SMALL OSCILLATIONS OF A COLLISIONLESS QUARK PLASMA

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The oscillations of a collisionless quark plasma are studied on the basis of the gauge covariant kinetic equations. The small oscillation approach provides the dispersion relations which coincide with those predicted by the finite-temperature QCD in one-loop approximation.

The long range of the chromodynamic forces suggests a rich spectrum of collective excitations of QCD plasma. Such excitations have been widely discussed, see e.g. the review in ref. [1], by means of the finite-temperature quantum field theory, where the positions of poles of the gluon propagator provide the dispersion relations of plasma waves. In this paper we consider the collisionless plasma oscillations on the basis of kinetic equations proposed by Heinz [2]. The derivation of these equations is given in ref. [3]. In fact, the approximation of classical collisionless plasma seems not very realistic for the QCD plasma because of the high density of the deconfined phase. Anyway it is interesting to consider the problem of plasma oscillations from a point of view different from that of the finite-temperature perturbative QCD. On the other hand, the kinetic theory approach provides results of transparent physical interpretation, which agree with those found in the one-loop approximation.

Recently Heinz [4] has attempted to study the quark-plasma waves in the framework of the variant of his kinetic theory [2,5], where the quark color is a continuous classical variable of the distribution function. However, as pointed out in my comment [6], the transport equations are not, in this case, gauge covariant. Heinz [7] and the authors of ref. [8] argue that the equations are gauge covariant because the color variable Q_a, $a=1,\dots,8$, transforms under infinitesimal local transformations as

$$Q_a \to Q_a + f_{abc}\, \omega_b(x) Q_c,$$

where $\omega_b(x)$ is the infinitesimal transformation parameter and f_{abc} is the group structure constant. If one accepts the above transformation law, the color cannot be treated as an independent variable of the distribution function since Q_a depends on a space point x [through $\omega_b(x)$]. Then the kinetic equations do not determine Q_a as a function of x. There is no quantum analog of the above transformation law since, in contrast to Heinz's statement [7], the Gell-Mann matrices do not change under gauge transformations.

Let me also note that Heinz's findings [4,9] concerning Landau damping of the plasma waves are not new. It was noticed many years ago that the Landau damping was absent for the time-like waves [10]; see also the handbook in ref [11].

To make this paper complete we briefly present the color kinetic theory of quarks [2,3].

Let us consider the gas of quarks interacting via the classical non-abelian SU(3) potentials $A_a^\mu(x)$. We neglect the thermal (non-virtual) gluons. For simplicity we treat quarks as spinless, and for the same reason the quarks are of one flavor only. The inclusion of several flavors is straightforward.

The quark (antiquark) distribution function is a two-color-index tensor $f_{ij}(p,x)$ [$\tilde{f}_{ij}(p,x)$], [$p=(E,\boldsymbol{p})$, $x=(t,\boldsymbol{x})$] which transforms with adjoint representation (as octet) under gauge transformations; the trace $f_{ii}(p,x)$ is gauge invariant. The distribution functions satisfy the transport equa-

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tions [‡1]

$$p^{\mu}D_{\mu}f(p,x)-\tfrac{1}{2}gp_{\mu}(\partial/\partial p^{\nu})\{F^{\mu\nu}(x),f(p,x)\}=0, \quad (1a)$$

$$p^{\mu}D_{\mu}\tilde{f}(p,x)+\tfrac{1}{2}gp_{\mu}(\partial/\partial p^{\nu})\{F^{\mu\nu}(x),\tilde{f}(p,x)\}=0, \quad (1b)$$

where D_μ is the covariant derivative in the adjoint representation, $F^{\mu\nu}=\frac{1}{2}\lambda^a F_a^{\mu\nu}$ is the stress tensor of the chromodynamic field (λ^a is the Gell-Mann matrix), which is generated by the color current in a self-consistent way,

$$D_\mu F^{\mu\nu}(x)=j^\nu(x), \quad (2)$$

with

$$j^\nu_{ij}(x)=\frac{g}{2}\int\frac{d^3p}{(2\pi)^3E}p^\nu\left[f_{ij}(p,x)-\tilde{f}_{ij}(p,x)-\tfrac{1}{3}\delta_{ij}\left(f_{kk}(p,x)-\tilde{f}_{kk}(p,x)\right)\right]. \quad (3)$$

