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On the neutron–proton correlations and deuteron production

Stanisław Mrówczyński [1]

High-Energy Department, Soltan Institute for Nuclear Studies, ul Hoza 69, PL-00-681 Warsaw, Poland

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The neutron–proton correlations and deuteron formation in nuclear collisions both appear due to the final state interactions Thus, we suggest simultaneous measurements of the neutron–proton correlation function and the deuteron formation rate We calculate in parallel the two quantities and express them through the space–time parameters of the particle source created in nucleus–nucleus collisions

The measurements of two-particle correlations are well known to provide information about space–time characteristics of particle sources in nuclear collisions for bombarding energies from tens of MeV [1] to hundreds of GeV [2] One usually deals with pairs of identical particles, protons at lower energies and pions at higher ones However, the correlations of non-identical particles [3] and the probabilities of bound state formations [4] (see also ref [5]) determine the space–time size of particle sources as well The neutron and proton are of particular interest here since one can study the two-particle correlation and bound state formation with the same particles It is important to stress that the correlation between neutron and proton with small relative momentum and the deuteron formation both appear due to the final state interaction [6–8]

The recent successful measurements of the neutron–neutron correlations [9] and the very first data on the neutron–proton (n–p) correlations [10] show that the n–p correlation function can be precisely measured We suggest to study this quantity simultaneously with the probability of the deuteron production, which can be easily measured at present The analysis of such data would provide, in particular, a valuable test of the method which uses the two-particle correlations to determine the space–time structure of particle sources If the method is indeed accurate the n–p pair in a scattering state and the deuteron should provide exactly the same space–time size of a source The derivation of the main formula of the method shows that it holds under rather restrictive conditions, which might be not always fulfilled in nuclear collisions (see e g ref [8])

The aim of this paper is to derive simple formulas, which can be used in the analysis of simultaneous measurements of the n–p correlation and deuteron production [#1] The n–p correlation function and the deuteron formation rate, which relates the deuteron production cross section to the cross section of the n–p pair production, were calculated in ref [3] and refs [4–7], respectively The calculations [4–7] assumed the simultaneous emission of the neutron and proton which further form a deuteron, while the analysis of two-particle correlations usually takes into account noninstantaneous particle emission Thus, we derive here the deuteron formation rate for noninstantaneous emission to make the two processes comparable The authors of ref [3], where the n–p correlation function was calculated were interested in relativistic systems and for this reason they used a rather complicated formalism of the Bethe–Salpeter amplitudes instead of wave functions. However, the n–p correlation function can be measured at present only for nonrelativistic neutrons Thus, we calculate the n–p correlation function using the familiar language of nonrelativistic wave functions We then find

[1] E-mail address MROW@PLEARN bitnet

[#1] The CHIC Collaboration led by Bo Jakobsson indeed plans such measurements that the correlation functions of the n–p pairs in a singlet and triplet state are qualitatively different from each other for sufficiently large sources We briefly discuss how this observation can be used to study the spin of a particle source

Let us start our considerations with discussing in parallel the cross sections of deuteron production and n–p emission These cross sections can be written as

$$\frac{d\sigma^{d}}{d^3\boldsymbol{P}} = \mathscr{A} \frac{d\tilde{\sigma}^{np}}{d^3(\frac{1}{2}\boldsymbol{P})d^3(\frac{1}{2}\boldsymbol{P})}, \qquad (1a)$$

$$\frac{d\sigma^{np}}{d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2} = \mathscr{R}(\boldsymbol{p}_1, \boldsymbol{p}_2) \frac{d\tilde{\sigma}^{np}}{d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2}, \qquad (1b)$$

where $d\tilde{\sigma}^{np}/d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2$ is the n–p cross section production which neglects particle correlations due to the final state interactions In the reference frame where a particle source is at rest, the deuteron formation rate $\mathscr{A}$ and the n–p correlation function $\mathscr{R}$ are expressed in the nonrelativistic limit as

$$\mathscr{A} = \tfrac{3}{4}(2\pi)^3 \int d^3r_1 dt_1 \int d^3r_2 dt_2$$
$$\times \mathscr{D}(\boldsymbol{r}_1, t_1)\mathscr{D}(\boldsymbol{r}_2, t_2)|\psi_d(\boldsymbol{r}'_1, \boldsymbol{r}'_2)|^2, \qquad (2a)$$

