Identity method to study chemical fluctuations in relativistic heavy-ion collisions

Marek Gaździcki
Goethe-Universität Frankfurt, Max-von-Laue Strasse 1, D-60438 Frankfurt am Main, Germany, and
Institute of Physics, Jan Kochanowski University, ul. Świętokrzyska 15, PL-25-406 Kielce, Poland

Katarzyna Grebieszkow and Maja Maćkowiak
Faculty of Physics, Warsaw University of Technology, ul. Koszykowa 75, PL-00-662 Warszawa, Poland

Stanisław Mrówczyński
Institute of Physics, Jan Kochanowski University, ul. Świętokrzyska 15, PL-25-406 Kielce, Poland and
Sołtan Institute for Nuclear Studies, ul. Hoża 69, PL-00-681 Warszawa, Poland
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Event-by-event fluctuations of the chemical composition of the hadronic final state of relativistic heavy-ion collisions carry valuable information on the properties of strongly interacting matter produced in the collisions. However, in experiments incomplete particle identification distorts the observed fluctuation signals. The effect is quantitatively studied and a new technique for measuring chemical fluctuations, the identity method, is proposed. The method fully eliminates the effect of incomplete particle identification. The application of the identity method to experimental data is explained.

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I. INTRODUCTION

Event-by-event fluctuations of chemical (particle-type) composition of hadronic final states of relativistic heavy-ion collisions are expected to be sensitive to properties of strongly interacting matter produced in the collisions [1]. Specific fluctuations can signal the onset of deconfinement when the collision energy becomes sufficiently high to create droplets of quark-gluon plasma [2]. At higher collision energies, where the quark-gluon phase is abundantly produced at the early collision stage, large chemical fluctuations can occur as the system hits the critical point of strongly interacting matter in the course of its temporal evolution [3,4]. It is thus certainly of interest to study event-by-event chemical fluctuations experimentally. First data coming from the CERN SPS [5–7] and BNL RHIC [8] were already published. The results are not very conclusive yet and more systematic measurements are needed. In addition, the question arises whether data analysis methods can be improved.

In real experiments it is impossible to determine uniquely the type of every detected particle. The identification requires measurements of particle electric charge and mass. Precise mass measurements are experimentally difficult and expensive. For this reason analyses of chemical fluctuations are usually performed in a limited acceptance where particle identification is relatively reliable. However, sensitivity to fluctuations of range larger than the acceptance window is then lost and signals from fluctuations of shorter range are usually diluted. Furthermore, it should be noted that results on fluctuations, unlike those on single-particle spectra, cannot be corrected for the limited acceptance. Often it is possible to enlarge the acceptance, but at the expense of a significant contamination of the sample by misidentified particles. The effect of particle misidentification can distort measured fluctuations. Thus, incomplete particle identification is a serious obstacle to the precise measurement of chemical fluctuations.

Although it is usually impossible to identify every detected particle, one can in general determine with high accuracy the percentage (averaged over many interactions) of, say, kaons among produced hadrons. This information will be shown to be sufficient to fully eliminate the effect of incomplete identification. In this paper we propose a new experimental technique called the *identity method* which achieves the goal independently of the specific properties of the chemical fluctuations under study.

In the study of chemical fluctuations the NA49 Collaboration [5–7] used the measure $\sigma_{\rm dyn}$, which is defined as the difference between fluctuations measured in real and *mixed* events. The effect of particle misidentification is accounted for by including it in the mixed events. The STAR Collaboration [8] used, in addition to the $\sigma_{\rm dyn}$ measure, the quantity $\nu_{\rm dyn}$. The latter one assumes that particles are uniquely identified.

It was suggested long ago [9,10] to quantify chemical fluctuations by the measure Φ [11], which proved to be efficient in experimental studies of event-by-event fluctuations of particle transverse momentum [12,13], electric charge [14], and quite recently of azimuthal angle [15]. The Φ measure, unlike $\sigma_{\rm dyn}$ and $\nu_{\rm dyn}$, is a strongly intensive measure of fluctuations. Namely, its magnitude is independent of the number and of the distribution (fluctuation) of the number of particle sources, if the sources are identical and independent from each other. This feature, which is discussed in detail in Ref. [16], is important in experimental studies of relativistic heavy-ion collisions where the collision centrality is never fully controllable. However, up to now it was unclear how to correct measurements of chemical Φ for the effect of particle misidentification.

The identity method, which is developed here, uses the fluctuation measure Ψ, a simple modification of Φ. The measure Ψ, similar to Φ, is strongly intensive but the modification makes it possible to correct the measurements for the effect of particle misidentification. Below we show that the measure Ψ can be factorized into a coefficient, which represents the effect of misidentification, and the quantity $\Psi_{\rm CI}$, which corresponds to the value Ψ would have for complete identification. The misidentification coefficient can be determined from the data in a model independent way. Therefore, the identity method provides the value of the fluctuation measure as it would be obtained in an experiment in which every particle is uniquely identified.

