Interaction of relativistic elementary atoms with matter. II. Numerical results

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The formulas of the cross sections of elementary-atom interaction with atoms of matter given in part I of this study are simplified to the form appropriate for numerical calculations. The cross sections are split into electric, magnetic, and spin parts. The numerical values of each part of the total and excitation cross sections of eight elementary atoms ($A_{2e}, A_{e\mu}, A_{e\pi}, A_{2\mu}, A_{\mu\pi}, A_{2\pi}, A_{\pi k}, A_{2k}$) interacting with five typical targets (C, Al, Cu, Ag, Pb) are given. The electric parts are found to give the dominant contribution to all cross sections except those of para-ortho and ortho-para transitions of the elementary atoms composed of two spin-$\frac{1}{2}$ particles. The results are compared to those published earlier. The points concerning the elementary-atom interaction with matter which are not studied by us are briefly reviewed. The magnetic form factors of elementary atoms are discussed.

I. INTRODUCTION

In this paper we continue our studies of elementary-atom interactions with matter. We give here the numerical values of cross sections of eight elementary atoms: A_{2e}, $A_{e\mu}$, $A_{e\pi}$, $A_{2\mu}$, $A_{\mu\pi}$, $A_{2\pi}$, $A_{\pi K}$, and A_{2K} interacting with C, Al, Cu, Ag, and Pb atoms.

Let us first recall some points from part I of our study.[1] The interaction process is considered in the frame where the elementary atom is initially at rest. Because the characteristic momentum transfer to the atom is of order of the inverse Bohr radius of the elementary atom, the initial and final atom motion is described nonrelativistically. The atom of matter is treated as a spinless structureless particle of a charge Ze. The electron screening of the nucleus electromagnetic field is taken into account by modification of the photon propagator.

Our first task is to rewrite the cross-section formulas given in part I using some simplifying assumptions and approximations. Since we are interested in the interaction of relativistic atoms it is assumed that the projectile velocity equals the velocity of light and the Lorentz γ factor is much greater than unity. Because the momentum transfer $\mathbf{q}$ is smaller than the initial momentum $\mathbf{k}_i$, the quantities of order of $Q/|\mathbf{k}_i|$ ($Q\equiv|\mathbf{q}|$) are neglected. For the same reason $\mathbf{q}$ is assumed perpendicular to the beam axis.[1] The initial atomic state is identified with the ground state, and the quantum numbers of this state are further suppressed to simplify the notation.

The paper is organized as follows. In Sec. II we consider the elementary atoms of spinless components. The excitation, elastic, and total cross sections are calculated. The atoms of one spinless and one spin-$\frac{1}{2}$ particle are discussed in Sec. III. The previously quoted cross sections are calculated; however the spin-flip processes are taken into account. The interaction of atoms composed of two spin-$\frac{1}{2}$ particles is studied in Sec. IV. In Sec. V our numerical results are discussed. Section VI is devoted to the brief review of the points which have not been studied in our papers. Finally we conclude our considerations.

II. THE ATOMS OF SPINLESS COMPONENTS

Keeping in mind the approximations and assumptions listed in Sec. I formula (I.33) [I.33 means Eq. (33) from paper I] can be rewritten in the form

$$d\sigma^{nlm}=\frac{Z^2e^4}{(2\pi)^2}|\Delta(q)|^2\left|F^{nlm}(\eta\mathbf{q})-F^{nlm}(\zeta\mathbf{q})-\mathbf{v}\cdot\left[\frac{1}{m_1}\mathbf{D}^{nlm}(\eta\mathbf{q})+\frac{1}{m_2}\mathbf{D}^{nlm}(\zeta\mathbf{q})\right]\right|^2 Q\,dQ\,d\phi\ ,$$

where we have used the form factors $\mathbf{D}$ (I.30) which are related to $\mathbf{G}$ by formula (I.32). All notations coincide with those from paper I ($c=\hbar=1$); however initial-state quantum numbers ($n=1$, $l=0$, $m=0$), as quoted previously, are suppressed. The quantization axis of the atomic orbital momentum is chosen along the momentum-transfer vector $\mathbf{q}$ which simultaneously defines the z axis of our coordinate system. Because $\mathbf{q}\perp\mathbf{v}$ and $\mathbf{v}^2=1$ one writes

$$\mathbf{v}\cdot\mathbf{D}^{nlm}(\mathbf{q})=\cos\phi D_x^{nlm}(\mathbf{q})+\sin\phi D_y^{nlm}(\mathbf{q})\ .$$

Using relations (A3) and (A4) given in the Appendix and observing that $F^{nlm}(\mathbf{q})=0$ for $m\neq 0$ one finds the atomic excitation cross section summarized over projections of the atomic orbital momentum

$$d\sigma^{nl}=d\sigma^{nl}_{\rm el}+d\sigma^{nl}_{\rm mag}\ , \tag{1}$$

where the electric and magnetic parts of the cross section read

$$d\sigma^{nl}_{\rm el}=\frac{Z^2e^4}{2\pi}\,|\,\Delta(q)\,|^{\,2}\,|\,F^{nlm=0}(\eta\mathbf{q})-F^{nlm=0}(\xi\mathbf{q})\,|^{\,2}Q\,dQ\ , \tag{2}$$

$$d\sigma^{nl}_{\rm mag}=\begin{cases}0 \quad \text{for } l=0\ ,\\ \dfrac{Z^2e^4}{\pi}\,|\,\Delta(q)\,|^{\,2}\left|\dfrac{1}{m_1}D_x^{nlm=1}(\eta\mathbf{q})+\dfrac{1}{m_2}D_x^{nlm=1}(\xi\mathbf{q})\right|^2 Q\,dQ \quad \text{for } l\geq 1\ .\end{cases} \tag{3}$$

The electric and magnetic cross sections relate to the interaction of the atomic component charges and currents with the electric and magnetic fields, respectively, generated by the fast atom of matter. The distinction of electric and magnetic cross sections is, of course, not Lorentz invariant. The cross sections (1)–(3), which do not depend on the choice of the quantization axis, are, as expected, ϕ independent.

