Transverse momentum versus multiplicity fluctuations in high-energy nuclear collisions

St. Mrówczyński,[1,2] M. Rybczyński,[1] and Z. Włodarczyk[1]
[1]*Institute of Physics, Świętokrzyska Academy, ul. Świętokrzyska 15, PL - 25-406 Kielce, Poland*
[2]*Sołtan Institute for Nuclear Studies, ul. Hoża 69, PL - 00-681 Warsaw, Poland*
(Received 13 July 2004; published 22 November 2004)

We discuss recently measured event-by-event fluctuations of transverse momentum and of multiplicity in relativistic heavy-ion collisions. It is shown that the nonmonotonic behavior of the p_T fluctuations as a function of collision centrality can be fully explained by the observed nonmonotonic multiplicity fluctuations. A possible mechanism responsible for the multiplicity fluctuations is also considered.

DOI: 10.1103/PhysRevC.70.054906 PACS number(s): 25.75.Gz

Event-by-event fluctuations of transverse momentum in heavy-ion collisions have been recently measured both at CERN SPS [1–3] and BNL RHIC [4–8]; see also the brief review in [9]. The data, which show a nontrivial behavior as a function of collision centrality, have been theoretically discussed from very different points of view [10–27], including complete or partial equilibration [12,13,20,22], critical phenomena [14,27], string or cluster percolation [23,25], and production of jets [11,26]. In spite of these efforts, a mechanism responsible for the fluctuations is far from being uniquely identified. Recently, the NA49 Collaboration published the very first data on multiplicity fluctuations as a function of collision centrality [28,29]. Unexpectedly, the ratio $\mathrm{Var}(N)/\langle N\rangle$, where $\mathrm{Var}(N)$ is the variance and $\langle N\rangle$ is the average multiplicity of negative particles, changes nonmonotonically when the number of wounded nucleons[1] grows. It is close to unity at fully peripheral ($N_w \leqslant 10$) and completely central ($N_w \geqslant 250$) collisions but it manifests a prominent peak at $N_w \approx 70$, as shown in Fig. 1(a). The measurement has been performed at the collision energy 158A GeV in the transverse momentum and pion rapidity intervals $(0.005, 1.5)$ GeV and (4.0, 5.5), respectively. The azimuthal acceptance has been also limited, and about 20% of all produced negative particles have been used in the analysis.

The aim of this paper is to show that the nontrivial behavior of transverse momentum fluctuations can be explained by the multiplicity fluctuations which enter the measures of p_T fluctuations. Specifically, we assume that in nucleus-nucleus collisions the event's transverse momentum is correlated to the event's multiplicity exactly as in the proton-proton interactions [30], and we express the Φ measure [31] of p_T fluctuations through the multiplicity fluctuations. It is convenient for our discussion to use data on the transverse momentum and multiplicity fluctuations measured in the same experimental conditions. For this reason, we choose the data obtained by the NA49 Collaboration which used the Φ measure [31] to quantify the fluctuations of transverse momentum.

Let us first introduce the measure. One defines the single-particle variable $z \overset{\mathrm{def}}{=} x - \overline{x}$ with the overline denoting averaging over a single-particle inclusive distribution. Here, we identify x with the particle transverse momentum p_T. The event variable Z, which is a multiparticle analog of z, is defined as $Z \overset{\mathrm{def}}{=} \sum_{i=1}^{N}(x_i - \overline{x})$, where the summation runs over particles from a given event. By construction, $\langle Z\rangle = 0$ where $\langle \cdots \rangle$ represents averaging over events. Finally, the Φ measure is defined in the following way:

$$\Phi(x) \overset{\mathrm{def}}{=} \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}} - \sqrt{\overline{z^2}}.$$

It is evident that $\Phi = 0$, when no interparticle correlations are present. Consequently, Φ is "deaf" to statistical noise. The measure also possesses a less trivial property. Namely, Φ is *independent* of the distribution of number of particle sources if the sources are identical and independent from each other [31,32]. Thus, the Φ measure is "blind" to the impact parameter variation as long as the "physics" does not change with the collision centrality. In particular, the Φ is independent of the impact parameter if the nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions.

