* way. The low-momentum partons are supposed to interact strongly and their density is substantially depleted due to the formation of massive hadron-like modes [#1]. In our further considerations, we treat quarks and gluons in the same way, although it is not quite clear at present whether the idea equally applies to quarks. Unfortunately, this issue cannot be clarified at present because most of the Monte Carlo calculations, in particular those for the inter-quark potential, deal with pure gluodynamics [#2].

The simplest way to formulate a model based on the presented idea is to assume, as in refs. [2-5], that there are no (deconfined) partons in the system with the momenta smaller than a critical value $K$, while

☆ This work was supported in part by the Deutsche Forschungsgemeinschaft under grant He1283/3-1 and by the Heraeus-Stiftung, Hanau.

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[#1] The idea of existence of such modes in the quark-gluon plasma has been considered by several authors [6].

[#2] In fact, the situation is even worse since, as shown in ref. [5], the modification of the quark spectrum appears important only in the case of a finite baryon density, which has not been studied in the lattice QCD due to serious difficulties of its formulation.

the partons with the momenta greater than $K$ behave as in a *perturbative* plasma and their momentum distribution is given by the Bose or Fermi distribution, respectively.

The cut-off model proved very successful in describing the Monte Carlo lattice data for the energy density and pressure [2-4]. And in this paper we continue these studies and consider the interquark potential, which has been recently calculated in lattice QCD [7,8]. Since the value of the cut-off parameter $K$ has been fixed in the previous studies [2-4], our present calculations contain no free parameter. It is also important to notice that the hadron-like modes, the nature of which is poorly known, very weakly influence the interquark potential because they are colourless.

The Debye screening mass, which enters the potential, is derived in the framework of the kinetic approach [9,10] developed in the context of quark-gluon oscillation studies. We compare the potential with Monte Carlo lattice data for the SU(2) [7] and SU(3) [8] gauge group. Encouraged by a very good agreement we discuss further applications of the model.

The potential $V$ acting between a heavy quark and antiquark, which is calculated on the lattice, is a combination of the singlet $V_1$ and adjoint $V_8$ [triplet for the SU(2) and octet for the SU(3) gauge group] potentials. For the SU($N$) group the potential $V$ is expressed as [11]

$$\exp\left(-\frac{V(r,T)}{T}\right)=\frac{1}{N^2}\exp\left(-\frac{V_1(r,T)}{T}\right)+\frac{N^2-1}{N^2}\exp\left(-\frac{V_8(r,T)}{T}\right), \tag{1}$$

where $r$ is the distance between the quark and the antiquark, $N$ is the number of colours and $T$ is the temperature. In the perturbative limit the singlet potential is [11]

$$V_1(r,T)=-\frac{g^2}{4\pi}\frac{N^2-1}{2N}\frac{\exp(-m_D r)}{r} = -(N^2-1)V_8(r,T), \tag{2}$$

where $g$ is the coupling constant and $m_D$ is the Debye screening mass. Assuming that $|T/V_{1,8}|\gg 1$, which is confirmed by the lattice data [7,8] for the range of $r$ and $T$ discussed below, one finds from eqs. (1), (2) [11]

$$-\frac{V(r,T)}{T}=\left(\frac{g^2}{4\pi}\right)^2\frac{N^2-1}{8N^2}\frac{\exp(-2m_D r)}{(rT)^2}. \tag{3}$$

The model discussed in this paper assumes that after *cutting-off* the low-momentum partons, the plasma can be treated in a perturbative way. Therefore, we will apply the formula (3) not only for the temperatures which are much greater than the deconfinement one ($T_c$), but also when $T\to T_c$. It should be stressed here that the derivation of eq. (3) [11] is not altered when the low-momentum partons are cut off.

