Deltas in hadron gas

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The relation between the author's transport theory approach to the systems with unstable particles and the S-matrix formulation of statistical mechanics is established. Then, the equilibrium characteristics of a classical gas of nucleons, deltas, and pions are studied. The finiteness of the delta decay width is taken into account. It is shown that if one treats the delta isobars as stable particles their number is significantly underestimated at the temperatures smaller than about 60 MeV.

I. INTRODUCTION

The classical (nonquantum) kinetic theory approach to the systems including unstable particles was developed in our recent paper.[1] The hadron gas is an example of a system where the presence of unstable particles, hadron resonances, essentially influences the properties of the gas. The decay width of hadron resonances is often greater than the temperature of the hadron systems, which suggests the importance of resonance instability. On the other hand, the lifetimes of the resonances can be longer than the average time intervals between successive collisions in the gas. So, the resonances should be treated in a manner similar to that for stable particles. The starting point of our approach has been the introduction of the profile function which is the generalization of the delta function that "keeps" a stable particle on the mass shell. Then the phase space element of the resonance with four-momentum p is chosen in the form $\Delta(p^2)d^4p$, where $\Delta(p^2)$ describes the mass smearing of the resonance. Using the profile function we have defined the resonance distribution function and macroscopical quantities. Kinetic equations have been formulated, where, besides binary collisions, resonance formation processes and resonance decays have been taken into account. It has been shown that the profile function can be uniquely determined through experimentally measurable quantities if the transition rates of the processes with the resonance involved satisfy the detailed balance condition or the bilateral normalization condition. For the resonance formation cross section in the Breit-Wigner form the profile function looks like

$$\Delta(M^2)=\frac{\Gamma}{2\pi M[(M-\overline{M})^2+\Gamma^2/4]}\ , \tag{1}$$

where $\overline{M}$ and Γ are the average resonance mass and the resonance decay width, respectively. Finally it has been demonstrated that the equilibrium characteristics of the resonances can be expressed in the form

$$\mathscr{O}_R=\int dM\, M\Delta(M^2)\mathscr{O}_{\rm st}(M)\ , \tag{2}$$

where $\mathscr{O}_{\rm st}(M)$ is the respective characteristic for stable particles with mass M.

The aim of this paper is twofold. In Sec. II we show that our model is, in the case of equilibrium systems, equivalent under certain assumptions to the S-matrix formulation of statistical mechanics by Dashen, Ma, and Berstein.[2] In their approach unstable particles (resonances) occur through the S-matrix elements of the resonance scattering of stable particles.

In Sec. III the equilibrium characteristics of a classical gas of nucleons, deltas, and pions are studied. We show that if one treats the delta isobars as stable particles their number can be significantly underestimated.

II. S-MATRIX EXPANSION OF THE GRAND POTENTIAL

To relate our kinetic theory approach to the Gibbs statistical mechanics, let us write the grand canonical potential of the system of stable and unstable particles. Only the interaction which leads to the resonance formation and resonance decay is included, i.e., the resonance scattering of stable particles is assumed to dominate two-body interaction. So, the grand potential reads

$$\Omega=\sum_i \Omega^i(m_i,\mu_i)+\sum_j \int dM\, M\Delta^j(M^2)\Omega^j(M,\mu_j)\ , \tag{3}$$

where $\Omega(m,\mu)$ is the grand canonical potential of the one-component ideal gas of particles with mass m and chemical potential μ. In the first term summation is performed over the sorts of stable particles while the second one is over the unstable particle sorts. Formula (3), of course, follows from (2).

The S-matrix expansion of the grand canonical potential for the system of relativistic particles is the following:

$$\Omega=\Omega_0-TV\sum_\nu a_\nu e^{\beta\mu\cdot\mathcal{N}}\ , \tag{4}$$

where Ω_0 is the grand potential of noninteracting particles, V is the volume of the system, $T=\beta^{-1}$ is the temperature, $\mathcal{N}$ is the set of conserved charges, and μ is the set of chemical potentials. $\nu=(\mathcal{N},\alpha)$, where α describes all quantum numbers required in fixing the system with $\mathcal{N}$ charges;