$\Phi(p_T)$ measured in nucleus-nucleus collisions at SPS energy as a function of centrality [2] is shown in Fig. 1(b). The measurement has been performed in exactly the same experimental conditions as that of multiplicity fluctuations shown in Fig. 1(a). As seen, both transverse momentum fluctuations expressed in terms of Φ and multiplicity fluctuations display a very similar centrality dependence, suggesting that they are related to each other.

In the very first paper, where the Φ measure was introduced [31], it was argued that the correlation between the event's multiplicity and transverse momentum is a main source of the p_T fluctuations as quantified by Φ. For the case of p-p interactions, the problem was then studied in detail in [18]. Following this paper, we introduce the correlation $\langle p_T\rangle$ vs N through the multiplicity-dependent temperature or slope parameter of the p_T distribution. Specifically, the single-particle transverse momentum distribution in the events of multiplicity N is chosen in the form suggested by the thermal model—i.e.,

$$P_{(N)}(p_T) \sim p_T \exp\left[-\frac{\sqrt{m^2 + p_T^2}}{T_N}\right], \tag{1}$$

where m is the particle mass while T_N is the multiplicity-dependent temperature. In Ref. [18], T_N was defined as

[1]A nucleon is called wounded if it interacts at least once in the course of a nucleus-nucleus collision. The number of wounded nucleons, N_w, approximately equals the number of participants, and we assume here that the equality holds.

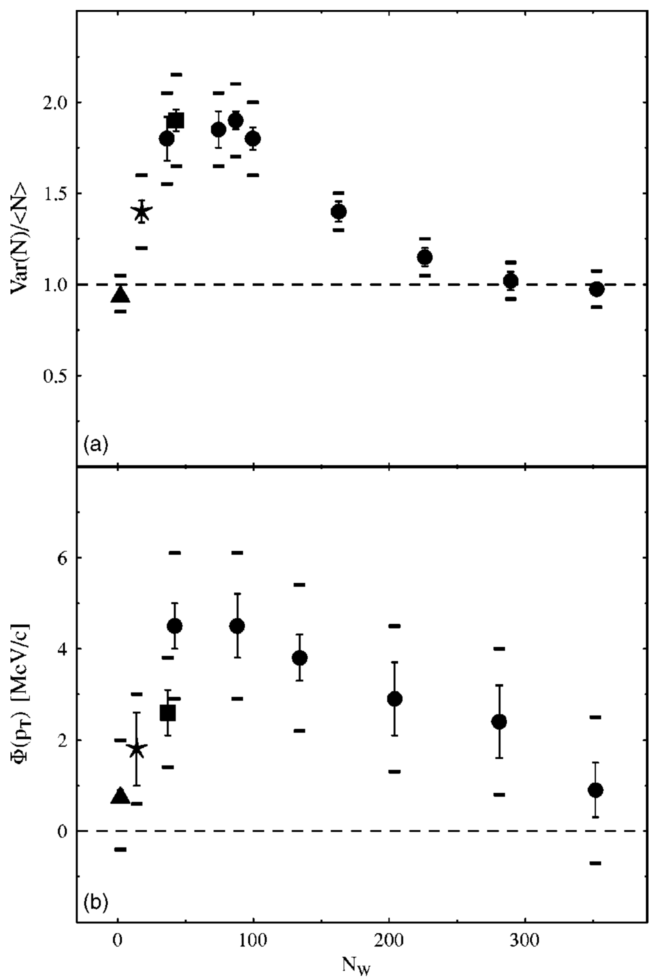

FIG. 1. Multiplicity (a) and transverse momentum (b) fluctuations of negative particles as a function of number of wounded nucleons. The triangles correspond to *p-p* collisions, asterisks to C-C, squares to Si-Si, and circles to Pb-Pb. There are denoted statistical errors with vertical bars and total errors including systematic uncertainties with horizontal dashes. The multiplicity data are taken from [28,29] while those on the transverse momentum from [2].