Before the identity method is presented, we introduce and discuss in Sec. II the fluctuation measures $\sigma_{\rm dyn}$, $\nu_{\rm dyn}$, Φ, and Ψ. In Sec. III we demonstrate by a Monte Carlo simulation how the effect of misidentification distorts the chemical fluctuations as quantified by $\nu_{\rm dyn}$, Φ, and Ψ. The identity method is formulated in Sec. IV. Instead of conclusions we present in the last section the steps required to apply the identity method to experimental data. To simplify the presentation, we consider chemical fluctuations of events composed only of kaons and pions. Clearly, kaons and pions can be replaced by particles of any other sort.

II. MEASURES OF FLUCTUATIONS

As mentioned in the Introduction, fluctuations of chemical composition of final states of relativistic heavy-ion collisions can be studied in several ways. The NA49 Collaboration [5–7] measured event-by-event fluctuations of the particle ratios K/π, K/p, p/π and determined the quantity $\sigma_{\rm dyn}$ defined as

$$\sigma_{\rm dyn} = {\rm sgn}\left(\sigma^2_{\rm data} - \sigma^2_{\rm mixed}\right)\sqrt{\left|\sigma^2_{\rm data} - \sigma^2_{\rm mixed}\right|}, \tag{1}$$

where $\sigma_{\rm data}$ and $\sigma_{\rm mixed}$ are the relative width (the width divided by the mean) of the event-by-event particle ratio distribution in, respectively, the data and artificially generated mixed events where every particle comes from a different real event. The fluctuations present in mixed events are attributable to the effect of particle misidentification and the statistical noise caused by the finite number of particles.

The STAR Collaboration used [8] the quantity $\nu_{\rm dyn}$ to measure chemical fluctuations. For the case of a two-component system of pions and kaons $\nu_{\rm dyn}$ is defined as

$$\nu_{\rm dyn} = \frac{\langle N_K(N_K-1)\rangle}{\langle N_K\rangle^2} + \frac{\langle N_\pi(N_\pi-1)\rangle}{\langle N_\pi\rangle^2} - 2\frac{\langle N_K N_\pi\rangle}{\langle N_K\rangle\langle N_\pi\rangle}, \tag{2}$$

where N_K and N_π are the numbers of kaons and pions in a given event and $\langle\cdots\rangle$ denotes averaging over events. The quantity $\nu_{\rm dyn}$ is defined in such a way that, in particular, it vanishes when the multiplicity distributions of pions and kaons are both Poissonian ($\langle N_i(N_i-1)\rangle = \langle N_i\rangle^2, i=\pi,\ K$) and independent from each other ($\langle N_K N_\pi\rangle = \langle N_K\rangle\langle N_\pi\rangle$). Thus, it is constructed to quantify the deviations of the fluctuations from the Poissonian noise. For large-enough particle multiplicities one finds the approximate relation $\nu_{\rm dyn} \approx \sigma^2_{\rm data} - \sigma^2_{\rm mixed}$, which gives $\nu_{\rm dyn} \approx {\rm sgn}(\sigma_{\rm dyn})\,\sigma^2_{\rm dyn}$ [8]. We note that the quantity $\nu_{\rm dyn}$ implicitly assumes unique identification of all particles.

As already noted, it was advocated long ago [9,10] to employ the measure Φ [11] to study chemical fluctuations. The measure is defined in the following way. One introduces the variable $z \equiv x - \overline{x}$, where x is a single-particle characteristic such as the transverse momentum or azimuthal angle. The overline denotes averaging over the single-particle inclusive distribution. The event variable Z, which is a multiparticle analog of z, is defined as $Z \equiv \sum_{i=1}^{N}(x_i - \overline{x})$, where the sum runs over the N particles in a given event. By construction, $\langle Z\rangle = 0$. The measure Φ is finally defined as

$$\Phi \equiv \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}} - \sqrt{\overline{z^2}}. \tag{3}$$

The measure Φ vanishes in the absence of interparticle correlations. This situation is discussed in some detail below for the case of chemical fluctuations. Here we note that the measure also possesses another important property: It is strongly intensive, which means that it is independent of the number and of the distribution (fluctuation) of the number of particle sources, if the sources are identical and independent from each other. In particular, if a nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions, then $\Phi_{AA} = \Phi_{NN}$. The strongly intensive property is a very valuable feature of Φ because centrality selection in relativistic heavy-ion collisions is never perfect and events of different numbers of particle sources are always mixed up. The strongly intensive property is also desirable when different centralities or different colliding systems are compared. For a discussion of strongly intensive quantities, see Ref. [16].

The analysis of chemical fluctuations can be performed with the help of Φ in two different but fully equivalent ways. In the first method [9], using the identity variable, chemical fluctuations are treated in analogy to fluctuations of transverse momentum. In the second method Φ is calculated from the moments of the multiplicity distributions [10].