The color indices are suppressed in eqs. (1), (2). Eq. (2) can be rewritten in the form

$$\partial_\mu F_a^{\mu\nu}-gf_{abc}A_{\mu b}F_c^{\mu\nu}=j_a^\nu, \quad (4)$$

where

$$j_a^\nu(x)=\frac{g}{2}\int\frac{d^3p}{(2\pi)^3E}p^\nu\lambda^a_{ji}\times\left[f_{ij}(p,x)-\tilde{f}_{ij}(p,x)\right]. \quad (5)$$

The gauge covariant set of eqs. (1), (2) can be found from the transport equation of the quark Wigner function [2,3] by means of the standard methods [12].

We are looking for the solutions of eqs. (1), (2) describing the small oscillations around the thermodynamical equilibrium state. Therefore the choose we distribution functions in the form

$$f_{ij}(p,x)=f^{eq}_{ij}(p)+\phi(p,x)\chi_{ij}(x),$$
$$\tilde{f}_{ij}(p,x)=\tilde{f}^{eq}_{ij}(p)+\tilde{\phi}(p,x)\tilde{\chi}_{ij}(x), \quad (6)$$

‡1 In the first version of this paper we have used the transport equations proposed by the author where the anticommutator $\frac{1}{2}\{F,f\}$ is replaced by the product Ff. Both sets of equations are equivalent for quasi-equilibrium plasma since, in this case, $[F,f]\approx 0$.

where f^{eq} and $\tilde{f}^{eq}$ are equilibrium distribution functions of quarks and antiquarks, respectively:

$$f^{eq}_{ij}(p)=n(p)\delta_{ij},\quad \tilde{f}^{eq}_{ij}(p)=\tilde{n}(p)\delta_{ij}.$$

One sees that the equilibrium distribution functions are gauge invariant and they give zero color current. It is assumed that

$$|\phi(p,x)\chi_{ij}(x)|\ll|n(p)|,$$
$$|(\partial\phi/\partial p^\nu)\chi_{ij}|\ll|\partial n/\partial p^\nu|,$$

and analogous relations for the antiquark functions. The factorized form of the function describing the plasma deviation from the equilibrium has been dictated by the following reasoning.

As follows from refs. [2,3], the matrices f_{ij} and $\tilde{f}_{ij}$ are hermitean. So are the $\phi\chi_{ij}$ and $\tilde{\phi}\tilde{\chi}_{ij}$ matrices. Because χ_{ij} transforms under the unitary local transformation $U(x)$ as

$$\chi(x)\to U(x)\chi(x)U^\dagger(x),$$

it can be made diagonal by means of the gauge transformation. Therefore the distribution function f of the form (6) can be diagonal due to the choice of gauge. The question arises whether the matrices χ and $\tilde{\chi}$ can be simultaneously diagonalized. We assume that it is the case (which demands $[\chi,\tilde{\chi}]=0$), however, the question remains open. So the matrices $\phi\chi$ and $\tilde{\phi}\tilde{\chi}$ read

$$\phi\chi=\begin{pmatrix}\alpha&0&0\\0&\beta&0\\0&0&\gamma\end{pmatrix},\quad \tilde{\phi}\tilde{\chi}=\begin{pmatrix}\tilde{\alpha}&0&0\\0&\tilde{\beta}&0\\0&0&\tilde{\gamma}\end{pmatrix}. \quad (7)$$