$$T_N = T + \Delta T(\langle N\rangle - N), \tag{2}$$

with ΔT controlling the correlation strength. The parametrization (2) was reasonable for proton-proton collisions where $\langle N\rangle$ is fixed, but it is not reasonable to study the centrality dependence in *A-A* collisions where $\langle N\rangle$ varies.

The correlation $\langle p_T\rangle$ vs N at SPS energy, which is directly observed [2,30] in *p-p* collisions, is most probably of simple kinematical origin. Namely, when the multiplicity of produced particles grows at fixed collision energy, there is less and less energy to be distributed among transverse degrees of freedom of produced particles. Consequently, the average event's p_T decreases when N grows. We expect that the correlation $\langle p_T\rangle$ vs N is also present in *A-A* collisions at fixed centrality as the number of wounded nucleons controls the amount of energy to be used for particle production. However, we replace the parametrization (2) by

$$T_N = T + \delta T\left(1 - \frac{N}{\langle N\rangle}\right), \tag{3}$$

with $\delta T = \Delta T\langle N\rangle$. Relation (3) correlates the slope parameter T_N to the event's multiplicity N at fixed $\langle N\rangle$. The parameters T and δT are assumed to be independent of the centrality while the average multiplicity $\langle N\rangle$ depends (roughly linearly) on N_w. As will be seen in our final formula (6), a small variation of T with the centrality does not much matter.

The inclusive transverse momentum distribution, which determines $\overline{z^2} = \overline{p_T^2} - \overline{p_T}^2$, reads

$$P_{\rm incl}(p_T) = \frac{1}{\langle N\rangle}\sum_N \mathcal{P}_N N P_{(N)}(p_T),$$

where $\mathcal{P}_N$ is the multiplicity distribution. The N-particle transverse momentum distribution in the events of multiplicity N is assumed to be the N product of $P_{(N)}(p_T)$. Therefore, all inter particle correlations different than $\langle p_T\rangle$ vs N are neglected here. Then, one easily finds

$$\langle Z^2\rangle = \sum_N \mathcal{P}_N \int_0^\infty dp_T^1 \cdots \int_0^\infty dp_T^N \times (p_T^1 + \cdots + p_T^N - N\overline{p_T})^2 P_{(N)}(p_T^1)\cdots P_{(N)}(p_T^N).$$

Assuming that the particles are massless and the correlation is weak—i.e., $T \gg \delta T$—the calculation of Φ can be performed analytically. The result is [18]

$$\Phi(p_T) = \sqrt{2}\frac{(\delta T)^2}{T\langle N\rangle^5}(\langle N^4\rangle\langle N\rangle^2 - 2\langle N^3\rangle\langle N^2\rangle\langle N\rangle - \langle N^3\rangle\langle N\rangle^2 + \langle N^2\rangle^3 + \langle N^2\rangle^2\langle N\rangle), \tag{4}$$

where terms of the third and higher powers of δT have been neglected. As seen, the lowest nonvanishing contribution to Φ is of the second order in δT.