$$a_\nu = \int \frac{d^3\bar{p}}{(2\pi)^3} dM\, e^{-\beta(\bar{p}^2+M^2)^{1/2}} \frac{1}{4\pi i} \mathrm{Tr}_\nu \left[S^{-1} \frac{\overleftrightarrow{\partial}}{\partial M} S \right]_c ,$$

where S is the S operator and the subscript c indicates that only the so-called connected diagrams are taken into account. Let us now discuss, as in Ref. 2, the hadron gas consisting of nucleons and pions. Suppose the scattering processes are dominated by πN resonance scattering which lead to the delta formation. We take into account only the collisions

$$\pi + \mathrm{N} \to \Delta \to \pi + \mathrm{N} \tag{5}$$

and ignore all other interaction processes. For such a system the first term in Eq. (4) is the grand canonical potential of the ideal gas of nucleons and pions, and the second term relates to the πN resonant interaction. For simplicity we neglect the role of particle spins, which are taken into account through the particle degeneration factors only. Taking the matrix element of process (5) in the Breit-Wigner form, one finds[2,3]

$$\begin{aligned}\frac{1}{4\pi i} \mathrm{Tr} \left[S^{-1} \frac{\overleftrightarrow{\partial}}{\partial M} S \right]_c &= -\mathrm{Re} \frac{\partial}{\partial M} \frac{\Gamma}{2\pi} (M - \overline{M} + i\Gamma/2)^{-1} \\ &\quad + \mathrm{Im} \frac{\Gamma^2}{2\pi} (M - \overline{M} - i\Gamma/2)^{-1} \\ &\quad \times \frac{\partial}{\partial M} (M - \overline{M} + i\Gamma/2)^{-1} .\end{aligned} \tag{6}$$

Substituting Eq. (6) in formula (4) we get the following grand canonical potential valid in the lowest order S-matrix expansion (4):

$$\Omega = \Omega_0 + g_\Delta z \int dM\, M \Delta(M^2) \times \left\{ -TV \int \frac{d^3\bar{p}}{(2\pi)^3} e^{-\beta(\bar{p}^2+M^2)^{1/2}} \right\} , \tag{7}$$

where g_Δ is the delta degeneration factor and $z = e^{\beta\mu}$ is the delta fugacity. We have assumed that the gas is symmetric, i.e., the total electric charge of the system equals (in natural units) half of the total baryon charge. In such a case the chemical potentials of all sorts of baryons, in particular, of the deltas with different electric charge, are equal to each other and the chemical potentials of pions vanish. The function $\Delta(M^2)$ from Eq. (7) is exactly equal to that from formula (1). In the parentheses one recognizes the grand canonical potential of the ideal gas of spinless particles with mass M. So, the form of potential (7) coincides with this one of (3). In that way we have shown that the idea of the profile function follows from the S-matrix formulation of statistical mechanics.

III. GAS CHARACTERISTICS

Keeping in mind formula (2) we discuss in this section the properties of a classical gas consisting of nucleons, deltas, and pions.

The Breit-Wigner formula (1) is correct for sufficiently narrow resonances, which is not the case for the isobars Δ. So, to describe the mass distribution of deltas we modify formula (1) as follows:

$$\Delta(M^2) = \zeta \frac{\Gamma}{M[(M - \overline{M})^2 + \Gamma^2/4]} \theta(M - m_\mathrm{N} - m_\pi) ,$$

where θ is the step function; m_N and m_π are the masses of nucleons and pions, respectively; $\overline{M} = 1232$ MeV; and $\Gamma = 115$ MeV. The coefficient ζ is found from the normalization condition

$$\int dM\, M \Delta(M^2) = 1 .$$

The baryon density of the system reads

$$\rho = z g_\Delta \int dM\, M \Delta(M^2) n(T,M) + z g_\mathrm{N} n(T, m_\mathrm{N}) , \tag{8}$$

with

$$n(T,m) = \frac{1}{2\pi^2} T m K_2(\beta m) ,$$

$g_\mathrm{N} = 4$, and $g_\Delta = 16$; K_ν is the McDonald function. As quoted previously, the fugacities of N and Δ are equal because the gas is assumed symmetric. The first term in Eq. (8) comes from the deltas while the second one comes from nucleons. Solving Eq. (8) with respect to z one finds the densities of deltas, ρ_Δ, and nucleons. In Fig. 1 we present the ratio of ρ_Δ to ρ as a function of temperature. In the case of symmetric gas this ratio is density independent. The dashed line is found under the assumption that the deltas are stable particles, i.e., $\Gamma = 0$. It is seen that particularly at low temperatures one strongly underesti-