Substituting the distribution functions f and $\tilde{f}$ of the form (6), (7) in eqs. (1a), (1b) and using the explicit expressions of the Gell-Mann matrices one finds the equations

$$p_\mu\partial^\mu\alpha+\tfrac{1}{2}g\left[F_3^{\mu\nu}+(1/\sqrt{3})F_8^{\mu\nu}\right]W_{\mu\nu}=0, \quad (8a)$$

$$p_\mu\partial^\mu\beta-\tfrac{1}{2}g\left[F_3^{\mu\nu}-(1/\sqrt{3})F_8^{\mu\nu}\right]W_{\mu\nu}=0, \quad (8b)$$

$$p_\mu\partial^\mu\gamma-(g/\sqrt{3})F_8^{\mu\nu}W_{\mu\nu}=0, \quad (8c)$$

and six algebraic equations which are trivially satisfied for the quasi-equilibrium plasma; $W_{\mu\nu}\equiv p_\mu\partial n/\partial p^\nu$.

The antiquark equations (9a)–(9c) can be found from eqs. (8a)–(8c) by means of the following replacement:

$$(\alpha,\ \beta,\ \gamma,\ n,\ F_a^{\mu\nu}) \to (\tilde{\alpha},\ \tilde{\beta},\ \tilde{\gamma},\ \tilde{n},\ -F_a^{\mu\nu}).$$

Putting f and $\tilde{f}$ of the form (6), (7) in eq. (5) we get

$$j_a^\nu(x) = 0, \quad \text{for} \quad a = 1, 2, 4, 5, 6, 7,$$

$$j_3^\nu(x) = \frac{g}{2} \int \frac{d^3 p}{(2\pi)^3 E} p^\nu (\alpha - \beta - \tilde{\alpha} + \tilde{\beta}),$$

$$j_8^\nu(x) = \frac{g}{2\sqrt{3}} \int \frac{d^3 p}{(2\pi)^3 E} p^\nu \times (\alpha + \beta - 2\gamma - \tilde{\alpha} - \tilde{\beta} + 2\tilde{\gamma}). \quad (10)$$

The next step in the small oscillation approach [11] is to assume that α, $\tilde{\alpha}$, β, $\tilde{\beta}$, γ, $\tilde{\gamma}$ and $F_3^{\mu\nu}$, $F_8^{\mu\nu}$ depend on x as $\exp(-ikx)$. However, the question arises as to whether the components α, $\tilde{\alpha}$, β, $\tilde{\beta}$, γ, $\tilde{\gamma}$ oscillate with the same frequency or whether there are various frequencies ω_α, ω_β, $\omega_\gamma \ldots$ for each component? It is seen from eq. (8c) that the field $F_8^{\mu\nu}$ has to oscillate with the frequency ω_γ. If this is so, it follows from eq. (8a) that the amplitude of this field vanishes because α oscillates with the frequency ω_α. Consequently γ vanishes. In this way one proves that the components α, $\tilde{\alpha}$, β, $\tilde{\beta}$, γ and $\tilde{\gamma}$ have to oscillate with the same frequency.

Let us now discuss eqs. (4) with the currents (10). We are looking for the solutions with $F_3^{\mu\nu}$ and $F_8^{\mu\nu}$ depending on x as $\exp(-ikx)$. The solution of this property reads

$$A_a^\mu(x) = 0 \quad \text{for} \quad a = 1, 2, 4, 5, 6, 7,$$
$$A_3^\mu(x) = a_3^\mu \exp(-ikx),$$
$$A_8^\mu(x) = a_8^\mu \exp(-ikx), \quad (11)$$

where a_3^μ and a_8^μ are constant four-vectors. To check the correctness of the above solution, one has to remember that $[\lambda_3, \lambda_8] = 0$ and the respective group structure constants vanish. Let us observe that because of (11) the stress tensors $F_3^{\mu\nu}$ and $F_8^{\mu\nu}$ are expressed through A_3^μ and A_8^μ, respectively, as the electrodynamic stress tensor through the electromagnetic four-potential.