We intend to express $\Phi(p_T)$ through $\mathrm{Var}(N)/\langle N\rangle$ but $\Phi(p_T)$, as given by Eq. (4), also depends on the third and fourth moments of the multiplicity distribution. It would be in the spirit of our minimalist approach to use the multiplicity distribution which maximizes the Shannon's information entropy $S \equiv \Sigma_N \mathcal{P}_N \ln \mathcal{P}_N$ [33] with $\langle N\rangle$ and $\mathrm{Var}(N)$ being fixed. The least biased method to obtain a statistical distribution was prompted by Jaynes [34]. An application of information theory to the phenomenology of high-energy collisions is discussed in [35]. The multiplicity distribution, which maximizes the entropy at fixed $\langle N\rangle$ and $\mathrm{Var}(N)$, is given by the formula

$$\mathcal{P}_N = \exp(a + bN + cN^2), \tag{5}$$

where the parameters a, b, and c are determined by the equations

$$\sum_N \mathcal{P}_N = 1, \quad \sum_N N\mathcal{P}_N = \langle N\rangle,$$

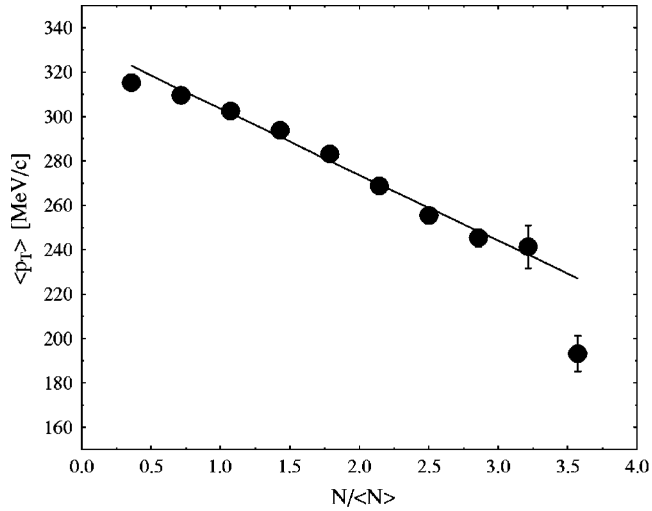

FIG. 2. The average transverse momentum of negatively charged particles produced in p-p collisions as a function of the event's negative particle multiplicity divided by the mean. The data are taken from [2] where the acceptance is precisely defined. The line corresponds to $T=137$ MeV and $\delta T=15.5$ MeV.

$$\sum_N (N-\langle N\rangle)^2\mathcal{P}_N = \mathrm{Var}(N).$$

Unfortunately, there are no simple analytic expressions of a, b, and c, and consequently the distribution (5) is very inconvenient to use. However, under the condition $\langle N\rangle \gg \sqrt{\mathrm{Var}(N)} \gg 1$, which is usually satisfied in A-A collisions at fixed centrality, the distribution (5) can be replaced by the continuous Gauss distribution. Then, we get the required relations $\langle (N-\langle N\rangle)^3\rangle = 0$ and $\langle (N-\langle N\rangle)^4\rangle = 3\langle (N-\langle N\rangle)^2\rangle^2$.

Using these relations, expression (4) obtains the form

$$\Phi(p_T) = \sqrt{2}\frac{(\delta T)^2}{T}\frac{\mathrm{Var}(N)}{\langle N\rangle}\left[1-\frac{1}{\langle N\rangle}+\frac{\mathrm{Var}^2(N)}{\langle N\rangle^4}+\frac{\mathrm{Var}(N)}{\langle N\rangle^3}\right].$$

Taking into account the already adopted assumption that $\langle N\rangle \gg \sqrt{\mathrm{Var}(N)} \gg 1$, we finally find

$$\Phi(p_T) = \sqrt{2}\frac{(\delta T)^2}{T}\frac{\mathrm{Var}(N)}{\langle N\rangle}. \tag{6}$$

When the negative binomial distribution, instead of the Gaussian, is used to describe the multiplicity distribution, one obtains the formula, which in the limit $\langle N\rangle \gg \sqrt{\mathrm{Var}(N)} \gg 1$, coincides with Eq. (6).