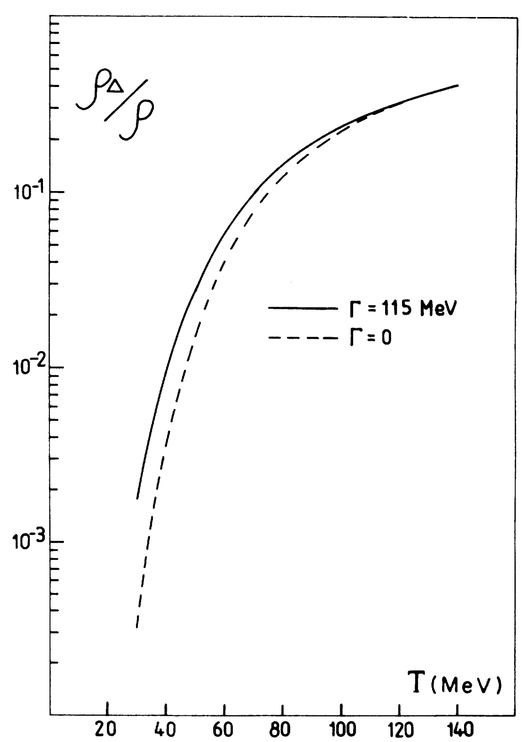

FIG. 1. The ratio of the delta density to the baryon density of the system versus temperature.

mates the density of the isobars if these particles are treated as stable ones. Our classical calculations cannot be extrapolated to the temperature $T \lesssim 30$ MeV since in such a case the quantum effects are essential.

The energy density of the system is

$$\epsilon = zg_\Delta \int dM\, M\Delta(M^2)u(T,M) + zg_N u(T,m_N) + g_\pi u(T,m_\pi) , \tag{9}$$

where $g_\pi = 3$ and

$$u(T,m) = \frac{1}{2\pi^2} T^2 m^2 [\beta m K_1(\beta m) + 3K_2(\beta m)] .$$

The first, second, and third terms of Eq. (9) come from the isobars, nucleons, and pions, respectively. The numerical calculations show that the energy density is completely insensitive to the value of Γ of the resonance. The reason is the following. At low temperatures, where the number of deltas strongly depends on the decay width, the absolute number of deltas is small as compared to the number of nucleons (see Fig. 1). Consequently the delta contribution to the energy density of the system is small. At high temperatures, where the number of deltas is comparable to that of nucleons, the description of resonances is, in practice, the same for $\Gamma = 0$ and $\Gamma = 115$ MeV (see Fig. 1).

In the framework of the thermodynamical model of the pion production in relativistic nucleus-nucleus collisions, the number of secondary pions (of all sorts) equals the number of pions plus the number of deltas present in the fireball at the moment in time of the fireball decay. The deltas are added since these isobars decay into pions which are finally registered. To discuss how the number of secondary pions changes if one takes into account the finiteness of the isobar decay width we have calculated the ratio

$$R = \frac{\rho_\pi + \rho_\Delta(\Gamma = 115 \text{ MeV})}{\rho_\pi + \rho_\Delta(\Gamma = 0)} ,$$

where ρ_π is the pion density. In Fig. 2 we present this ratio as a function of temperature for three values of the baryon density measured in the units of normal nuclear density $\rho_0 = 0.17\ \text{fm}^{-3}$. It is seen that at low temperatures, R significantly exceeds unity. As is known, the thermodynamical model overestimates, at least by a factor of 2, the multiplicity of secondary pions in nucleus-nucleus collisions.[4,5] The proper description of deltas makes this difficulty even more serious.

It is known that about 80% of the produced pions come from the delta decays[6] in nucleon-nucleon collisions at a few GeV/c incident momentum. So it is of physical interest to consider the ratio of the delta density to the density of pions in the hadron gas. In Fig. 3 we present this ratio versus temperature for three values of the baryon density. It is seen that the form of the ratio as a function of temperature essentially changes if one takes into account the finiteness of the delta decay width. The ratio monotonically decreases when temperature increases, which is not the case for $\Gamma = 0$.