We introduce the new functions

$$\eta(p, x) = \alpha(p, x) - \tilde{\alpha}(p, x),$$
$$\zeta(p, x) = \beta(p, x) - \tilde{\beta}(p, x),$$
$$\varphi(p, x) = \gamma(p, x) - \tilde{\gamma}(p, x). \quad (12)$$

From eqs. (8a)–(8c) and (9a)–(9c) we get the set of kinetic equations which we write down in the more familiar three-vector notation with the chromoelectric and chromomagnetic field $\boldsymbol{E}_3$, $\boldsymbol{E}_8$ and $\boldsymbol{B}_3$, $\boldsymbol{B}_8$. Then the terms $(\boldsymbol{p} \times \boldsymbol{B}) \cdot \partial n/\partial \boldsymbol{p}$ occur. For the isotropic plasma considered here such terms equal zero [11] because $\partial n/\partial \boldsymbol{p}$ is parallel to $\boldsymbol{p}$. For this reason the magnetic fields play no role for the isotropic plasma. Therefore the set of kinetic equations looks like

$$(\partial/\partial t + \boldsymbol{V} \cdot \nabla)\eta + \tfrac{1}{2} g [\boldsymbol{E}_3 + (1/\sqrt{3})\boldsymbol{E}_8] \cdot \boldsymbol{\Omega} = 0,$$
$$(\partial/\partial t + \boldsymbol{V} \cdot \nabla)\zeta - \tfrac{1}{2} g [\boldsymbol{E}_3 - (1/\sqrt{3})\boldsymbol{E}_8] \cdot \boldsymbol{\Omega} = 0,$$
$$(\partial/\partial t + \boldsymbol{V} \cdot \nabla)\varphi - (1/\sqrt{3}) g \boldsymbol{E}_8 \cdot \boldsymbol{\Omega} = 0,$$
$$\operatorname{div} \boldsymbol{E}_3 = \rho_3 = \frac{g}{2} \int \frac{d^3 p}{(2\pi)^3} (\eta - \zeta),$$
$$\operatorname{div} \boldsymbol{E}_8 = \rho_8 = \frac{g}{2\sqrt{3}} \int \frac{d^3 p}{(2\pi)^3} (\eta + \zeta - 2\varphi), \quad (13)$$

where $\boldsymbol{V} \equiv \boldsymbol{p}/E$ and $\boldsymbol{\Omega} \equiv (\partial/\partial \boldsymbol{p})(n + \tilde{n})$.

It is seen from the above equations that the problem of the small oscillations of the quark plasma closely resembles that one of the electromagnetic plasma. Let us note here that analogous considerations for the quark plasma with SU(2) color group provide equations which exactly coincide with those of the electron–positron plasma.

We introduce now the polarization vectors $\boldsymbol{P}_3$, $\boldsymbol{P}_8$ defined as

$$\operatorname{div} \boldsymbol{P}_a = -\rho_a, \quad \partial \boldsymbol{P}_a/\partial t = \boldsymbol{j}_a, \quad a = 3, 8. \quad (14)$$

and the chromoelectric inductions

$$\boldsymbol{D}_a = \boldsymbol{E}_a + \boldsymbol{P}_a, \quad a = 3, 8. \quad (15)$$

The definition (14) is self-consistent if

$$\partial \rho_2/\partial t + \operatorname{div} \boldsymbol{j}_a = 0, \quad a = 3, 8,$$

which is the case because the distribution functions are assumed diagonal. The chromoelectric permeability tensor ε_{ab}^{ij} (the indices i, j denote here cartesian space axes) is defined as

$$D_a^i(k) = \varepsilon_{ab}^{ij}(k) E_b^j(k), \quad a = 3, 8, \tag{16}$$

where $\boldsymbol{D}_a(k)$ and $\boldsymbol{E}_a(k)$ are Fourier transformed $\boldsymbol{D}_a(x)$ and $\boldsymbol{E}_a(k)$ fields. One sees that