The values of the parameters T and δT for p-p collisions can be obtained from the NA49 data published in [2]. Following [18], we have computed the average p_T at fixed N, using the distribution (1) with T_N given by Eq. (3). Comparing the results of our calculations with the experimental data [2], which are shown in Fig. 2, we have found $T=137$ MeV and $\delta T=15.5$ MeV. We note that T and δT are essentially independent from each other when the experimental data are fitted as δT determines the slope of the curve shown in Fig. 2 while T controls its vertical position. For $T=137$ MeV and $\delta T=15.5$ MeV, the coefficient in formula (6) equals

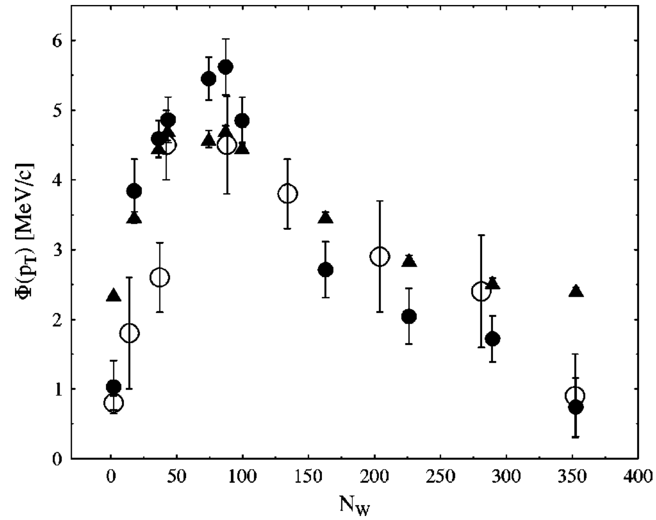

FIG. 3. $\Phi(p_T)$ as a function of number of wounded nucleons. The open circles correspond to the NA49 data [2], and the solid circles show the results of our simulation while the triangles present the prediction of the analytical formula (6) with the numerical coefficient given by Eq. (7).

$$\sqrt{2}\frac{(\delta T)^2}{T} \approx 2.48 \text{ MeV}. \tag{7}$$

In Fig. 3 we compare the experimental values of $\Phi(p_T)$ with the predictions of formula (6) with the numerical coefficient given by Eq. (7). As seen, the agreement is quite satisfactory.[2] However, the analytic result (6) has been derived under several rather rough approximations. Therefore, we have also performed a Monte Carlo simulation which is free of these approximations. For every nucleus-nucleus collision at a given centrality, we have first generated its multiplicity, using the negative binomial distribution with the mean value and variance as in the experimental data [28,29]. Further, we have attributed the transverse momentum from the distribution (1) with $T=137$ MeV and $\delta T=15.5$ MeV to each particle assuming, as in the experimental analysis [2], that all particles are pions. Having a sample of events for every centrality, the Φ measure has been computed. The statistical errors have been determined by means of the subsample method. The results of our simulation are confronted with the experimental data in Fig. 3. As seen, there is a perfect agreement.

[2]In Ref. [18] the parameters T and δT were estimated as 167 and 8.2 MeV, using the data on p-p collisions at 205 GeV [30]. The data [2] were not available at that time. Then, $\sqrt{2}(\delta T)^2/T \approx 0.57$, and the value of $\Phi(p_T)$ calculated by means of formula (6) is dramatically underestimated. It was also concluded in [18] that the p_T vs N correlation produces too small a value of $\Phi(p_T)$ to explain the experimental value. Now, this conclusion must be revoked. The discrepancy between the data [30] and [2] can be easily explained. The data [30] were collected at higher collision energy in the full phase space while the NA49 measurement [2] was performed, as already mentioned, in the forward rapidity window (4, 5.5).

Our calculations explicitly take into account only the p_T vs N correlation. However, other correlations, in particular those due to quantum statistics, are not entirely neglected. Since we use the experimental value of $\mathrm{Var}(N)/\langle N\rangle$, all correlations which contribute to this quantity are effectively taken into account in our estimate of $\Phi(p_T)$.