We next describe the first method for the example of a two-component system of pions and kaons. One defines the single-particle variable x as $x = w_K$, where w_K is called the *kaon identity* and $w^i_K = 1$ if the ith particle is a kaon and $w^i_K = 0$ if the ith particle is a pion. This implies unique particle identification. One then directly uses the definition (3) to evaluate Φ.

Let us now discuss the most important case for which the measure Φ of chemical fluctuations vanishes. Because the inclusive distribution of w_K equals

$$P(w_K) = \begin{cases} \dfrac{\langle N_\pi\rangle}{\langle N\rangle} & \text{for} \quad w_K = 0, \\[2ex] \dfrac{\langle N_K\rangle}{\langle N\rangle} & \text{for} \quad w_K = 1, \end{cases} \tag{4}$$

one finds

$$\overline{z^2} \equiv \overline{w_K^2} - \overline{w_K}^2 = \frac{\langle N_K\rangle}{\langle N\rangle}\left(1 - \frac{\langle N_K\rangle}{\langle N\rangle}\right). \tag{5}$$

When interparticle correlations are absent, the distribution of particle identities in events of multiplicity N reads

$$P_N\left(w^1_K, w^2_K, \ldots, w^N_K\right) = \mathcal{P}_N P\left(w^1_K\right) P\left(w^2_K\right)\cdots P\left(w^N_K\right), \tag{6}$$

where $\mathcal{P}_N$ is an arbitrary multiplicity distribution of particles of any type. One shows that $\langle Z^2\rangle$ computed with the event distribution (6) equals $\langle Z^2\rangle = \langle N\rangle\overline{z^2}$ and, consequently, $\Phi = 0$.

In the second method the measure Φ of chemical fluctuations is obtained from the moments of the experimentally measured multiplicity distributions of kaons and pions. As shown in Ref. [10], one has

$$\overline{z^2} = \frac{\langle N_K\rangle\langle N_\pi\rangle}{\langle N\rangle^2}, \tag{7}$$

$$\frac{\langle Z^2\rangle}{\langle N\rangle} = \frac{\langle N_\pi\rangle^2}{\langle N\rangle^3}\left(\langle N_K^2\rangle - \langle N_K\rangle^2\right) + \frac{\langle N_K\rangle^2}{\langle N\rangle^3}\left(\langle N_\pi^2\rangle - \langle N_\pi\rangle^2\right) - 2\,\frac{\langle N_K\rangle\langle N_\pi\rangle}{\langle N\rangle^3}\left(\langle N_K N_\pi\rangle - \langle N_K\rangle\langle N_\pi\rangle\right), \tag{8}$$

which substituted in Eq. (3) gives the measure Φ.

The formulas (7) and (8) clearly show that Φ, like $\nu_{\rm dyn}$, vanishes when the multiplicity distributions of pions and kaons are both Poissonian and independent from each other. However, more generally, Φ vanishes for any multiplicity distribution provided it satisfies (6). The distribution (6) leads to the multiplicity distribution of the form

$$\mathcal{P}_{N_K N_\pi} = \mathcal{P}_{N_K+N_\pi} \times \binom{N_K+N_\pi}{N_K}\left(\frac{\langle N_K\rangle}{\langle N\rangle}\right)^{N_K}\left(1-\frac{\langle N_K\rangle}{\langle N\rangle}\right)^{N_\pi}, \tag{9}$$

with the moments

$$\langle N_K(N_K-1)\rangle = \frac{\langle N_K\rangle^2}{\langle N\rangle^2}\langle N(N-1)\rangle, \tag{10}$$

$$\langle N_\pi(N_\pi-1)\rangle = \frac{\langle N_\pi\rangle^2}{\langle N\rangle^2}\langle N(N-1)\rangle, \tag{11}$$

$$\langle N_K N_\pi\rangle = \frac{\langle N_K\rangle\langle N_\pi\rangle}{\langle N\rangle^2}\langle N(N-1)\rangle. \tag{12}$$

One checks that Φ and $\nu_{\rm dyn}$ both vanish when these moments are substituted into Eq. (8) and Eq. (2), respectively.