For the atoms composed of a particle and antiparticle formulas (2) and (3) are

$$d\sigma^{nl}_{\rm el}=[1-(-1)^l]\frac{Z^2e^4}{\pi}\,|\,\Delta(q)F^{nlm=0}(\tfrac{1}{2}\mathbf{q})\,|^{\,2}Q\,dQ\ ,$$

$$d\sigma^{nl}_{\rm mag}=[1-(-1)^l]\frac{2Z^2e^4}{\pi m^2}\,|\,\Delta(q)D_x^{nlm=1}(\tfrac{1}{2}\mathbf{q})\,|^{\,2}Q\,dQ\ .$$

Using Eq. (I.37) one finds the total cross section which we also split into electric and magnetic parts:

$$d\sigma^{\rm tot}_{\rm el}=\frac{Z^2e^4}{\pi}\,|\,\Delta(q)\,|^{\,2}[1-F^{100}(\mathbf{q})]Q\,dQ\ , \tag{4}$$

$$d\sigma^{\rm tot}_{\rm mag}=\frac{Z^2e^4}{2\pi}\,|\,\Delta(q)\,|^{\,2}\left[\left[\frac{1}{m_1^{\,2}}+\frac{1}{m_2^{\,2}}\right]K(\mathbf{v},\mathbf{q}=0)+\frac{2}{m_1m_2}K(\mathbf{v},\mathbf{q})\right]Q\,dQ\ . \tag{5}$$

The total cross sections (4) and (5) do not include the radiation and pair-creation processes which are of higher order in the coupling constant and, consequently, can be neglected here.

For numerical calculations we have used the photon propagator $\Delta(q)$ in the Thomas-Fermi-Molier form[2]

$$\Delta(q)=4\pi\sum_{i=1}^{3}\frac{\alpha_i}{\mathbf{q}^{\,2}+\beta_i^{\,2}} \tag{6}$$

with

$$\beta_i=\frac{m_eb_i}{121}Z^{1/3}$$

and

$$b_1=6.0\ ,\quad b_2=1.2\ ,\quad b_3=0.3\ ,$$

$$\alpha_1=0.10\ ,\quad \alpha_2=0.55\ ,\quad \alpha_3=0.35\ ,$$

where m_e is the electron mass. Let us recall that $q^2\simeq-\mathbf{q}^2$ (Ref. 1).

Substituting in Eqs. (1)–(5) the photon propagator (6) and the formulas of the atomic form factors given in Appendix and Ref. 5, the cross sections (1)–(5) have been numerically integrated from 0 to ∞. The results for the atoms $A_{2\pi}$, A_{2K}, and $A_{\pi K}$ are given in Tables I, II, and III.

TABLE I. The cross sections (in cm^2) of $A_{2\pi}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

$\pi\pi$		C	Al	Cu	Ag	Pb
1S	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
2S	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
2P	Electric	1.5×10^{-22}	6.6×10^{-22}	3.0×10^{-21}	7.3×10^{-21}	2.1×10^{-20}
	Magnetic	5.9×10^{-25}	2.2×10^{-24}	8.5×10^{-24}	1.8×10^{-23}	4.5×10^{-23}
3S	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
2P	Electric	2.6×10^{-23}	1.1×10^{-22}	5.1×10^{-22}	1.3×10^{-21}	3.6×10^{-21}
	Magnetic	1.3×10^{-25}	5.1×10^{-25}	1.9×10^{-24}	4.2×10^{-24}	1.0×10^{-23}
Total	Electric	3.1×10^{-22}	1.3×10^{-21}	6.1×10^{-21}	1.5×10^{-20}	4.4×10^{-20}
	Magnetic	2.5×10^{-24}	9.6×10^{-24}	3.6×10^{-23}	7.9×10^{-23}	1.9×10^{-22}

TABLE II. The cross sections (in cm^2) of A_{2K} atom interaction with atoms of C, Al, Cu, Ag, and Pb.