$$\mathscr{R}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \tfrac{1}{4}\mathscr{R}^{s}(\boldsymbol{p}_1, \boldsymbol{p}_2) + \tfrac{3}{4}\mathscr{R}^{t}(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

with

$$\mathscr{R}^{s,t}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \int d^3r_1 dt_1 \int d^3r_2 dt_2$$
$$\times \mathscr{D}(\boldsymbol{r}_1, t_1)\mathscr{D}(\boldsymbol{r}_2, t_2)|\psi^{s,t}_{np}(\boldsymbol{r}'_1, \boldsymbol{r}'_2)|^2, \qquad (2b)$$

where the source function $\mathscr{D}(\boldsymbol{r}, t)$, which is normalized as $\int d^3r\, \mathscr{D}(\boldsymbol{r}, t) = 1$, gives the probability to emit a nucleon from a space–time point $(\boldsymbol{r}, t)$ [#2] ψ_d, ψ^{s}_{np} and ψ^{t}_{np} are the wave functions of a deuteron, of an n–p pair in a singlet state and in a triplet state, respectively, $\boldsymbol{r}'_i \equiv \boldsymbol{r}_i - (\boldsymbol{p}_i/m)t_i$, $i = 1, 2$ with m being the nucleon mass The emitted nucleons are assumed unpolarized and consequently, the correlation function $\mathscr{R}$ is a sum of the singlet and triplet correlation functions $\mathscr{R}^{s,t}$ with weight coefficients $\frac{1}{4}$ and $\frac{3}{4}$, respectively The origin of the coefficient $\frac{3}{4}$ in eq (2a) is similar The correlation function $\mathscr{R}$ and the deuteron formation rate $\mathscr{A}$ have been discussed separately in numerous papers, see the review [1] for $\mathscr{R}$ and refs [4,5] for $\mathscr{A}$ The form of $\mathscr{R}$ expressed by eq (2b) has been written for the first time in ref [11] for p–p correlations The modification of (2b), which is now frequently used in data analysis, is described in ref [12] Simultaneous discussion of the particle correlations and bound state formation, which stresses common character of both phenomena, is present in refs [6–8] Ref [8] provides the most systematic derivation of eqs (1) and (2) in the nonrelativistic limit One also finds in ref [8] the discussion of assumptions and approximations which leads to these equations

Let us choose the following gaussian parametrization of a source

$$\mathscr{D}(\boldsymbol{r}, t) = (2\pi r_0^2)^{-3/2}(2\pi\tau^2)^{-1/2}$$
$$\times \exp\left(-\frac{\boldsymbol{r}^2}{2r_0^2} - \frac{t^2}{2\tau^2}\right), \qquad (3)$$

which gives the mean radius (life time) squared of a source as $\langle \boldsymbol{r}^2 \rangle = 3r_0^2$ and $\langle t^2 \rangle = 3\tau^2$ The parametrization (3) allows one to factorize the integrals over the center-of-mass and relative coordinates, which are defined as $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)$, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $T = \frac{1}{2}(t_1 + t_2)$, $t = t_1 - t_2$, $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$, $\boldsymbol{q} = \frac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2)$ The wave functions are written down in the form

$$\psi_i(\boldsymbol{r}_1, \boldsymbol{r}_2) = \exp(i\boldsymbol{P}\boldsymbol{R})\phi_i(\boldsymbol{r}), \qquad (4)$$

where the index i labels the wave functions of the deuteron, the n–p pair in a singlet state and the n–p pair in triplet state, respectively

Substituting eqs (3) and (4) into eqs (2) one gets, after integration over $\boldsymbol{R}$ and T

$$\mathscr{A} = \tfrac{3}{4}(2\pi)^3 \int d^3r\, dt\, \mathscr{D}_r(\boldsymbol{r}, t)|\phi_d(\boldsymbol{r}')|^2, \qquad (5a)$$

$$\mathscr{R}^{s,t}(\boldsymbol{q}, \boldsymbol{P}) = \int d^3r\, dt\, \mathscr{D}_r(\boldsymbol{r}, t)|\phi^{s,t}_{np}(\boldsymbol{r}')|^2, \qquad (5b)$$

where $\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{v}t$ with $\boldsymbol{v}$ being the center-of-mass velocity equal to $\boldsymbol{P}/2m$ and

$$\mathscr{D}_r(\boldsymbol{r}, t) = (4\pi r_0^2)^{-3/2}(4\pi\tau^2)^{-1/2}$$
$$\times \exp\left(-\frac{\boldsymbol{r}^2}{4r_0^2} - \frac{t^2}{4\tau^2}\right) \qquad (6)$$