The quantity which determines all chromodynamic characteristics of a system, in particular the Debye screening mass, is the chromodynamic dielectric tensor, which for an isotropic medium can be decomposed into its longitudinal and transversal parts. According to the approach of ref. [10], which is valid for a small coupling constant, these parts read in the collisionless limit

$$\epsilon_L(k)=1+\frac{g^2}{2\omega\boldsymbol{k}^2}\int\frac{d^3p}{(2\pi)^3}\frac{\boldsymbol{v}\cdot\boldsymbol{k}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}+i0^+}\boldsymbol{k}\cdot\frac{\partial f(p)}{\partial\boldsymbol{p}}, \tag{4a}$$

$$\epsilon_T(k)=1+\frac{g^2}{4\omega}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}+i0^+}\times\left(\boldsymbol{v}\cdot\frac{\partial f(p)}{\partial\boldsymbol{p}}-\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\boldsymbol{k}^2}\boldsymbol{k}\cdot\frac{\partial f(p)}{\partial\boldsymbol{p}}\right) \tag{4b}$$

with

$$f(p)=N_f[n(p)+\bar{n}(p)]+2Nn_g(p),$$

where $k=(\omega,\boldsymbol{k})$ is the wave four-vector, $p=(E,\boldsymbol{p})$ is the four-momentum of a massless parton and $\boldsymbol{v}=\boldsymbol{p}/E$ is the parton velocity, $N_f$ is the number of quark flavours; $n(p)$, $\bar{n}(p)$ and $n_g(p)$ are the distribution functions of quarks, antiquarks and gluons, respectively. According to the cut-off model [2-5] we choose these functions in the following form:

$$n(p)=\bar{n}(p)=\frac{2}{\exp(E/T)+1}\Theta(E-K), \tag{5a}$$

$$n_g(p)=\frac{2}{\exp(E/T)-1}\Theta(E-K), \tag{5b}$$

where the coefficients 2 take into account the spin degrees of freedom of quarks and gluons. The plasma is assumed baryonless in our consideration. Because the hadron-like modes are colourless, they do not contribute to the dielectric tensor within the approximations discussed in detail in ref. [10].

One can explicitly calculate both components of the dielectric tensor substituting the distribution functions (5) in eq. (4). The result is

$$\epsilon_L(k) = 1 + \frac{3\omega_0^2}{k^2} \times \left[1 - \frac{\omega}{2|k|}\left(\ln\left|\frac{|k|+\omega}{|k|-\omega}\right| - i\pi\Theta(|k|-\omega)\right)\right], \tag{6a}$$

$$\epsilon_T(k) = 1 - \frac{3\omega_0^2}{2k^2} \times \left[1 - \left(\frac{\omega}{2|k|} - \frac{|k|}{2\omega}\right)\left(\ln\left|\frac{|k|+\omega}{|k|-\omega}\right| - i\pi\Theta(|k|-\omega)\right)\right], \tag{6b}$$

with the plasma frequency $\omega_0$, which equals

$$\omega_0^2 = \frac{g^2T^2}{3\pi^2}\left[N_f I_+(K/T) + N I_-(K/T)\right], \tag{7}$$

where

$$I_\pm(a) = \int_a^\infty \frac{dx\, x^2 \exp(x)}{[\exp(x) \pm 1]^2} - \frac{a^2}{\exp(a) \pm 1}.$$

One immediately notices that we have received the well-known formulas of the dielectric tensor of the plasma of massless particles (see e.g. refs. [9,10,12]) and the only modification appears in the expression of the plasma frequency (7), which for the *normal* ($K=0$) quark-gluon plasma reads (see e.g. refs. [9,10])

$$\omega_0^2 = \tfrac{1}{18} g^2T^2(N_f + 2N). \tag{8}$$

Obviously eq. (7) with $K=0$ gives eq. (8).

The functions $I_\pm(a)$ can be approximated as

$$I_+(a) = \tfrac{1}{6}\pi^2 - \tfrac{1}{2}a^2, \quad I_-(a) = \tfrac{1}{3}\pi^2 - 2a \quad \text{for } \frac{1}{a} \gg 1$$

and

$$I_\pm(a) = 2(a+1)\exp(-a) \quad \text{for } \exp(a) \gg 1.$$

Consequently, the plasma frequency (7) can be rewritten as

$$\omega_0^2 = \tfrac{1}{18} g^2T^2(N_f + 2N) - \frac{g^2TK}{3\pi^2}\left(N_f\frac{K}{2T} + 2N\right)$$

for $T/K \gg 1$

and

$$\omega_0^2 = \frac{2g^2T^2(N_f+N)}{3\pi^2}\left(\frac{K}{T}+1\right)\exp\left(-\frac{K}{T}\right)$$

for $\exp\left(\frac{K}{T}\right) \gg 1$.