It appears convenient for our further considerations to modify Φ to the form

$$\Psi = \frac{\langle Z^2\rangle}{\langle N\rangle} - \overline{z^2}, \tag{13}$$

which preserves the properties of Φ; it vanishes in the absence of interparticle correlations and it is strongly intensive. The measure Ψ will be used to formulate the identity method for the study of chemical fluctuations. When expressed through moments of the multiplicity distribution, it equals

$$\Psi = \frac{1}{\langle N\rangle^3}\left[\langle N_\pi^2\rangle\langle N_K\rangle^2 + \langle N_\pi\rangle^2\langle N_K^2\rangle - 2\langle N_\pi\rangle\langle N_K\rangle\langle N_\pi N_K\rangle - \langle N_\pi\rangle^2\langle N_K\rangle - \langle N_\pi\rangle\langle N_K\rangle^2\right]. \tag{14}$$

Comparing Eq. (2)) to Eq. (14) one finds that Ψ and $\nu_{\rm dyn}$ are proportional to each other:

$$\Psi = \frac{\langle N_\pi\rangle^2\langle N_K\rangle^2}{\langle N\rangle^3}\,\nu_{\rm dyn}. \tag{15}$$

We note here that $\nu_{\rm dyn}$ is not intensive but it becomes even strongly intensive when multiplied by $\langle N\rangle$, $\langle N_K\rangle$, or $\langle N_\pi\rangle$.

III. EFFECT OF MISIDENTIFICATION

As mentioned in the Introduction, complete identification of every particle is impossible. In this section we show how the incomplete particle identification influences the magnitudes of fluctuation measures. For this purpose we considered a simple model of chemical fluctuations where the multiplicity of pions is Poissonian with a mean value of 100 and the number of kaons is 20% of the number of pions (strict correlation of the numbers of kaons and pions). Actually, N_K is taken as the integer number closest to $N_\pi/5$, which is smaller or equal to $N_\pi/5$. The fluctuation measures can be easily computed analytically for the model but our aim here is to simulate the effect of incomplete particle identification.

There are many experimental techniques to measure particle mass. We discuss here the effect of misidentification referring to measurements of energy loss, dE/dx, in a detector material. This method is applied by the experiments NA49, NA61, and STAR. The detectors are equipped with Time Projection Chambers in which dE/dx is measured. The value of dE/dx can be used to identify particles because it depends on both particle mass m and momentum p in the combination of velocity ($\beta = p/\sqrt{m^2+p^2}$). In the case of large separation of the energy loss distributions of pions and kaons, as schematically shown in Fig. 1(a), almost unique particle identification is possible. This is, however, not possible when the measured pion and kaon dE/dx distributions overlap, as illustrated in Fig. 1(b).

Performing the Monte Carlo simulations we assumed that the dE/dx distributions of pions and kaons are Gaussians centered, respectively, at 1.4 and 1.2 in arbitrary units. They are normalized to the mean multiplicity of pions and of kaons, respectively. To quantify the bias caused by particle misidentification a simple particle identification scheme is used, namely, a particle is identified as a pion if $dE/dx > 1.3$ and as a kaon if $dE/dx \leqslant 1.3$. The width σ of both Gaussians is chosen to be the same but its value is varied from 0 to 0.08. With growing width of the peaks of the dE/dx distribution, the fraction of misidentified particles obviously increases. The results of the simulation are illustrated in Fig. 2, where the fluctuation measures Φ, Ψ, and $\nu_{\rm dyn}$ are shown as a function of σ. As seen, the magnitudes of Φ, Ψ, and $\nu_{\rm dyn}$ decrease as the fraction of misidentified particles grows and the measures vanish when particle identification becomes totally random. This sizable and experimentally unavoidable effect was the main motivation to develop the identity method, which fully eliminates the problem.

IV. IDENTITY METHOD

The identity method, which is described here for a two-component system of pions and kaons, utilizes the measure Ψ defined by Eq. (13). However, the kaon identity w_K is not