A finite Δ width had been incorporated previously, see, e.g. Ref. 7, in the model calculations of hot hadronic

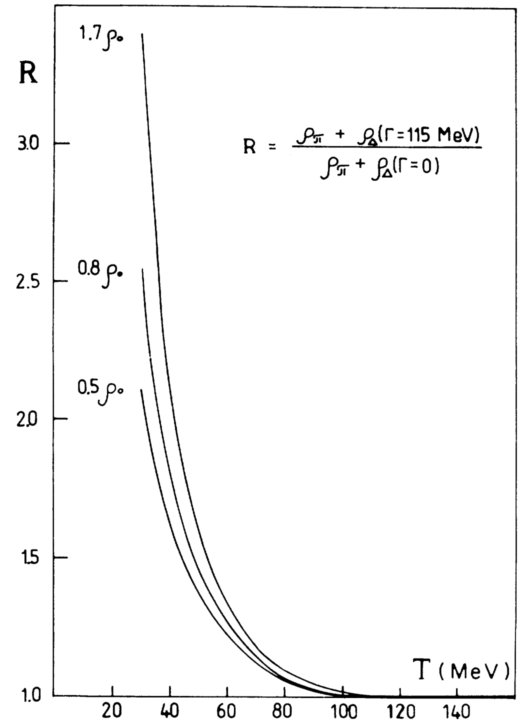

FIG. 2. The ratio R as a function of temperature for three values of the baryon density.

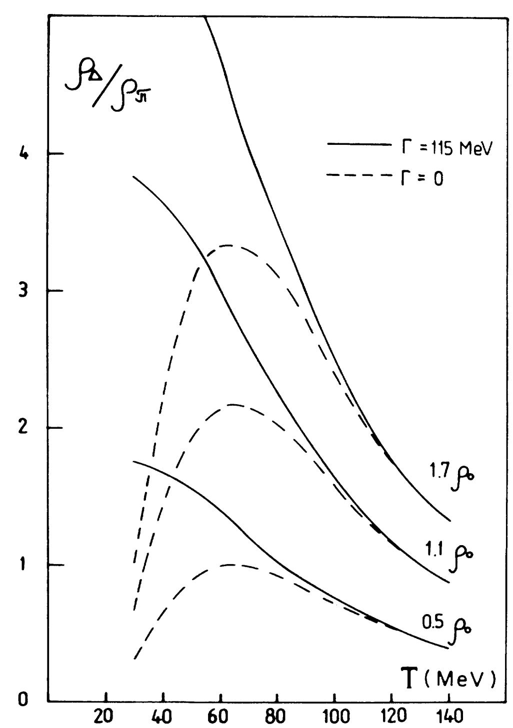

FIG. 3. The ratio of the delta density to the density of pions versus temperature for three values of the baryon density.

matter by discretizing the Δ-mass distribution, i.e., the mass spectrum had been approximated by several discrete masses at regular intervals from threshold. The statistical weight of each component had been found from the Breit-Wigner formula and the total weight had been normalized to 1. This procedure is, of course, approximately equivalent to ours.

IV. CONCLUSIONS

Establishing the relation between our transport theory approach and the *S*-matrix formulation of statistical mechanics, we have made the idea of the profile function more convincing. On the other hand, limitations of this concept are also better seen. The *S*-matrix expansion of the grand potential[2] is rapidly covergent for classical systems. In the region where quantum effects are important, the expansion practically fails since one should take into account numerous multiparticle diagrams for a realistic description of the system. An analogous situation occurs with our approach. To make the idea of the profile function adequate for quantum systems one should include the dependence of this function on the medium, i.e., the unstable particle lifetime should be density and temperature dependent.

We have applied our model to the description of the hadron gas of nucleons, pions, and deltas. It has been shown that if one treats the isobars as stable particles their number in the gas is significantly underestimated, particularly at temperatures smaller than about 60 MeV.

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[3]There is a minor misprint in Eq. (7.11) of Ref. 2. The second term of the right-hand side of this equation should be additionally divided by π. It should be also remembered that in Ref. 2 Γ denotes the decay half-width while in our paper Γ is the decay width.

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