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Comments

Comments are short papers which comment on papers of other authors previously published in **Physical Review C**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract.*

Applicability of transport theory of gases to the description of excited nuclear matter

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An important role for three-particle collisions in nuclear matter is shown. The idea of a mean field is discussed. It is argued that the applicability of gas theory methods to the description of excited nuclear matter is very questionable.

It seems to be very natural to describe a high-energy nucleus-nucleus interaction as a superposition of binary nucleon-nucleon collisions. In fact, such a picture has been exploited in the Monte Carlo intranuclear cascade models and in the models where the kinetic equation has been used in an explicit form. For a long list of references see the review by Nagamiya and Gyulassy.[1] The majority of authors have tried to reproduce experimental data, particularly inclusive spectra of secondary particles. Usually the assumptions of model calculations are justified by an agreement of these calculations with experimental data. Recently, more subtle problems have been discussed in the literature.[2,3] Namely, there have been attempts to describe the time evolution of a colliding nuclear system and to extract such characteristics as the equation of state of highly excited matter occurring at the early stages of collisions. In my opinion, such an analysis demands a careful discussion of the applicability of the methods based on the Boltzmann equation with only binary collisions taken into account. Let me note that in both papers nuclear matter at double or triple normal density has been considered. Lack of discussion in the quoted papers provokes me to write this Comment. Below I consider the role of three-particle collisions and the idea of mean field.

A simple estimate shows that the volume related to a strong interaction radius is very close to the volume per nucleon in nuclear matter at normal density. Thus, one suspects that, besides binary collisions, collisions with more than two particles in the initial state play an important role in transport phenomena. Let me consider the problem more carefully. Because the temperature of nuclear matter does not exceed a value of about 140 MeV, I treat nucleons as nonrelativistic particles in the local rest frame of nuclear matter. Restricting myself to temperatures higher than 50 MeV and densities not higher than double normal, I neglect quantum effects. Simple considerations have led me to the following numbers of two- and three-particle collisions per unit time per unit volume:

$$\begin{aligned} C_2 &= \int d^3p_1\, d^3p_2\, \sigma |\mathbf{v}_1 - \mathbf{v}_2| f(\mathbf{r},\mathbf{p}_1) f(\mathbf{r},\mathbf{p}_2) \ , \\ C_3 &= \frac{4}{3\sqrt{\pi}} \int d^3p_1\, d^3p_2\, d^3p_3\, \sigma^{5/2} |\mathbf{v}_1 - \mathbf{v}_2| \\ &\quad \times f(\mathbf{r},\mathbf{p}_1) f(\mathbf{r},\mathbf{p}_2) f(\mathbf{r},\mathbf{p}_3) \ , \end{aligned} \tag{1}$$

where σ is the total nucleon-nucleon cross section, $\mathbf{p}_i$ and $\mathbf{v}_i$ are the momentum and velocity of an ith nucleon, and $f(\mathbf{r},\mathbf{p})$ is the distribution function. I have assumed that three particles undergo interaction when all of them occur simultaneously in the interaction sphere of radius equal to $\sqrt{\sigma/\pi}$.

Substituting the Maxwell-Boltzmann distribution functions into (1) and assuming that σ depends weakly on $\mathbf{p}$ [when compared with $f(\mathbf{r},\mathbf{p})$], one finds the numbers of two- and three-particle collisions for a gas in equilibrium

$$C_2 = \frac{4}{\sqrt{\pi}} \sigma \rho^2 \left(\frac{T}{m} \right)^{1/2} \ ,$$

$$C_3 = \frac{16}{3\pi} \sigma^{5/2} \rho^3 \left(\frac{T}{m} \right)^{1/2} \ ,$$

where ρ and T are the density and temperature of the gas and m is the nucleon mass. Let me notice that the ratio C_3/C_2 is independent of the temperature and

$$\frac{C_3}{C_2} = \frac{4}{3\sqrt{\pi}} \sigma^{3/2} \rho \ .$$

As one can expect, the coefficient in front of the density in the above formula equals the volume of the interaction sphere. For $\sigma = 35$ mb and $\rho = \rho_0$, where ρ_0 is the normal nuclear density equal to 0.16 fm^{-3}, one finds $C_3/C_2 = 0.79$, while the Boltzmann equation is applicable when this ratio is much less than unity. For $\rho > 1.5\rho_0$ the number of three-nucleon collisions is greater than that of two nucleons.

The role of nonbinary collisions in relativistic heavy-ion interactions has been earlier studied within the intranuclear cascade model.[4] It has been found that the contribution from such collisions reaches up to 60% for ^{12}C-^{12}C interactions.

In spite of the above arguments one can say that the theory of zero temperature nuclear matter teaches us that multiparticle interactions are not of principal importance. However, it should be stressed that the arguments coming from the theory of cold nuclear matter are inadequate for the excited matter discussed in this note. As observed by Weisskopf,[5] the Pauli principle plays a crucial role for determining the characteristics of cold nuclear matter. Due to

Pauli blocking, the effects of collisions, in particular multinucleon collisions, are highly suppressed, and the motion of a nucleon is similar to that of a free particle. Such a situation does not occur for excited matter.

One may hope that multiparticle effects can be taken into account by introducing the mean field to the kinetic equation. The idea of mean field comes from the theory of electromagnetic plasma, where this field is usually introduced under the assumption that the average interparticle distance is much smaller than the Debye radius, i.e., the effective range of the forces. An analogous situation never arises for nuclear matter. In the case of cold matter one can assume that a nucleon moves in the smooth potential because of the quantum effects quoted previously. Thus, the introduction of the mean field is justified. The Pauli blocking is negligible for excited matter and the idea of this field is less supported. In spite of this ambiguity the introduction of the mean field cannot solve the problem of the proper treatment of multiparticle collisions. The point is that the mean field, in contrast to the collisions, does not lead to entropy production.[6] On the other hand, due to the introduction of the mean field, collective flow effects can be significantly overestimated.

The presented discussion leads me to the conclusion that the applicability of the transport theory of gases for the description of excited nuclear matter is very questionable. Thus, the methods of this theory have to be used with reservations.

At the end let me comment on the recent paper by Malfliet.[7] To describe nucleus-nucleus collisions, he has used the methods known from the theory of liquids. Namely, the Enskog equation has been applied. The attractive forces between nucleons have been ascribed to the mean field, while the hard core repulsive potential occurred in the collision term. In this way only a small part of the nucleon-nucleon cross section has been related to the collisions. In my opinion, the Malfliet approach is the best one which can be done now. However, because of the important phenomenological input the predictive power of this approach is rather modest.

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