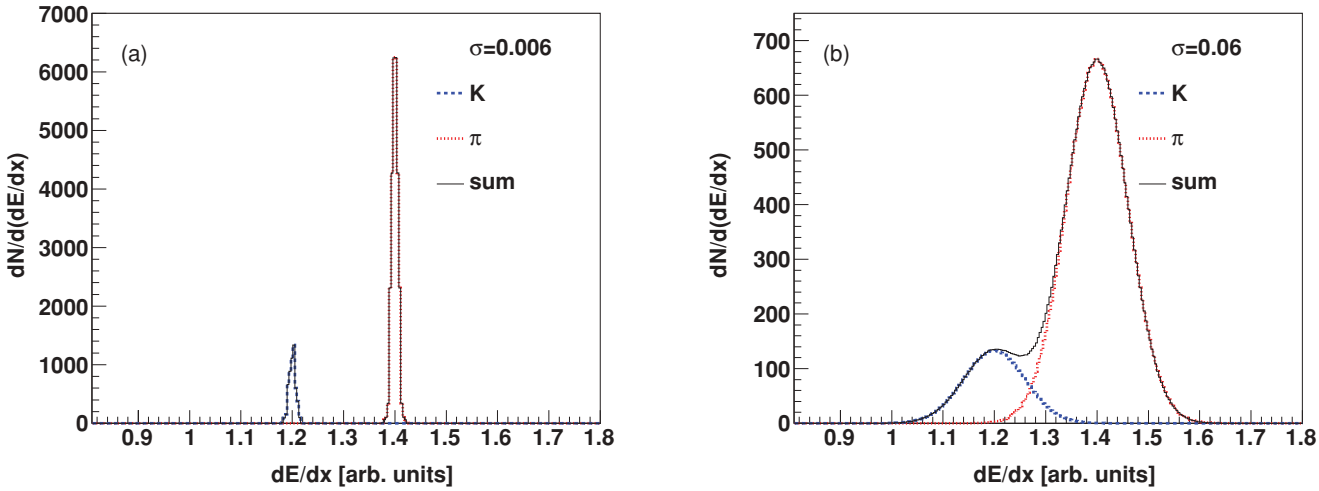

FIG. 1. (Color online) The distribution of dE/dx with nonoverlapping peaks of pions and kaons (a), which allows unique particle identification, and the distribution with overlapping peaks (b), which does not allow unique identification.

limited to either 1 or 0 anymore, but can take any value from the interval [0, 1].

In the previous section we assumed that particles are identified according to the energy-loss distribution. To make the presentation of the identity method more general we assume here that particle identification is achieved by measurement of particle mass not specifying the particular experimental technique which is used for this purpose. Because any measurement is of finite resolution, we deal with continuous distributions of observed masses of pions and kaons, which are denoted as $\rho_\pi(m)$ and $\rho_K(m)$, respectively. They are normalized as

$$\int dm\, \rho_\pi(m) = \langle N_\pi \rangle, \qquad \int dm\, \rho_K(m) = \langle N_K \rangle. \quad (16)$$

The kaon identity is defined as

$$w_K(m) \equiv \frac{\rho_K(m)}{\rho(m)}, \quad (17)$$

where $\rho(m) \equiv \rho_\pi(m) + \rho_K(m)$, and is normalized as

$$\int dm\, \rho(m) = \langle N \rangle \equiv \langle N_\pi \rangle + \langle N_K \rangle. \quad (18)$$

If the distributions $\rho_\pi(m)$ and $\rho_K(m)$ do not overlap, the particles can be uniquely identified and $w_K = 0$ for a pion and $w_K = 1$ for a kaon. When the distributions $\rho_\pi(m)$ and $\rho_K(m)$ overlap, w_K can take the value of any real number from [0, 1]. Figure 3 illustrates the latter case. The mass distributions are shown in Fig. 3(a) and the distribution of kaon identity in Fig. 3(b). The peaks close to 0 and 1 in Fig. 3(b) correspond to the mass regions in which pions, respectively kaons, are well

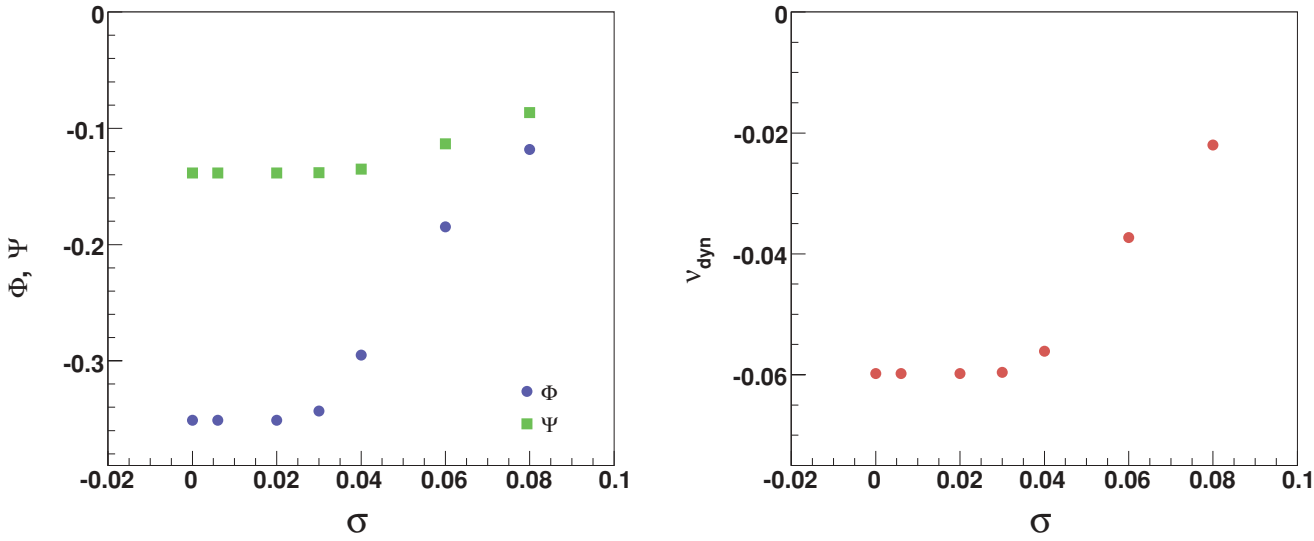

FIG. 2. (Color online) The measures of chemical fluctuations Φ and Ψ (a) and $\nu_{\rm dyn}$ (b) as functions of the width of the energy-loss distribution.