The screening mass is defined through the equation (see e.g. ref. [12])

$$\epsilon_L(\omega=0, k) = 1 + \frac{m_D^2}{k^2}$$

and, then, eq. (6a) provides

$$m_D = \sqrt{3}\,\omega_0. \tag{9}$$

In the further calculations, we express the coupling constant through the QCD scale parameter $\Lambda$ by means of the well-known formula, which for pure gluodynamics is [13]

$$\frac{g^2}{4\pi} = \frac{6\pi}{\frac{11}{2}N\ln(M^2/\Lambda^2)} \tag{10}$$

with the mass parameter $M$ defined for the plasma as [13]

$$M^2 = \tfrac{4}{3}\langle k^2\rangle,$$

where $\langle k^2\rangle$ is the thermal average of the gluon squared momentum [#3]. In the cut-off model one finds

$$M^2 = \tfrac{4}{3}T^2\frac{\int_a^\infty dx\, x^4[\exp(x)-1]^{-1}}{\int_a^\infty dx\, x^2[\exp(x)-1]^{-1}}, \tag{11}$$

where $a = K/T$. When the cut-off parameter $K$ goes to zero

$$M = 4\left(\frac{\zeta(5)}{\zeta(3)}\right)^{1/2} T \cong 3.715\ T.$$

[#3] In fact, the form of the coupling constant in thermal QCD and its role in perturbative expansion is presently somewhat controversial due to recent studies of this problem; see ref. [14] and references therein.

If $\exp(K/T) \gg 1$ one can well approximate the expression (11) as

$$M^2 = \frac{4}{3} \frac{K^4 + 4K^3 T + 12K^2 T^2 + 24KT^3 + 24T^4}{K^2 + 2KT + 2T^2}.$$

It is important to stress that within the cut-off model, the coupling constant (10), which governs the high-momentum partons, is small even at temperatures close to $\Lambda$ if $K \gg T$. Therefore, the perturbative expansion seems applicable in this case.

Let us now confront the potential (3) with the Monte Carlo lattice data.

*1. SU(2) gauge group.* The cut-off model has been used to describe very precise SU(2) data for the energy density and pressure [3]. Since the deconfinement phase transition in the SU(2) quarkless plasma is of the second order, it has been assumed that the cut-off parameter varies with the temperature as $(T-T_c)^\alpha$, where $T_c$ is the critical temperature. Fitting the data, the following relation has been found [3]:

$$\frac{K}{T_c} = 2.9 \left( \frac{T_c}{T - T_c} \right)^{0.30}. \qquad (12)$$

As can be seen, the parameter $K$ goes to zero when the temperature increases and $K$ diverges to infinity when the temperature approaches the critical one. Therefore, at the critical temperature deconfined gluons disappear completely. Substituting eq. (12) into eq. (7) and using eq. (9), one finds that the screening length (inverse Debye mass), as expected, goes to infinity at the critical temperature.

To compare the *continuum* potential (3) with the lattice data one has to relate the *continuum* QCD scale parameter $\Lambda$ to the lattice scale parameter $\Lambda_L$. It has been proved in ref. [15] that $\Lambda = 57.5\Lambda_L$. If the temperature and the scale parameter $\Lambda$ are measured in units of the critical temperature, which equals $37.9\Lambda_L$ [3], we do not need to know the actual value of $\Lambda_L$.

In fig. 1 we present the interquark potential for two temperatures of the plasma [7]. The points are the result of the extrapolation procedure to an infinite continuous system. The solid lines correspond to the cut-off model, while the dashed lines represent the standard perturbative calculations ($K=0$). One sees that, in contrast to the latter ones, the cut-off model describes the data very well. And it should be stressed that the calculations contain no free parameter.

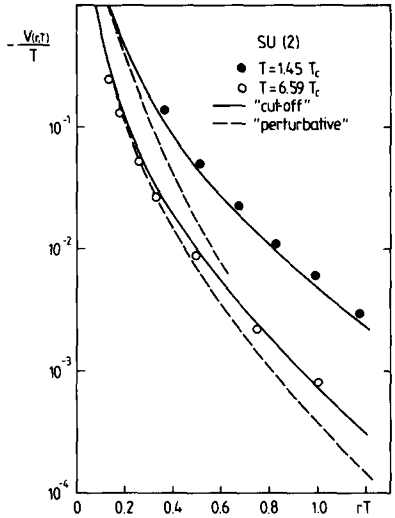


Fig. 1. The interquark potential for the SU(2) gauge group for two temperatures of the pure gluon plasma. The points are Monte Carlo data extrapolated to an infinite system [7]. The solid lines correspond to the cut-off model, while the dashed ones represent the standard perturbative calculations ($K=0$).