KK		C	Al	Cu	Ag	Pb
$1S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
$2S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
$2P$	Electric	1.5×10^{-23}	7.0×10^{-23}	3.4×10^{-22}	8.6×10^{-22}	2.5×10^{-21}
	Magnetic	7.8×10^{-26}	3.4×10^{-25}	1.6×10^{-24}	4.0×10^{-24}	1.1×10^{-23}
$3S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
$3P$	Electric	2.6×10^{-24}	1.2×10^{-23}	5.7×10^{-23}	1.5×10^{-22}	4.3×10^{-22}
	Magnetic	1.7×10^{-26}	7.8×10^{-26}	3.6×10^{-25}	8.9×10^{-25}	2.5×10^{-24}
Total	Electric	3.0×10^{-23}	1.4×10^{-22}	6.7×10^{-22}	1.7×10^{-21}	5.1×10^{-21}
	Magnetic	3.3×10^{-25}	1.5×10^{-24}	6.8×10^{-24}	1.7×10^{-23}	4.8×10^{-23}

III. THE ATOMS OF SPIN-0 AND SPIN-$\frac{1}{2}$ COMPONENTS

In the case of the interaction of the spin-0-$\frac{1}{2}$ atom the excitation cross section reads

$$d\sigma^{nl}=d\sigma^{nl}_{\rm el}+d\sigma^{nl}_{\rm mag}+d\sigma^{nl}_{\rm spin}\ ,$$

where except for the previously discussed electric and magnetic cross sections given by formulas (2) and (3) there is the cross section related to the spin-flip process (I.35) which is

$$d\sigma^{nl}_{\rm spin}=\frac{Z^2e^4}{8\pi}\,|\,\Delta(q)F^{nlm=0}(\zeta\mathbf{q})\,|^2\frac{Q^3}{m_2{}^2}dQ\ . \tag{7}$$

The spin-flip processes contribute to the total cross section as

$$d\sigma^{\rm tot}_{\rm spin}=\frac{Z^2e^4}{8\pi}\,|\,\Delta(q)\,|^2\frac{Q^3}{m_2{}^2}dQ\ . \tag{8}$$

As quoted in paper I the integrated cross section (8) is logarithmically divergent at high momentum transfer.

The results of numerical calculations for $A_{e\pi}$ and $A_{\mu\pi}$ atoms are given in Tables IV and V. The cross section (8) is integrated from $Q=0$ to $Q=100/a$ where a is the Bohr atom radius.

IV. THE ATOMS OF SPIN-$\frac{1}{2}$ PARTICLES

In this section we discuss the spin-$\frac{1}{2}$-$\frac{1}{2}$ atoms. Our discussion is split into two parts. The atoms initially in a para state (the atom spin equals zero) are considered in Sec. IV A while the atoms in ortho state (the spin equals unity) in Sec. IV B.

A. Para atoms

If the atomic spin is conserved in the interaction process the excitation cross section is described by formulas (1)–(3). The cross section of the atomic excitation associated with the para-ortho transition found from Eq. (I.36) reads

$$d\sigma^{nl}_{\text{para-ortho}}=\frac{Z^2e^4}{8\pi}\,|\,\Delta(q)\,|^2\times\left|\frac{1}{m_1}F^{nlm=0}(\eta\mathbf{q})+\frac{1}{m_2}F^{nlm=0}(\zeta\mathbf{q})\right|^2Q^3dQ\ , \tag{9}$$

TABLE III. The cross sections (in cm^2) of $A_{\pi K}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

πK		C	Al	Cu	Ag	Pb
$1S$	Electric	3.4×10^{-24}	1.6×10^{-23}	8.0×10^{-23}	2.1×10^{-22}	6.4×10^{-22}
	Magnetic	0.0	0.0	0.0	0.0	0.0
$2S$	Electric	7.4×10^{-25}	3.5×10^{-24}	1.7×10^{-23}	4.5×10^{-23}	1.4×10^{-22}
	Magnetic	0.0	0.0	0.0	0.0	0.0
$2P$	Electric	6.6×10^{-23}	2.9×10^{-22}	1.3×10^{-21}	3.3×10^{-21}	9.3×10^{-21}
	Magnetic	3.1×10^{-25}	1.3×10^{-24}	5.3×10^{-24}	1.2×10^{-23}	3.1×10^{-23}
$3S$	Electric	1.5×10^{-25}	7.1×10^{-25}	3.5×10^{-24}	9.2×10^{-24}	2.8×10^{-23}
	Magnetic	0.0	0.0	0.0	0.0	0.0
$3P$	Electric	1.1×10^{-23}	5.0×10^{-23}	2.3×10^{-22}	5.7×10^{-22}	1.6×10^{-21}
	Magnetic	7.1×10^{-26}	2.9×10^{-25}	1.2×10^{-24}	2.8×10^{-24}	7.0×10^{-24}
Total	Electric	1.4×10^{-22}	6.2×10^{-22}	2.9×10^{-21}	7.3×10^{-21}	2.1×10^{-20}
	Magnetic	1.3×10^{-24}	5.5×10^{-24}	2.3×10^{-23}	5.2×10^{-23}	1.3×10^{-22}

TABLE IV. The cross sections (in cm^2) of $A_{e\pi}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