Further, we calculate separately the deuteron formation rate and the correlation function

(1) *The deuteron formation rate* After changing the variables $(\boldsymbol{r}, t) \to (\boldsymbol{r} - \boldsymbol{v}t, t)$ in eq (5a), one easily performs the integration over time and then the in-

[#2] It should be understood here that the coordinates $(\boldsymbol{r}, t)$ determine the position of a nucleon wave-package center tegration over polar and azimuthal angles assuming that the deuteron wave function is spherically symmetric The results reads

$$\mathscr{A}=\frac{3\pi^3}{r_0 v\tau}\int_0^\infty \mathrm{d}r\, r|\phi_\mathrm{d}(r)|^2 \exp\left(-\frac{r^2}{4r_0^2}\right)\mathrm{erfi}(ar)\,, \tag{7}$$

where $r\equiv|\boldsymbol{r}|$ and

$$a\equiv\frac{v\tau}{2r_0\sqrt{r_0^2+v^2\tau^2}}\,,\quad \mathrm{erfi}(x)\equiv\frac{2}{\sqrt{\pi}}\int_0^x \mathrm{d}t\exp(t^2)$$

We use in further calculations the deuteron wave function in the Hulthén form

$$\phi_\mathrm{d}(\boldsymbol{r})=\left(\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}\right)^{1/2}\frac{\exp(-\alpha r)-\exp(-\beta r)}{r}\,, \tag{8}$$

with $\alpha=0\ 23\ \mathrm{fm}^{-1}$ and $\beta=1\ 61\ \mathrm{fm}^{-1}$ [13] The calculation of the integral (7) with the wave function (8) has been performed numerically and the formation rate $\mathscr{A}$ as a function of the source size parameter r_0 is presented in fig 1 for several values of the parameter $v\tau$ Let us stress here the source life-time parameter τ and the deuteron velocity v enter the formation rate $\mathscr{A}$ only as a product $v\tau$ Physically it means that the formation rate (the correlation function) does not depend directly on the source life time but on the distance the particle pair traverses during the source life time

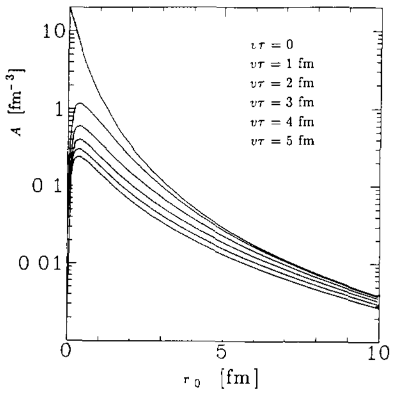

Fig 1 The deuteron formation rate as a function of r_0 for several values of $v\tau$ The value of $v\tau$ increases from the highest line ($v\tau=0$) to the lowest line ($v\tau=5$ fm)

The function $\mathscr{A}(r_0, v\tau)$ shown in fig 1 manifests specific mathematical properties Namely,

$$\mathscr{A}(r_0\to0,\ v\tau=0)\to6\pi^3|\phi_\mathrm{d}(r=0)|^2=3\pi^2\alpha\beta(\alpha+\beta)$$

$$\mathscr{A}(r_0\to0,\ v\tau>0)\to0$$

These limits are useful to check numerical calculations

If the size of a particle source is comparable with its life time, r_0 is much greater than τ in the nonrelativistic limit, where $v\ll1$ Then, one can well approximate the deuteron formation rate (7) by its value at $v\tau=0$ In this case, one can perform a completely analytic calculation and the result is

$$\mathscr{A}_0=\frac{3\pi^2}{2r_0^2}\frac{\alpha\beta(\alpha+\beta)}{(\alpha-\beta)^2}\times[F(2\alpha r_0)-2F((\alpha+\beta)r_0)+F(2\beta r_0)]\,, \tag{9}$$

where

$$F(x)\equiv\exp(x^2)\,\mathrm{erfc}(x)\,,$$

$$\mathrm{erfc}(x)\equiv\frac{2}{\sqrt{\pi}}\int_x^\infty \mathrm{d}t\exp(-t^2)$$