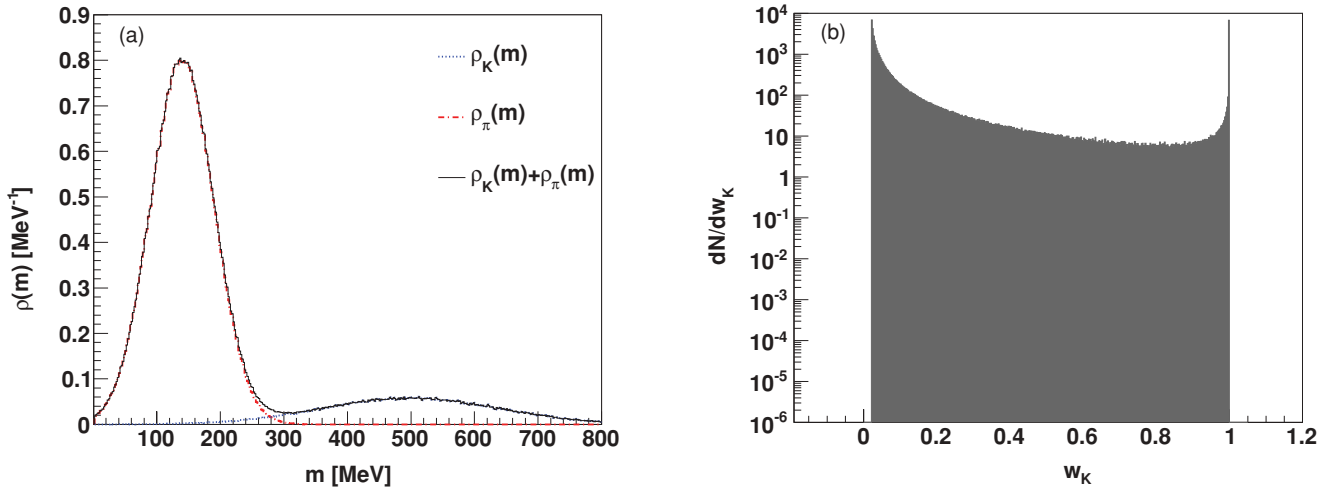

FIG. 3. (Color online) The distributions of observed masses of pions $\rho_\pi(m)$, of kaons $\rho_K(m)$, and their sum $\rho(m)$ (a) and the corresponding distribution of kaon identity w_K (b).

identified. The w_K values around 0.5 correspond to particles for which the measured mass is in the transition region between the kaon and pion peaks ($m \approx 280$ MeV) in the distribution $\rho(m)$ shown in Fig. 3(a).

Let us now explain how the fluctuation measure Ψ is calculated once the mass distributions were experimentally obtained. The single-particle variable entering Eq. (13) is defined as in Sec. II: $z \equiv w_K - \overline{w_K}$. The bar denotes the inclusive average which is computed as follows:

$$\overline{w_K} \equiv \frac{1}{\langle N\rangle}\int dm\,\rho(m)\,w_K(m) = \frac{1}{\langle N\rangle}\int dm\,\rho_K(m) = \frac{\langle N_K\rangle}{\langle N\rangle}. \tag{19}$$

Analogously, one finds $\overline{w_K^2}$ and $\overline{z^2} \equiv \overline{w_K^2} - \overline{w_K}^2$. The quantity $\langle Z^2\rangle$ is obtained as

$$\langle Z^2\rangle = \frac{1}{N_{\rm ev}}\sum_{n=1}^{N_{\rm ev}}\left(\sum_{i=1}^{N_n} w_K^i - N_n\,\overline{w_K}\right)^2, \tag{20}$$

where $N_{\rm ev}$ is the number of events and N_n is the multiplicity of the nth event. Substituting $\langle Z^2\rangle$, $\overline{z^2}$, and $\langle N\rangle$ into Eq. (13), one finds the measure Ψ, the magnitude of which, however, is biased by the effect of particle misidentification. Next we discuss the correction procedure.

As shown in the Appendix, the measure Ψ can be expressed through the moments of the multiplicity distributions of pions and kaons as

$$\Psi = A\left(\frac{\langle N_K\rangle}{\langle N\rangle} - \overline{u_K}\right)^2, \tag{21}$$

where

$$A \equiv \frac{1}{\langle N\rangle}\left[\langle N_\pi^2\rangle\frac{\langle N_K\rangle^2}{\langle N_\pi\rangle^2} + \langle N_K^2\rangle - \langle N_K\rangle - \frac{\langle N_K\rangle^2}{\langle N_\pi\rangle} - 2\langle N_\pi N_K\rangle\frac{\langle N_K\rangle}{\langle N_\pi\rangle}\right], \tag{22}$$

and

$$\overline{u_K} \equiv \frac{1}{\langle N_K\rangle}\int dm\,\rho_K(m)\,w_K(m). \tag{23}$$

In the case of complete particle identification (CI) the distributions $\rho_\pi(m)$ and $\rho_K(m)$ do not overlap and thus $\overline{u_K} = 1$. Then, the result for Ψ is

$$\Psi_{\rm CI} = A\left(\frac{\langle N_K\rangle}{\langle N\rangle} - 1\right)^2, \tag{24}$$

which is equivalent to the expression (14).