$e\pi$		C	Al	Cu	Ag	Pb
$1S$	Electric	9.9×10^{-20}	3.6×10^{-19}	1.3×10^{-18}	2.8×10^{-18}	6.7×10^{-18}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	1.1×10^{-24}	3.1×10^{-24}	8.8×10^{-24}	1.6×10^{-23}	3.1×10^{-23}
$2S$	Electric	4.2×10^{-21}	1.2×10^{-20}	3.4×10^{-20}	6.1×10^{-20}	1.2×10^{-19}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	7.3×10^{-26}	2.3×10^{-25}	7.1×10^{-25}	1.4×10^{-24}	2.8×10^{-24}
$2P$	Electric	1.5×10^{-20}	3.9×10^{-20}	9.8×10^{-20}	1.6×10^{-19}	2.9×10^{-19}
	Magnetic	6.1×10^{-25}	1.4×10^{-24}	3.0×10^{-24}	4.6×10^{-24}	7.6×10^{-24}
	Spin	1.3×10^{-25}	3.6×10^{-25}	1.0×10^{-24}	1.8×10^{-24}	3.5×10^{-24}
$3S$	Electric	1.0×10^{-21}	3.0×10^{-21}	8.6×10^{-21}	1.6×10^{-20}	3.0×10^{-20}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	2.0×10^{-26}	6.3×10^{-26}	2.0×10^{-25}	3.8×10^{-25}	7.8×10^{-25}
$3P$	Electric	3.8×10^{-21}	9.9×10^{-21}	2.5×10^{-20}	4.3×10^{-20}	7.7×10^{-20}
	Magnetic	1.6×10^{-25}	3.5×10^{-25}	7.8×10^{-25}	1.2×10^{-24}	2.1×10^{-24}
	Spin	3.6×10^{-26}	1.1×10^{-25}	3.0×10^{-25}	5.5×10^{-25}	1.1×10^{-24}
Total	Electric	3.0×10^{-19}	1.0×10^{-18}	3.5×10^{-18}	7.2×10^{-18}	1.7×10^{-17}
	Magnetic	6.7×10^{-24}	1.9×10^{-23}	5.5×10^{-23}	1.0×10^{-22}	2.2×10^{-22}
	Spin	5.9×10^{-23}	2.6×10^{-22}	1.2×10^{-21}	2.9×10^{-21}	8.2×10^{-21}

where the summation over final-state spin projections has been performed.

The para-ortho transitions give the following contribution to the total cross section [Eq. (I.39)]:

$$d\sigma^{\rm tot}_{\rm para\text{-}ortho}=\frac{Z^2e^4}{8\pi}\,|\,\Delta(q)\,|^2 \times\left[\frac{1}{m_1{}^2}+\frac{1}{m_2{}^2}+\frac{2}{m_1m_2}F^{100}(\mathbf{q})\right]Q^3dQ\ , \tag{10}$$

which is divergent at high momentum transfer if integrated over Q.

The results of numerical calculations for A_{2e}, $A_{e\mu}$, and $A_{2\mu}$ atoms are presented in Tables VI, VII, and VIII. The cross section (10) is integrated up to $100/a$.

B. Ortho atoms

In the case of the atomic excitation with the spin (not spin projection) conserved, one has to add to the previously derived cross sections (1)–(3) the cross section related to the changes of the atomic spin projection [Eq. (I.36)]:

$$d\sigma^{nl}_{\rm spin}=\frac{Z^2e^4}{12\pi}\,|\,\Delta(q)\,|^2 \times\left|\frac{1}{m_1}F^{nlm=0}(\eta\mathbf{q})-\frac{1}{m_2}F^{nlm=0}(\xi\mathbf{q})\right|^2 Q^3dQ\ , \tag{11}$$

where the summation over final-state spin projections and averaging over initial spin projections have been performed.

TABLE V. The cross sections (in cm^2) of $A_{\mu\pi}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

$\mu\pi$		C	Al	Cu	Ag	Pb
$1S$	Electric	7.4×10^{-25}	3.5×10^{-24}	1.7×10^{-23}	4.5×10^{-23}	1.4×10^{-22}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	1.7×10^{-27}	7.2×10^{-27}	3.3×10^{-26}	8.2×10^{-26}	2.3×10^{-25}
$2S$	Electric	1.7×10^{-25}	8.1×10^{-25}	4.0×10^{-24}	1.0×10^{-23}	3.1×10^{-23}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	1.3×10^{-29}	6.1×10^{-29}	3.1×10^{-28}	8.0×10^{-28}	2.4×10^{-27}
$2P$	Electric	1.9×10^{-22}	8.3×10^{-22}	3.7×10^{-21}	9.2×10^{-21}	2.6×10^{-20}
	Magnetic	7.1×10^{-25}	2.6×10^{-24}	9.6×10^{-24}	2.1×10^{-23}	4.9×10^{-23}
	Spin	5.2×10^{-27}	2.4×10^{-28}	1.2×10^{-27}	3.1×10^{-27}	9.5×10^{-27}
$3S$	Electric	3.6×10^{-26}	1.7×10^{-25}	8.4×10^{-25}	2.2×10^{-24}	6.6×10^{-24}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	3.2×10^{-30}	1.5×10^{-29}	7.5×10^{-29}	2.0×10^{-28}	6.0×10^{-28}
$3P$	Electric	3.3×10^{-23}	1.4×10^{-22}	6.5×10^{-22}	1.6×10^{-21}	4.6×10^{-21}
	Magnetic	1.6×10^{-25}	5.9×10^{-25}	2.2×10^{-24}	4.7×10^{-24}	1.1×10^{-23}
	Spin	1.2×10^{-29}	5.7×10^{-29}	2.8×10^{-28}	7.4×10^{-28}	2.2×10^{-27}
Total	Electric	4.0×10^{-22}	1.7×10^{-21}	7.9×10^{-21}	2.0×10^{-20}	5.6×10^{-20}
	Magnetic	3.0×10^{-24}	1.1×10^{-23}	4.1×10^{-23}	8.9×10^{-23}	2.1×10^{-22}
	Spin	3.5×10^{-27}	1.6×10^{-26}	7.5×10^{-26}	1.9×10^{-25}	5.7×10^{-25}