The multiplicity fluctuations strongly influence the p_T fluctuations expressed in terms of Φ, as the measure Φ, depends on the particle multiplicity distribution. It should be stressed that other fluctuation measures, such as F used by the PHENIX Collaboration [4,5] or $\Delta\sigma$ and σ_{dyn} by STAR [6,7], are also influenced by multiplicity fluctuations. Therefore, our main result (6) can be easily translated for F, $\Delta\sigma$, or σ_{dyn}.

A specific pattern of the p_T fluctuations has been explained by the observed multiplicity fluctuations. Before closing our considerations we briefly consider a possible origin of the nonmonotonic dependence of $\mathrm{Var}(N)/\langle N\rangle$ on the collision centrality. For this purpose we first derive a well-known formula which relates particle number fluctuations to interparticle correlations. The average multiplicity can be written as

$$\langle N\rangle = \int_V d^3r\rho(\mathbf{r}).$$

V is the volume of the interaction zone (fireball), where the particles are produced, and $\rho(\mathbf{r})$ is the particle density. The average multiplicity of produced particles is known to be roughly proportional to the number of wounded nucleons, N_w [36]. Since N_w is in turn proportional to the volume V, we have $\langle N\rangle=\overline{\rho}V$ with $\overline{\rho}$ being constant. The second moment of the multiplicity distribution can be written as

$$\langle N(N-1)\rangle = \int_V d^3r_1\int_V d^3r_2\rho_2(\mathbf{r}_1,\mathbf{r}_2),$$

where $\rho_2(\mathbf{r}_1,\mathbf{r}_2)$ is the two-particle density. Defining the correlation function $\nu(\mathbf{r}_1-\mathbf{r}_2)$ through the equation

$$\rho_2(\mathbf{r}_1,\mathbf{r}_2) = \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)[1+\nu(\mathbf{r}_1-\mathbf{r}_2)],$$

we get the desired formula

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = 1 + \overline{\rho}\int_V d^3r\nu(\mathbf{r}), \tag{8}$$

which tells us that the multiplicity distribution is Poissonian if particles are independent from each other $[\nu(\mathbf{r})=0]$. The pattern seen in Fig. 1(a) clearly shows that the particles are correlated at the stage of production.

For further discussion we assume that the fireball is spherically symmetric and that its radius equals $R\approx r_0N_w^{1/3}$ with $r_0\approx 1$ fm. Then, formula (8) reads

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = 1 + 4\pi\,\overline{\rho}\int_0^R dr\, r^2\nu(r). \tag{9}$$

It is not difficult to invent a correlation function $\nu(r)$ which when substituted into Eq. (9) reproduces the data shown in Fig. 1(a). Various functions are discussed in [37]. Here we only describe the qualitative features of $\nu(r)$. The correlation function has to be positive at small distances (attractive interaction) and negative at larger ones (repulsive interaction). The sign of the correlation changes at $r\approx 4$ fm which corresponds to $N_w\approx 70$ when $\mathrm{Var}(N)/\langle N\rangle$ reaches its maximum. For $r\gtrsim(300)^{1/3}\approx 7$ fm the correlation function vanishes. A physical mechanism responsible for such a correlation function is rather unclear but some possibilities, which include a combination of strong and electromagnetic interactions, percolation, dipole-dipole interactions, and nonextensive thermodynamics, are discussed in [37].

We conclude our considerations as follows. A nontrivial behavior of transverse momentum fluctuations as a function of collision centrality can be fully explained by the centrality dependence of multiplicity fluctuations if the mean transverse momentum is correlated to the particle multiplicity in nucleus-nucleus collisions as in the proton-proton interactions. This correlation is most probably of simple kinematic origin. Our observation seems to exclude various exotic explanations of transverse momentum fluctuations. However, a mechanism responsible for multiplicity fluctuations still needs to be clarified.