(2) *The correlation function* We calculate the correlation function $\mathscr{R}$ following ref [3], where the relativistic system has been considered

If the source radius is significantly greater than the n–p interaction range, the wave function of the n–p pair (in a scattering state) can be approximated by its asymptotic form which is

$$\phi_\mathrm{np}^{s\,t}(\boldsymbol{r})=\exp(\mathrm{i}\boldsymbol{qr})+f^{s,t}(q)\frac{\exp(\mathrm{i}qr)}{r}\,, \tag{10}$$

where $q\equiv|\boldsymbol{q}|$ and $f^{s,t}(\boldsymbol{q})$ is the scattering amplitude The correction due to the nonasymptotic states of the n–p pairs are commented on at the end of our paper The scattering amplitude is chosen as

$$f^{s,t}(q)=\frac{-a^{s\,t}}{1-\frac{1}{2}d^{s,t}a^{s,t}q^2+\mathrm{i}qa^{s,t}}\,, \tag{11}$$

where $a^{s,t}$ ($d^{s\,t}$) is the scattering length (effective range) of the n–p scattering, $a^s=-23\ 7$ fm, $d^s=2\ 7$ fm and $a^t=5\ 4$ fm, $d^t=1\ 7$ fm [14] [#3] The amplitude (11) takes into account only the s-wave scatter-

[#3] The authors of ref [3] use the opposite sign convention of the scattering lengths ing This is justified as long as "small" relative momenta are considered

Substituting the wave function (10) into eq (5b) and changing the variables $(\boldsymbol{r}, t) \to (\boldsymbol{r}-\boldsymbol{v}t, t)$ one finds

$$\mathscr{R}^{s,t}(\boldsymbol{q}, \boldsymbol{P}) = 1 + 2\,\mathrm{Re}\Big(f^{s,t}(q) \times \int d^3r\, dt\, r^{-1} \mathscr{D}_r(\boldsymbol{r}'', t) \exp(-i\boldsymbol{q}\boldsymbol{r} + iqr)\Big) + |f^{s,t}(q)|^2 \int d^3r\, dt\, r^{-2} \mathscr{D}_r(\boldsymbol{r}'', t), \quad (12)$$

where $\boldsymbol{r}'' \equiv \boldsymbol{r} + \boldsymbol{v}t$ The computation of the third term on the right-hand-side of eq (12) is rather straightforward, while before calculation of the second term on the right-hand-side of eq (12) we should choose the physical observables When $v\tau = 0$, the correlation function depends only on the magnitude of the vector $\boldsymbol{q}$ In the general case when $v\tau > 0$, the correlation function (12) depends on the magnitudes of two vectors $\boldsymbol{q}$ and $\boldsymbol{P}$ and their relative orientation Thus, we choose as arguments of the correlation function the pair velocity v, the relative longitudinal momentum q_L parallel to $\boldsymbol{P}$ and the transverse moment q_T perpendicular to $\boldsymbol{P}$ We then introduce cylindrical coordinates (ρ, z, ϕ) with the z-axis parallel to $\boldsymbol{P}$, and perform the integration over time and azimuthal angle Then, one finds

$$\mathscr{R}^{s,t}(q_L, q_T, v) = 1 + \left(2\sqrt{\pi}\, r_0^2 \sqrt{r_0^2 + v^2\tau^2}\right)^{-1} \times \int_0^\infty d\rho\, \rho \int_{-\infty}^{\infty} dz\, (\rho^2+z^2)^{-1/2} J_0(q_T\rho) \times \exp\left(-\frac{\rho^2+z^2}{4r_0^2} + \frac{v^2\tau^2 z^2}{4r_0^2(r_0^2+v^2\tau^2)}\right) \times \left[\mathrm{Re} f^{s,t}(q) \cos\left(q_L z - q\sqrt{\rho^2+z^2}\right) + \mathrm{Im} f^{s,t}(q) \sin\left(q_L z - q\sqrt{\rho^2+z^2}\right)\right] + |f^{s,t}(q)|^2 \frac{1}{2r_0 v\tau} \operatorname{arcsh} \sqrt{\frac{v^2\tau^2}{r_0^2+v^2\tau^2}}, \quad (13)$$