TABLE VI. The cross sections (in cm^2) of positronium interaction with atoms of C, Al, Cu, Ag, and Pb.

	ee	C	Al	Cu	Ag	Pb
1*S*	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	4.4×10^{-24}	1.2×10^{-23}	3.5×10^{-23}	6.4×10^{-23}	1.2×10^{-22}
2*S*	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	2.9×10^{-25}	9.2×10^{-25}	2.8×10^{-24}	5.4×10^{-24}	1.1×10^{-23}
2*P*	Electric	6.1×10^{-20}	1.5×10^{-19}	3.8×10^{-19}	6.3×10^{-19}	1.1×10^{-18}
	Magnetic	6.1×10^{-25}	1.3×10^{-24}	2.9×10^{-24}	4.6×10^{-24}	7.6×10^{-24}
	Spin	3.4×10^{-25}	9.6×10^{-25}	2.7×10^{-24}	4.9×10^{-24}	9.2×10^{-24}
	Para-ortho	0.0	0.0	0.0	0.0	0.0
3*S*	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	7.9×10^{-26}	2.5×10^{-25}	7.9×10^{-25}	1.5×10^{-24}	3.1×10^{-24}
3*P*	Electric	1.5×10^{-20}	3.9×10^{-20}	9.9×10^{-20}	1.7×10^{-19}	3.0×10^{-19}
	Magnetic	1.6×10^{-25}	3.5×10^{-25}	7.8×10^{-25}	1.2×10^{-24}	2.0×10^{-24}
	Spin	9.6×10^{-26}	2.8×10^{-25}	8.1×10^{-25}	1.5×10^{-24}	2.8×10^{-24}
	Para-ortho	0.0	0.0	0.0	0.0	0.0
Total	Electric	4.6×10^{-19}	1.5×10^{-18}	4.7×10^{-18}	9.5×10^{-18}	2.1×10^{-17}
	Magnetic	5.0×10^{-24}	1.3×10^{-23}	3.7×10^{-23}	6.7×10^{-23}	1.4×10^{-22}
	Spin	6.2×10^{-23}	2.6×10^{-22}	1.2×10^{-21}	2.9×10^{-21}	7.9×10^{-21}
	Para-ortho	9.6×10^{-23}	4.0×10^{-22}	1.8×10^{-21}	4.3×10^{-21}	1.2×10^{-20}

TABLE VII. The cross sections (in cm^2) of $A_{e\mu}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

	$e\mu$	C	Al	Cu	Ag	Pb
1*S*	Electric	9.9×10^{-20}	3.6×10^{-19}	1.3×10^{-18}	2.8×10^{-18}	6.7×10^{-18}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	7.1×10^{-25}	2.0×10^{-24}	5.7×10^{-24}	1.0×10^{-23}	2.0×10^{-23}
	Para-ortho	1.1×10^{-24}	3.2×10^{-24}	9.0×10^{-24}	1.6×10^{-23}	3.2×10^{-23}
2*S*	Electric	4.2×10^{-21}	1.2×10^{-20}	3.4×10^{-20}	6.1×10^{-20}	1.2×10^{-19}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	4.9×10^{-26}	1.5×10^{-25}	4.7×10^{-25}	9.1×10^{-25}	1.9×10^{-24}
	Para-ortho	7.3×10^{-26}	2.3×10^{-25}	7.1×10^{-25}	1.4×10^{-24}	2.8×10^{-24}
2*P*	Electric	1.6×10^{-20}	4.0×10^{-20}	9.9×10^{-20}	1.7×10^{-19}	2.9×10^{-19}
	Magnetic	6.1×10^{-25}	1.4×10^{-24}	3.0×10^{-24}	4.6×10^{-24}	7.6×10^{-24}
	Spin	8.4×10^{-26}	2.4×10^{-25}	6.8×10^{-25}	1.2×10^{-24}	2.3×10^{-24}
	Para-ortho	1.3×10^{-25}	3.6×10^{-25}	1.0×10^{-24}	1.8×10^{-24}	3.5×10^{-24}
3*S*	Electric	1.0×10^{-21}	3.0×10^{-21}	8.6×10^{-21}	1.6×10^{-20}	3.0×10^{-20}
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	1.3×10^{-26}	4.2×10^{-26}	1.3×10^{-25}	2.5×10^{-25}	5.2×10^{-25}
	Para-ortho	2.0×10^{-26}	6.3×10^{-26}	2.0×10^{-25}	3.8×10^{-25}	7.8×10^{-25}
3*P*	Electric	3.8×10^{-21}	1.0×10^{-20}	2.6×10^{-20}	4.3×10^{-20}	7.8×10^{-20}
	Magnetic	1.6×10^{-25}	3.5×10^{-25}	7.8×10^{-25}	1.2×10^{-24}	2.1×10^{-24}
	Spin	2.4×10^{-26}	7.1×10^{-26}	2.0×10^{-25}	3.7×10^{-25}	7.1×10^{-25}
	Para-ortho	3.6×10^{-26}	1.1×10^{-25}	3.0×10^{-25}	5.5×10^{-25}	1.1×10^{-24}
Total	Electric	3.0×10^{-19}	1.0×10^{-18}	3.5×10^{-18}	7.2×10^{-18}	1.7×10^{-17}
	Magnetic	6.7×10^{-24}	1.9×10^{-23}	5.5×10^{-23}	1.0×10^{-22}	2.2×10^{-22}
	Spin	4.0×10^{-23}	1.7×10^{-22}	7.8×10^{-22}	1.9×10^{-21}	5.4×10^{-21}
	Para-ortho	5.9×10^{-23}	2.6×10^{-22}	1.2×10^{-21}	2.9×10^{-21}	8.2×10^{-21}

TABLE VIII. The cross sections (in cm^2) of $A_{2\mu}$ atom interaction with atoms of C, Al, Cu, Ag, and Pb.