We are very grateful to Kasia Grebieszkow, Marek Gaździcki, and Peter Seyboth for fruitful discussions.

[1] NA49 Collaboration, H. Appelshauser *et al.*, Phys. Lett. B **459**, 679 (1999).
[2] NA49 Collaboration, T. Anticic *et al.*, Phys. Rev. C **70**, 034902 (2004).
[3] CERES Collaboration, D. Adamova *et al.*, Nucl. Phys. **A727**, 97 (2003).
[4] PHENIX Collaboration, K. Adcox *et al.*, Phys. Rev. C **66**, 024901 (2002).
[5] PHENIX Collaboration, S. S. Adler *et al.*, nucl-ex/0310005.
[6] STAR Collaboration, S. A. Voloshin, nucl-ex/0109006.
[7] STAR Collaboration, J. Adams *et al.*, nucl-ex/0308033.
[8] STAR Collaboration, C. A. Pruneau, nucl-ex/0401016.
[9] J. T. Mitchell, J. Phys. G **30**, S819 (2004).
[10] M. Gaździcki, A. Leonidov, and G. Roland, Eur. Phys. J. C **6**, 365 (1999).
[11] F. Liu, A. Tai, M. Gaździcki, and R. Stock, Eur. Phys. J. C **8**, 649 (1999).
[12] M. Bleicher *et al.*, Phys. Lett. B **435**, 9 (1998).
[13] St. Mrówczyński, Phys. Lett. B **439**, 6 (1998).
[14] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. D **60**, 114028 (1999).
[15] A. Capella, E. G. Ferreiro, and A. B. Kaidalov, Eur. Phys. J. C **11**, 163 (1999).
[16] G. Baym and H. Heiselberg, Phys. Lett. B **469**, 7 (1999).

[17] A. Dumitru and R. D. Pisarski, Phys. Lett. B **504**, 282 (2001).
[18] R. Korus and St. Mrówczyński, Phys. Rev. C **64**, 054906 (2001).
[19] R. Korus, St. Mrówczyński, M. Rybczyński, and Z. Włodarczyk, Phys. Rev. C **64**, 054908 (2001).
[20] M. A. Stephanov, Phys. Rev. D **65**, 096008 (2002).
[21] S. A. Voloshin, nucl-th/0206052.
[22] S. Gavin, Phys. Rev. Lett. **92**, 162301 (2004).
[23] J. Dias de Deus and A. Rodrigues, hep-ph/0308011.
[24] M. Rybczyński, Z. Włodarczyk, and G. Wilk, Acta Phys. Pol. B **35**, 819 (2004).
[25] E. G. Ferreiro, F. del Moral, and C. Pajares, Phys. Rev. C **69**, 034901 (2004).
[26] Q. J. Liu and T. A. Trainor, Phys. Lett. B **567**, 184 (2003).
[27] M. A. Stephanov, Prog. Theor. Phys. Suppl. **153**, 139 (2004).
[28] NA49 Collaboration, M. Gaździcki *et al.*, J. Phys. G **30**, S701 (2004).
[29] NA49 Collaboration, M. Rybczyński *et al.*, nucl-ex/0409009.
[30] T. Kafka *et al.*, Phys. Rev. D **16**, 1261 (1977).
[31] M. Gaździcki and St. Mrówczyński, Z. Phys. C **54**, 127 (1992).
[32] St. Mrówczyński, Phys. Lett. B **465**, 8 (1999).
[33] C. E. Shanon, Bell Syst. Tech. J. **27**, 620 (1948).
[34] E. T. Jaynes, Phys. Rev. **106**, 620 (1957).
[35] G. Wilk and Z. Włodarczyk, Phys. Rev. D **43**, 794 (1991).
[36] NA49 Collaboration, S. V. Afanasiev *et al.*, Phys. Rev. C **66**, 054902 (2002).
[37] M. Rybczyński and Z. Włodarczyk, nucl-th/0408023.