where $q \equiv |\boldsymbol{q}| = \sqrt{q_T^2 + q_L^2}$ and J_0 is the Bessel function, which appears due to the integration over azimuthal angle At $q=0$, the correlation function (13) can be computed analytically, and the result is

$$\mathscr{R}^{s,t}(q=0, v) = 1 - \frac{a^{s,t}}{\sqrt{\pi}\, v\tau} \ln \frac{\sqrt{r_0^2+v^2\tau^2}+v\tau}{\sqrt{r_0^2+v^2\tau^2}-v\tau} + \frac{(a^{s,t})^2}{2r_0 v\tau} \operatorname{arcsh} \sqrt{\frac{v^2\tau^2}{r_0^2+v^2\tau^2}} \quad (14)$$

In the general case when $q \neq 0$ the integrations in eq (13) have to be performed numerically Examples of such calculations are displayed in figs 2 and 3

As expected, the n-p correlation function manifests a prominent peak at zero relative momentum In fact, this peak appears because the singlet correlation function gives a dominant contribution to the spin average correlation function As we show below,

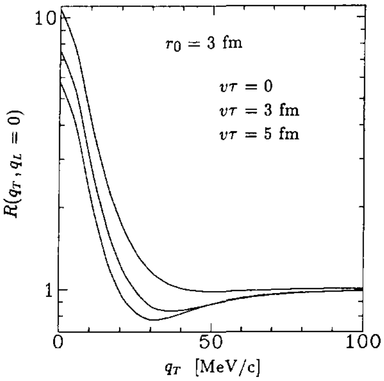

Fig 2 The n-p correlation function versus q_T at $q_L = 0$ for several values of $v\tau$ and $r_0 = 3$ fm The value of $v\tau$ increases from the highest line to the lowest line

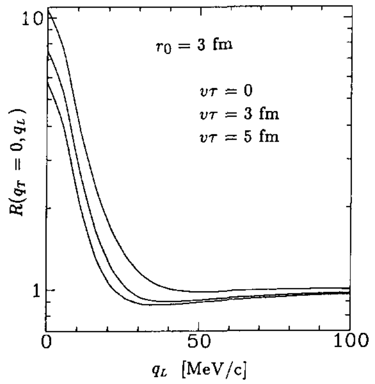

Fig 3 The n-p correlation function versus q_L at $q_T = 0$ for several values of $v\tau$ and $r_0 = 3$ fm The value of $v\tau$ increases from the highest line to the lowest line

the triplet correlation function has a minimum at $q=0$ for sufficiently large sources Let us also observe here that the q_T- and q_L-dependences of $\mathscr{R}$ are rather similar to each other It is not surprising since these dependences are exactly identical when $v\tau=0$

When $r_0 \gg v\tau$, one can well approximate the correlation function (13) by the function at $v\tau=0$ In this case, the correlation function depends only on q and the computation can be performed analytically using spherical coordinates The result is

$$\mathscr{R}_0^{s,t}(q)=1+\operatorname{Re}f^{s,t}(q)\frac{1}{2r_0^2 q}\times\exp(-4r_0^2q^2)\operatorname{erfi}(2r_0q)-\operatorname{Im}f^{s,t}(q)\frac{1}{2r_0^2q}[1-\exp(-4r_0^2q^2)]+|f^{s,t}(q)|^2\frac{1}{2r_0^2} \quad (15)$$

The correlation function (15) is shown in figs 4–6 In figs 5 and 6 we present not only the spin average correlation function but also the correlation functions of n–p pairs in singlet ($S=0$) and triplet ($S=1$) states One sees that for large source radii these functions are qualitatively different from each other While the singlet correlation function has a maximum at $q=0$, the triplet function has a minimum there In the channel $S=1$ there are two physical phenomena which compete with each other These are the scattering on attractive potential and the deuteron formation The first mechanism leads to the positive correlation, while the formation of deuterons depletes the sample of n–p pairs and effectively leads to the negative correlation It appears that for large source radii, the n–p correlation function is dominated by the deuteron formation The relative contribution of the first mechanism grows as the source radius goes to zero At $r_0 \simeq 3.5$ fm, both mechanisms cancel each other, and no correlation is seen When the radius decreases further the scattering mechanism dominates and the triplet correlation is positive as in the singlet case Since the singlet interaction of an n–p pair is much stronger than the triplet one, the behaviour of the average correlation function is always dominated by the singlet one