	$\mu\mu$	C	Al	Cu	Ag	Pb
$1S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	6.7×10^{-27}	2.9×10^{-26}	1.3×10^{-25}	3.3×10^{-25}	9.3×10^{-25}
$2S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	5.2×10^{-29}	2.5×10^{-28}	1.2×10^{-27}	3.2×10^{-27}	9.7×10^{-27}
$2P$	Electric	2.4×10^{-22}	1.0×10^{-21}	4.6×10^{-21}	1.1×10^{-20}	3.2×10^{-20}
	Magnetic	8.2×10^{-25}	2.9×10^{-24}	1.1×10^{-23}	2.3×10^{-23}	5.3×10^{-23}
	Spin	1.4×10^{-28}	6.5×10^{-28}	3.2×10^{-27}	8.4×10^{-27}	2.5×10^{-26}
	Para-ortho	0.0	0.0	0.0	0.0	0.0
$3S$	Electric	0.0	0.0	0.0	0.0	0.0
	Magnetic	0.0	0.0	0.0	0.0	0.0
	Spin	0.0	0.0	0.0	0.0	0.0
	Para-ortho	1.3×10^{-29}	6.0×10^{-29}	3.0×10^{-28}	7.9×10^{-28}	2.4×10^{-27}
$3P$	Electric	4.2×10^{-23}	1.8×10^{-22}	8.1×10^{-22}	2.0×10^{-21}	5.7×10^{-21}
	Magnetic	1.8×10^{-25}	6.6×10^{-25}	2.4×10^{-24}	5.1×10^{-24}	1.2×10^{-23}
	Spin	3.3×10^{-29}	1.5×10^{-28}	7.6×10^{-28}	2.0×10^{-27}	6.0×10^{-27}
	Para-ortho	0.0	0.0	0.0	0.0	0.0
Total	Electric	5.0×10^{-22}	2.1×10^{-21}	9.8×10^{-21}	2.4×10^{-20}	6.9×10^{-20}
	Magnetic	3.5×10^{-24}	1.3×10^{-23}	4.5×10^{-23}	9.7×10^{-23}	2.3×10^{-22}
	Spin	1.9×10^{-27}	8.7×10^{-27}	4.3×10^{-26}	1.1×10^{-25}	3.4×10^{-25}
	Para-ortho	1.4×10^{-26}	5.8×10^{-26}	2.8×10^{-25}	7.0×10^{-25}	2.0×10^{-24}

The cross section (11) gives the contribution to the total cross section:

$$d\sigma^{\rm tot}_{\rm spin}=\frac{Z^2e^4}{12\pi}\,|\,\Delta(q)\,|^2 \times\left[\frac{1}{m_1{}^2}+\frac{1}{m_2{}^2}-\frac{2}{m_1m_2}F^{100}(\mathbf{q})\right]Q^3dQ\ . \quad (12)$$

The cross section (12) integrated over Q is logarithmically divergent.

The cross section of the atomic excitation associated with the ortho-para transition is

$$d\sigma^{nl}_{\rm ortho\text{-}para}=\tfrac{1}{3}d\sigma^{nl}_{\rm para\text{-}ortho}\ , \quad (13)$$

where the cross section $d\sigma^{nl}_{\rm para\text{-}ortho}$ is given by Eq. (9). The respective total cross section is identically expressed through $d\sigma^{\rm tot}_{\rm para\text{-}ortho}$.

The results of numerical calculations for A_{2e}, $A_{e\mu}$, and $A_{2\mu}$ atoms are presented in Tables VI, VII, and VIII. The cross section (12) is integrated up to $Q=100/a$. The cross sections of ortho-para transitions which are not included in the tables can be found from relation (13).

V. DISCUSSION OF THE NUMERICAL RESULTS

Some of the cross sections presented in Tables I–VIII have been calculated previously. Dulian and Kotzinian[3] have given the electric parts of excitation and total cross sections.[4] They have also studied the transitions from ortho to para positronium. Pak and Tarasov[6] have calculated the positronium total cross section taking into account the electric contribution only. Finally, most of electric cross sections presented here have been given in Ref. 5.

The cross sections from Tables I–VIII coincide with those calculated earlier except the elastic cross sections of $A_{e\pi}$ from Ref. 3. These cross sections are, in our opinion, underestimated in Ref. 3 by a factor of about 1.4–1.8. One should also remember that the cross sections given in Ref. 3 should be averaged over initial states and summarized over orbital momentum and spin projections to compare them to our results from Tables IV and VI.

Small discrepancies between the cross sections presented here and in Ref. 5 are due to numerical uncertainties. The values given here are of higher precision.

Let us now discuss the contents of Tables I–VIII. One observes that except para-ortho transitions the electric parts give dominant contributions to the cross sections. The cross sections related to the spin-flip of the atomic components are smaller than the respective electric cross sections by about 5 orders of magnitude. This result can be easily understood. The differential cross section of spin transition differs from the respective electric one by a factor Q^2/m^2 (for equal-mass-component atoms), which, for characteristic momentum transfer equal to the inverse Bohr radius, is $e^4/4\approx1.3\times10^{-5}$.