Although particle-by-particle identification is usually difficult, statistical identification is reliable. In the latter case, we do not know whether a given particle is a kaon, but we know the average numbers of kaons and of pions. We introduce the concept of *random identification*, which assumes that for every particle the probability of being a kaon equals $\langle N_K\rangle/\langle N\rangle$. Such a situation is described by mass distributions of the form

$$\rho_i(m) = \begin{cases} 0 & \text{for} \quad m < m_{\rm min}, \\ \frac{\langle N_i\rangle}{m_{\rm max}-m_{\rm min}} & \text{for} \quad m_{\rm min} \leqslant m \leqslant m_{\rm max}, \\ 0 & \text{for} \quad m_{\rm max} < m, \end{cases}$$

where $i = \pi, K$ and $m_{\rm min}$ and $m_{\rm max}$ denote lower and upper limits of the measured mass range, respectively. With this distribution,

$$\overline{u_K} = \overline{w_K} = \frac{\langle N_K\rangle}{\langle N\rangle}. \tag{25}$$

When $\overline{u_K} = \langle N_K\rangle/\langle N\rangle$ is substituted into Eq. (21), $\Psi = 0$. Thus, the measure Ψ vanishes when particle identification is random.

Finally, we arrive at the crucial point of the considerations. It appears that the measure Ψ can be expressed as

$$\Psi = \Psi_{\rm CI}\left(1 - \frac{V_I}{V_R}\right)^2, \tag{26}$$

where $\Psi_{\rm CI}$ is the measure Ψ for the complete identification, as given by Eq. (24). The quantities V_I and V_R are the values of

the integral

$$V \equiv \int dm\, \rho(m)\, w_K(m)\, [1 - w_K(m)], \tag{27}$$

evaluated for the cases of imperfect and random identification, respectively.

One proves the equality (26) by observing that

$$V_I = \langle N_K \rangle (1 - \overline{u_K}), \qquad V_R = \frac{\langle N_K \rangle \langle N_\pi \rangle}{\langle N \rangle}. \tag{28}$$

Substituting Eqs. (24) and (28) into the equality (26), one obtains the formula (21). Actually, the relation (26) was first discovered by performing various numerical simulations and only then was it proven analytically. Equation (26) allows one to experimentally obtain $\Psi_{\rm CI}$ from Ψ. Thus, the effect of misidentification is fully corrected by the factor $(1 - V_I/V_R)^2$ which measures the quality of the applied procedure of particle identification. The factor is independent of the correlations under study and can be determined from experimental data.

V. EXPERIMENTAL PROCEDURE

The application of the identity method to experimental data is not difficult. We present here a step-by-step procedure to obtain the measure $\Psi_{\rm CI}$ of fluctuations of kaons (or any other selected particle type) with respect to all particles (represented by the sum of kaons and pions in the previous sections). It is important to note that if misidentification occurs between more than two particle types, the identity method does not allow to study relative fluctuations of two of them, for example, of kaons and pions in the presence of protons.

We come back to the specific, but typical, example considered in Sec. III of particle identification via measurements of particle energy loss in the detector material. The energy loss dE/dx is denoted by X. The energy loss of particles of a given type depends on the particle mass and momentum (via the velocity) and detector characteristics. The distribution of X is usually a multidimensional function, which can be determined experimentally by averaging over particles from many interactions. The energy loss distribution, which for a given particle momentum is typically fitted by a sum of four Gaussians corresponding to electrons, pions, kaons, and (anti-)protons, allows one to determine the average multiplicities of kaons and of all particles. Having obtained this information, one should proceed as follows.

(i) Extract the energy-loss distribution of kaons $\rho_K(X)$ from the inclusive distribution $\rho(X)$. The distributions should be normalized as

$$\int dX\, \rho(X) = \langle N \rangle, \qquad \int dX\, \rho_K(X) = \langle N_K \rangle.$$

(ii) Determine the kaon identity

$$w_K(X) = \frac{\rho_K(X)}{\rho(X)}$$

for every registered particle.

(iii) Compute the fluctuation measure Ψ from the definition (13). This is the *raw* value of Ψ, which is not corrected yet for the effect of misidentification.