In the case of a spinning source, the number of n–p pairs in a triplet state (parallel spins) is expected to be larger than $\frac{3}{4}$ of all pairs Consequently, the cor-

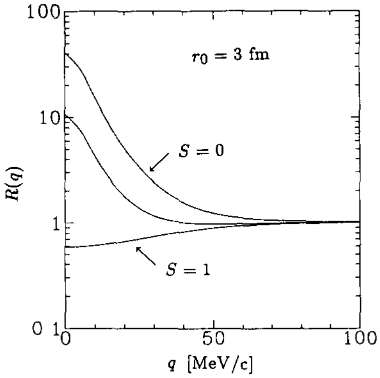

Fig 5 The singlet, triplet and spin average n–p correlation functions versus q at $v\tau=0$ and $r_0=3$ fm

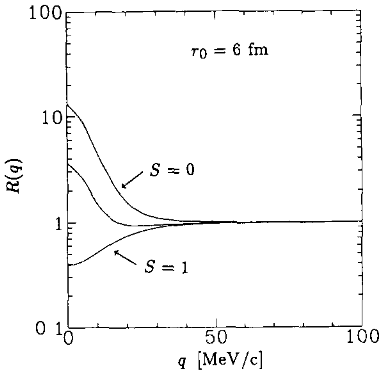

Fig 6 The singlet, triplet and spin average n–p correlation functions versus q at $v\tau=0$ and $r_0=6$ fm

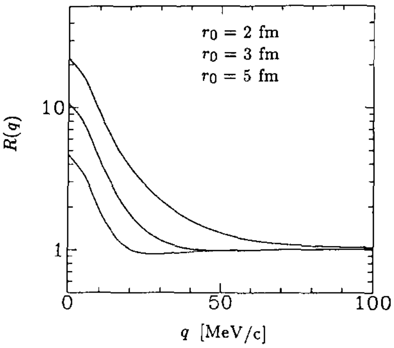

Fig 4 The n–p correlation function versus q at $v\tau=0$ for several values of r_0 The value of r_0 increases from the highest line to the lowest line

relation function averaged over spin might be significantly changed when compared to that of a spinless source The simultaneous measurements of the n–p correlation function and the deuteron formation rate, which is also sensitive to the fraction of n–p pairs in the triplet state, allows one, at least in principle, to determine the source spin This issue, however, demands further studies

In our derivation of the n–p correlation function, the asymptotic form of the n–p pair wave function has been used for all separations of proton and neutron The correction due to the nonasymptotic states has been discussed in ref [3] It was found there that the term of the correlation function, which is proportional to $|f^{s,t}(q)|^2$, should be multiplied by the factor

$$1-\frac{1}{2\sqrt{\pi}}\frac{d^{s,t}}{r_0}$$

to correct the correlation function

Our calculations of the deuteron formation rate and n–p correlation function neglect the Coulomb interaction between the proton and the charged source This can be important for *slow* protons or/and sources with *large* charges The role of the Coulomb effects has been recently studied in ref [15]

Let us briefly summarize our considerations Recent measurements of the n–n and n–p correlations [9,10] show that the n–p correlation function can be determined very precisely Since the n–p correlations and the deuteron formation both appear due to final state interactions, we suggest simultaneous analysis of these phenomena We have calculated the deuteron formation rate $\mathcal{A}$ and the n–p correlation function $\mathcal{R}$ expressing them both through the particle source parameters When $r_0 \gg v\tau$, i e the source radius is much greater than the product of the source life time and nucleon velocity in the source rest frame, $\mathcal{A}$ and $\mathcal{R}$ can be well approximated by the analytic expressions (9) and (15), respectively, found for $v\tau=0$ The qualitatively different behaviour of the singlet and triplet correlation functions opens the possibility to study the spin characteristics of particle sources in nuclear collisions

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