The magnetic cross sections are smaller than the respective electric ones by 5 orders of magnitude for the atoms of small reduced mass such as positronium and by 2 orders for the atoms of heavier reduced mass such as pionium, $A_{2\pi}$, or kaonium, A_{2K}. It occurs due to the increase of electric current related to the motion of atom

components when the Bohr radius decreases.

The cross-section increase with the target charge Z can be well parametrized as Z^{α} and this parametrization can be used to find the cross sections for the target elements which are not calculated in our paper. For example, the total cross sections of positronium and kaonium read

$$\sigma_{2e}^{\text{tot}} \approx 3.4 \times 10^{-20} Z^{1.46} \quad (\text{cm}^2)$$

and

$$\sigma_{2K}^{\text{tot}} \approx 9.0 \times 10^{-25} Z^{1.96} \quad (\text{cm}^2) .$$

As seen the exponent α increases with the elementary-atom reduced mass, and for kaonium it is close to its maximum value, 2. It reflects the fact that only small impact-parameter collisions give a significant contribution to the cross section if the elementary-atom Bohr radius is much smaller than the screening length of the target atom. Then the electron screening is of no importance and the Z dependence of the cross section is as that one of Coulomb field.

Because the electric cross sections are identical to those found in nonrelativistic calculations[5] (with $\mathbf{v}^2 = 1$), further remarks on the cross sections can be found in Ref. 5.

VI. CONCLUSIONS

The calculations of elementary-atom interaction with matter presented here do not solve the problem completely.

From the pure theoretical point of view our approach with its nonrelativistic description of the elementary atom is not very elegant. On the other hand, this description leads to certain difficulties in studying the spin-flip processes (divergences of the integrated cross sections).

There are also important points which are beyond the scope of our paper. First let us note that an atom of matter has been treated as structureless in our considerations. Therefore the excitations of the atom have not been taken into account. In fact the cross sections of the (incoherent) processes associated with the matter atom excitation are much smaller than the cross sections of processes without excitations (coherent processes) for sufficiently heavy atoms of matter because the coherent cross sections are roughly proportional to Z^2 while the incoherent ones are proportional to Z. The incoherent interactions have been studied by Pak and Tarasov,[6,7] and they have shown that the ratio of positronium total incoherent to coherent cross sections is 0.93 for $Z=6$ and decreases to 0.14 for $Z=60$. They have also demonstrated that the ratio is just Z^{-1} for the elementary atoms of Bohr radius, much less than the hydrogen-atom Bohr radius.

The elementary-atom ionization cross sections have not been given by us due to the difficulties in calculating the respective form factors which have to be found by means of the so-called exact Coulomb wave functions of the continuous spectrum. Let us note that one cannot use the plane wave to describe the ionized atom because in this case the integration over the plane-wave momentum is equivalent to the summation over a complete set of wave functions, and the resulting cross sections equals the total, not ionization, cross section. This point has not been realized by the authors of Ref. 3 (see Ref. 4).

Pak and Tarasov[7] have calculated the ionization cross section of positronium using the approximate formula for the summation of the cross sections of excitations to the discrete spectrum states. The cross section of interest has been found as the difference of the total cross section and the summarized excitation one.

The probability of particle interaction in the target of a thickness l can be usually found from the formula

$$W(l) = 1 - \exp(-\rho \sigma^{\text{tot}} l) , \tag{14}$$

where ρ is the density of atoms in the target. Therefore the knowledge of the total cross section is sufficient to determine the probability W. Formula (14) is valid under the assumption that the particle collisions with atoms of the target are independent from one another. Nemenov has observed[8] that for the ultrarelativistic positronia, as those registered in the experiment,[9] the characteristic time of positronium internal motion (or order of the inverse binding energy) can be much longer than the time interval between successive collisions in the target. Therefore the collisions are not independent from one another, and the probability of the positronium destruction is significantly smaller than the one predicted by formula (14). The phenomenon, which is analogous to the famous Landau-Pomeranchuk effect,[10] has been called the positronium superpenetrability. Lyuboshitz and Podgoretsky[11] have shown that for the targets, thick when compared to the mean free path, the probability for observing the positronium in the bound state does not exponentially decrease with the target thickness but it is proportional to l^{-1}. Further analysis of the positronium penetrability of thick and thin targets has been recently performed by Pak and Tarasov[12] and the results will be compared with experiment in the near future.[13]

After the brief review of the points which have not been studied by us let us recapitulate our considerations.

The cross sections of relativistic elementary atoms interacting with atoms of matter have been derived in the Born approximation. The problem has been considered in the reference frame where the elementary atom is initially at rest and the atom of matter is a fast particle. Because the characteristic momentum transfer to the atom is of order of the inverse Bohr radius of the atom, the initial and final atom states have been treated nonrelativistically. Each cross section has been split, due to certain approximations, into electric, magnetic, and, if any, spin parts. It has been possible because the respective amplitudes do not interfere. Although the cross sections integrated over momentum transfer are, of course, Lorentz invariant the above quoted parts of them are not. Therefore our terminology relates to the particular choice of the reference frame. The electric part corresponds to the interaction of electric charges of atomic components with the electric field of the projectile. The electric parts of cross sections found in relativistic and nonrelativistic[5] approaches are exactly the same. The magnetic parts reflect the interaction of the current generated by the atomic component motion with the projectile magnetic field. The spin cross sections relate to the spin-flip processes. In the case of the atom composed of

two spin-$\frac{1}{2}$ particles the inverse of the atomic component spin projection leads to the change of the atomic spin projection without or with the change of the atomic total spin. In the latter case one deals with the para-ortho or ortho-para transitions. The magnetic and spin parts of cross sections, which automatically arise in the relativistic approach, are absent in the nonrelativistic one. However the numerical calculations presented in Tables I–VIII explicitly show that these relativistic corrections are negligible when compared to the electric part background.