*2. SU(3) gauge group.* In fig. 2 we present the interquark potential calculated on a $12^3 \times 4$ lattice for the pure SU(3) gluodynamics [8]. Unfortunately, the calculations have been performed only on a lattice of one size, and consequently one cannot perform an extrapolation to an infinite system. It is also important to notice that the critical temperature obeys an asymptotic scaling law only for lattices with the number of sites in the time direction greater than about 9 [2]. The potential has been calculated for three temperatures in ref. [8] and the authors have taken into account the scaling violation when the temperatures have been expressed in the units of the critical temperature. Since a finite-size analysis of the potential has not been performed, it seems more consistent, in our opinion, to give the values of temperature, which neglect the scaling violation. For this reason the temperatures shown in fig. 2 differ from those given in ref. [8]. (Using the critical value of the lattice coupling $\beta$ given in ref. [8], we have found that for the lattice with four sites in the time direction the critical temperature equals $75.2\Lambda_L$. For the lattices which satisfy the asymptotic scaling law $T_c = 51.3\Lambda_L$ [2].)

The solid lines in fig. 2 represent the cut-off model,

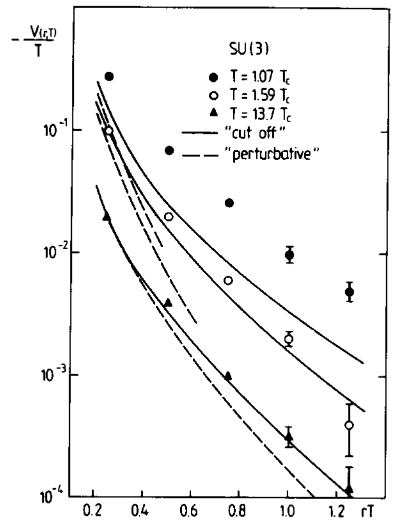


Fig. 2. The interquark potential for the SU(3) gauge group for three temperatures of the pure gluon plasma. The points are Monte Carlo data obtained on a $12^3 \times 4$ lattice [8]. The solid lines correspond to the cut-off model, while the dashed ones represent the standard perturbative calculations ($K = 0$).

while the dashed lines are the standard perturbative calculations. Because the SU(3) system undergoes a first-order phase transition, we expect that the cut-off parameter remains finite when the temperature equals the deconfinement one. Therefore, we have taken the value of $K$ independent of temperature and equal $3.4 T_c$. This value has been found in ref. [4], where the pressure of the SU(3) plasma has been considered [#4]. As in the case of the SU(2) group, we have used the relation connecting the continuum QCD scale parameter with the lattice one. This relation for the SU(3) group reads $\Lambda = 83.5 \Lambda_L$ [15].

Unfortunately, the results presented in fig. 2 are not very conclusive because the Monte Carlo data are not corrected for finite-size effects. The analysis of these effects performed for the SU(2) case [7] shows that they are important for temperatures much greater than the critical one because then the volume of the lattice system is small, and when the temperature approaches the critical one due to the increase of the correlation length, which diverges at $T = T_c$ (because of the second-order phase transition). The situation in the SU(3) case is probably similar to that in the SU(2) one; however, the reason for the large finite size effects at $T$ near $T_c$ is different. When the temperature of the plasma is higher but close to the critical one, it is expected (because of the first-order phase transition) that a metastable hadron phase, which can appear, affects more substantially a small lattice system than a big one because the hadron-phase life-time increases when the system size decreases.

Due to large finite-size effects at high $T$, a very good agreement of the cut-off model with the data for $T = 13.7 T_c$ seems to be rather accidental. For such a high temperature one expects that the standard perturbative model should describe the data well. Then, the discrepancy seen in fig. 2 might be a measure of the finite size effects. The difference between the cut-off model and the perturbative calculations would be reduced, if we assumed, as in the SU(2) case, a decrease of the cut-off parameter $K$ with temperature. The highest significance has, in our opinion, the quite good description of the Monte Carlo data by the cut-off model for $T = 1.59 T_c$, since we expect that the finite-size effects are smallest in this case. The lattice volume is rather large, and a possible existence of the metastable hadron phase is strongly damped. A substantial underestimation of the lattice potential by the cut-off model for $T = 1.07 T_c$ might be due to the metastable hadron phase, where a confining potential acts.