(iv) Knowing the mean multiplicities, compute the quantity

$$V_R = \frac{\langle N_K \rangle (\langle N \rangle - \langle N_K \rangle)}{\langle N \rangle}.$$

(v) Using all particles, calculate the integral

$$V_I = \int dX \rho(X)\, w_K(X)\, [1 - w_K(X)].$$

(vi) Determine the fluctuation measure $\Psi_{\rm CI}$, which is free of the effect of misidentification, as

$$\Psi_{\rm CI} = \frac{\Psi}{(1 - V_I/V_R)^2}.$$

The experimental data accumulated by the NA49 Collaboration are currently under analysis using the identity method proposed in this paper [17].

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APPENDIX

We express here the measure Ψ through the moments of multiplicity distributions of pions and kaons. For this purpose one writes $\langle Z^2 \rangle$ as

$$\begin{aligned}\langle Z^2 \rangle = \sum_{N_\pi=0}^{\infty} \sum_{N_K=0}^{\infty} \mathcal{P}_{N_\pi N_K} \int dm_1^\pi P_\pi(m_1^\pi) \int dm_2^\pi P_\pi(m_2^\pi) \cdots \\ \times \int dm_{N_\pi}^\pi P_\pi(m_{N_\pi}^\pi) \int dm_1^K P_K(m_1^K) \\ \times \int dm_2^K P_K(m_2^K) \cdots \int dm_{N_K}^K P_K(m_{N_K}^K) [w_K(m_1^\pi) \\ + w_K(m_2^\pi) + \cdots + w_K(m_{N_\pi}^\pi) + w_K(m_1^K) \\ + w_K(m_2^K) + \cdots + w_K(m_{N_K}^K) - (N_\pi + N_K)\overline{w_K}]^2,\end{aligned} \tag{A1}$$

where $\mathcal{P}_{N_\pi N_K}$ is the multiplicity distribution of pions and kaons; $P_\pi(m) \equiv \rho_\pi(m)/\langle N_\pi \rangle$ and $P_K(m) \equiv \rho_K(m)/\langle N_K \rangle$ are the mass distributions of pions and kaons, respectively.

Equation (A1) gives

$$\begin{aligned}\langle Z^2 \rangle = \langle N_\pi \rangle \overline{u_\pi^2} + \langle N_K \rangle \overline{u_K^2} + \langle N^2 \rangle \langle N_\pi (N_{\pi-1}) \rangle \overline{w_K}^2 \overline{u_\pi}^2 \\ + \langle N_K (N_K - 1) \rangle \overline{u_K}^2 + 2 \langle N_\pi N_K \rangle \overline{u_\pi}\, \overline{u_K} \\ - 2 \langle N N_\pi \rangle \overline{w_K}\, \overline{u_\pi} - 2 \langle N N_K \rangle \overline{w_K}\, \overline{u_K},\end{aligned} \tag{A2}$$

where

$$\overline{u_\pi^n} \equiv \frac{1}{\langle N_\pi \rangle} \int dm\, \rho_\pi(m)\, w_K^n(m),$$
$$\overline{u_K^n} \equiv \frac{1}{\langle N_K \rangle} \int dm\, \rho_K(m)\, w_K^n(m), \tag{A3}$$

with $n = 1, 2$. Because

$$\frac{\langle N_\pi \rangle}{\langle N \rangle}\, \overline{u_\pi^n} + \frac{\langle N_K \rangle}{\langle N \rangle}\, \overline{u_K^n} = \overline{w_K^n}, \tag{A4}$$

Eq. (A2) provides

$$\langle Z^2 \rangle = \langle N \rangle \overline{w_K^2} + \left[\langle N^2 \rangle + \langle N_\pi (N_\pi - 1) \rangle \frac{\langle N \rangle^2}{\langle N_\pi \rangle^2} - 2 \langle N N_\pi \rangle \frac{\langle N \rangle}{\langle N_\pi \rangle} \right] \overline{w_K}^2$$
$$- 2 \left[\langle N_\pi (N_\pi - 1) \rangle \frac{\langle N \rangle \langle N_K \rangle}{\langle N_\pi \rangle^2} - \langle N N_\pi \rangle \frac{\langle N_K \rangle}{\langle N_\pi \rangle} + \langle N N_K \rangle - \langle N_\pi N_K \rangle \frac{\langle N \rangle}{\langle N_\pi \rangle} \right] \overline{w_K}\, \overline{u_K}$$
$$+ \left[\langle N_\pi (N_\pi - 1) \rangle \frac{\langle N_K \rangle^2}{\langle N_\pi \rangle^2} + \langle N_K (N_K - 1) \rangle - 2 \langle N_\pi N_K \rangle \frac{\langle N_K \rangle}{\langle N_\pi \rangle} \right] \overline{u_K}^2. \tag{A5}$$

Keeping in mind that $\overline{z^2} = \overline{w_K^2} - \overline{w_K}^2$ and $\overline{w_K} = \langle N_K \rangle / \langle N \rangle$, one finds after somewhat lengthy calculations the formula (21) with A given by Eq. (22).

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