$$\varepsilon_{ab}^{ij}(k) = \delta_{ab} \varepsilon^{ij}(k).$$

Using the definitions (14)–(16) and substituting in eqs. (13) η, ζ, φ, $\boldsymbol{E}_3$ and $\boldsymbol{E}_8$ which depend on x as $\exp(-\mathrm{i}kx)$ $[k = (\omega, \boldsymbol{k})]$, one finds

$$\varepsilon^{ij}(k) = \delta_{ij} - \frac{g^2}{2\omega} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{V^i \Omega^j}{\boldsymbol{k} \cdot \boldsymbol{V} - \omega - \mathrm{i}0^+}, \tag{17}$$

where the infinitesimal parameter 0^+ has been introduced to make the integral (17) well defined [11]. The chromoelectric permeability can be split into the longitudinal ε_{L} and transverse ε_{T} parts

$$\varepsilon^{ij} = \varepsilon_{\mathrm{T}}\left(\delta_{ij} - k^i k^j / \boldsymbol{k}^2\right) + \varepsilon_{\mathrm{L}} k^i k^j / \boldsymbol{k}^2.$$

Because the equations of motion of the fields $F_3^{\mu\nu}$ and $F_8^{\mu\nu}$ coincide (for small oscillations) with the Maxwell equations of the electrodynamic fields, the dispersion relations of longitudinal and transverse plasma oscillations, respectively, are defined by the well-known equations [11]

$$\varepsilon_{\mathrm{L}}(k) = 0, \quad \varepsilon_{\mathrm{T}}(k) - \boldsymbol{k}^2/\omega^2 = 0. \tag{18}$$

The formula (17) (with $g^2 = 2e^2$) is identical to that of the electrodynamic plasma. Therefore the solutions of eqs. (18) can be found in the literature [11].

As an illustration we consider the hot plasma of massless quarks and antiquarks. The dispersion relations read

longitudinal modes:

$$\omega^2 = \omega_0^2 + \tfrac{3}{5}\boldsymbol{k}^2 \quad \text{for} \quad \omega_0^2 \gg \boldsymbol{k}^2,$$

$$\omega^2 = \boldsymbol{k}^2\left[1 + 4\exp\left(-2\boldsymbol{k}^2/3\omega_0^2 - 2\right)\right] \quad \text{for} \quad \omega_0^2 \ll \boldsymbol{k}^2, \tag{19}$$

transverse modes:

$$\omega^2 = \omega_0^2 + \tfrac{6}{5}\boldsymbol{k}^2 \quad \text{for} \quad \omega_0^2 \gg \boldsymbol{k}^2,$$

$$\omega^2 = \tfrac{3}{2}\omega_0^2 + \boldsymbol{k}^2 \quad \text{for} \quad \omega_0^2 \ll \boldsymbol{k}^2, \tag{20}$$

where the Langmuir frequency ω_0 for the classical plasma of zero baryon charge of N_{f} flavours is

$$\omega_0^2 = (2/3\pi^2) N_{\mathrm{f}} g^2 T^2. \tag{21a}$$

For the Fermi–Dirac distribution function of quarks the plasma frequency reads

$$\omega_0^2 = \tfrac{1}{18} N_{\mathrm{f}} g^2 T^2. \tag{21b}$$

Because the longitudinal and transverse oscillations are time-like ($\omega^2 > \boldsymbol{k}^2$), the phase velocity of the waves is greater than the velocity of light. For this reason the Landau damping is absent. However, it does not mean that there is no energy dissipation of the plasma waves. Because at the temperature of about 200 MeV the plasma frequency is much greater than the quark mass, the quark–antiquark pair creation can be an effective damping mechanism, although this mechanism cannot be studied in the framework of the transport equations without collision terms.

The formulae (19), (20) exactly coincide with those found on the basis of the finite temperature QCD in the one-loop approximation [1,13]. Our value of the plasma frequency (21b) deviates from that given in ref. [13] $[\omega_0^2 = \tfrac{1}{18}(N_{\mathrm{f}} + 6) g^2 T^2]$ because the self-interactions of gluons (gluon loops) are neglected in our considerations.

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