The interquark potential obtained in the Monte Carlo QCD has been fitted with the formula $\exp(-\mu r)/r^d$ with $\mu$ and $d$ as parameters in refs. [7,8,16] [#5]. The parameter $d$ equals about 2 for high temperatures as follows from eq. (3); however, it seems to decrease when $T \to T_c$. It has been suggested in ref. [16] that $d = 1$ in this case, and some theoretical arguments have been given in favour of such a dependence. At present it is rather premature to draw the final conclusion because the Monte Carlo data corrected for the final-size effects are not available yet. However, the cut-off model will be in trouble if indeed $d = 1$. In any case we find the results presented above,

[#4] In fact, a plasma with dynamical quarks has been discussed in ref. [4]. Since the comparison of the potential with the SU(3) data is rather crude because of the finite-size effects, we believe that this does not make a substantial difference.

[#5] The screening mass has been identified with $\mu$ in these papers, while according to our model, see eq. (3), $m_D$ equals $\frac{1}{2}\mu$ assuming that $d \cong 2$.

in particular those concerning the SU(2) group, rather encouraging. Therefore, let us briefly speculate on further consequences of the cut-off model.

As discussed above, the dielectric tensor of the plasma with cut-off low-momentum modes coincides, up to the plasma frequency, with the one of the *normal* ($K=0$) plasma. Therefore, the dispersion relations also coincide but the plasma frequency is (at the same temperature) smaller for the cut-off plasma. Let us also observe that a damping of the oscillations is probably strongly influenced by the lack of low-momentum partons. As discussed in refs. [17,10] the dominant damping mechanism (in the *normal* plasma) is the plasmon decay into a gluon pair. The plasmon with energy $\omega_0$ and zero momentum (in the thermostat rest frame) decays into two gluons of energies equal $\frac{1}{2}\omega_0$. The presence of gluons with energies $\frac{1}{2}\omega_0$ in the plasma increases the width of this decay due to the boson *attraction* in momentum space. On the other hand their presence leads to plasmon formation. As shown in ref. [10] the plasmon decay is of order of $g$ and the plasmon formation is of the same order. However, the net plasmon decay width which takes into account both loss and gain processes by plasmon decay and plasmon formation is of order of $g^2$. Consequently, the damping rate ($\gamma$) is of this order.

How does this reasoning change when there are no (deconfined) partons with momentum smaller than $K$? When the plasmon energy is greater than $2K$, the reasoning remains unchanged. However, if $\frac{1}{2}\omega_0$ is smaller than the cut-off parameter $K$, there is no boson *attraction* and no plasmon formation. The plasmon decays as in vacuum. Simple considerations analogous to those presented in ref. [10] show that the plasmon decay into gluons, or into (massless) quarks is of order $g^4$. Therefore, the plasmon-decay contribution to the damping rate is of the same order as that of binary collisions of plasma partons. We cannot judge which is the dominant mechanism of the damping; however, $\gamma$ seems to be of order $g^4$. In conclusion, the oscillations of the plasma with cut-off low-momentum modes are probably very similar to those of the *normal* one; however, the plasma frequency (the plasmon mass) is smaller and the low-frequency oscillations are weakly damped. Unfortunately, the picture of plasma oscillation presented is rather speculative because it completely neglects the coupling of the massive modes to colourful partons.

Let us summarize our considerations. We have discussed the potential acting between a heavy (static) quark and antiquark embedded in the quark–gluon plasma using a *nonperturbative* plasma model, where low-momentum partons are converted into colourless hadron-like modes. In the framework of the kinetic approach, we have calculated the dielectric tensor of such a plasma and then the Debye (screening) mass, which enters the interquark potential. A detailed comparison with the Monte Carlo lattice data has been performed. In the case of the SU(2) gauge group we have found a very good agreement. The model seems also to describe the SU(3) data; however, the comparison is obscured due to the lack of an analysis of the finite-size effects of the SU(3) data. At the end, we have briefly discussed oscillations of the plasma with cut-off low-momentum modes.

We are indebted to Ulrich Heinz for discussions and a critical reading of the manuscript. Very helpful conversations with Keijo Kajantie and Helmut Satz are also gratefully acknowledged. One of us (M.I.G.) expresses his gratitude for the warm hospitality at the Physics Department of the University of Marburg, where this work was completed.

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