The only qualitatively new relativistic effects are the para-ortho and ortho-para transitions which can be of practical importance due to the difference of the para and ortho states of elementary atoms. For example, the lifetime of ortho-positronium is greater by 3 orders of magnitude than that of para-positronium.[14]

We conclude our study as follows. Most of the results presented here are of pure theoretical rather than practical significance. We have elucidated in detail the problem of the elementary-atom interaction with atom of matter. In particular the role of relativistic effects has been determined. We have explicitly proved the correctness of the calculations from Refs. 3, 6, 11, and 12, where only the electric part of the interaction has been taken into account.

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APPENDIX

In this appendix we discuss the form factors $\mathbf{D}^{nlm}(\mathbf{q})$ and $K_{100}(\mathbf{v},\mathbf{q})$ defined as

$$\mathbf{D}^{nlm}(\mathbf{q})=i\int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}}\phi^*_{nlm}(\mathbf{r})\nabla\phi_{100}(\mathbf{r}) , \tag{A1}$$

$$K_{100}(\mathbf{v},\mathbf{q})=\int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}}\,|\,\mathbf{v}\cdot\nabla\phi_{100}(\mathbf{r})\,|^2 , \tag{A2}$$

where ϕ_{nlm} and ϕ_{100} are the hydrogenlike atom wave functions. The quantization axis of the atomic orbital momentum is chosen along the vector $\mathbf{q}$ which defines the z axis of the coordinate system. Substituting the explicit expression of the atomic ground-state wave function in Eq. (A1) one finds

$$\mathbf{D}^{nlm}(\mathbf{q})=\frac{1}{i\sqrt{\pi}}\int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}-r}\phi^*_{nlm}(\mathbf{r})\begin{pmatrix}\sin\Theta\cos\phi\\ \sin\Theta\sin\phi\\ \cos\Theta\end{pmatrix} ,$$

where the expressions in the column relate to the x, y, and z components of $\mathbf{D}^{nlm}$ vector denoted as D_x^{nlm}, D_y^{nlm}, and D_z^{nlm}. In this appendix we use the so-called atomic units where the atom Bohr radius equals unity.

Because the ϕ dependence of the function ϕ_{nlm} is $e^{im\phi}$ one easily performs the integration over ϕ. Then one finds that

$$\begin{aligned} &D_x^{nlm}(\mathbf{q})\neq 0 \quad \text{for } m=\pm 1 ,\\ &D_y^{nlm}(\mathbf{q})\neq 0 \quad \text{for } m=\pm 1 ,\\ &D_z^{nlm}(\mathbf{q})\neq 0 \quad \text{for } m=0 . \end{aligned} \tag{A3}$$

Otherwise the components of $\mathbf{D}^{nlm}(\mathbf{q})$ equal zero. One also trivially derives the relations

$$\begin{aligned} &D_x^{nlm=1}(\mathbf{q})=D_x^{nlm=-1}(\mathbf{q}) ,\\ &D_y^{nlm=1}(\mathbf{q})=-D_y^{nlm=-1}(\mathbf{q}) ,\\ &D_y^{nlm=\pm 1}(\mathbf{q})=\pm iD_x^{nlm=\pm 1}(\mathbf{q}) . \end{aligned} \tag{A4}$$

In Table IX we give the formulas of the x and z components of some form factors $\mathbf{D}^{nlm}$. The components which are not given in the table can be found from relations (A4).

Substituting the explicit expression of ϕ_{100} in Eq. (A2) we get

TABLE IX. $\mathbf{D}^{nlm}(\mathbf{q})$ form factors.

		$n=1$	$n=2$	$n=3$
$l=0$	x	0	0	0
$m=0$	z	$\frac{2^3Q}{(2^2+Q^2)^2}$	$\frac{2^{11/2}Q}{(3^2+2^2Q^2)^3}(3+2^2Q^2)$	$-\frac{2^33^{3/2}}{(2^4+3^2Q^2)^4}[2^7+3^2Q^2(2^3-3^3Q^2)]$
$l=1$	x		0	0
$m=0$	z		$\frac{2^{9/2}3}{(3^2+2^2Q^2)^3}(3^2-2^2Q^2)$	$\frac{2^{13/2}3}{(2^4+3^2Q^2)^4}[7\times 2^7-3^2Q^2(2^3+3^3Q^2)]$
$l=1$	x		$\frac{-i2^4}{(3^2+2^2Q^2)^2}$	$\frac{-i2^43}{(2^4+3^2Q^2)^3}(2^3+3^2Q^2)$
$m=1$	z		0	0
$l=2$	x			0
$m=0$	z			$-\frac{2^{15/2}3^{3/2}Q}{(2^4+3^2Q^2)^4}(2^3-3^2Q^2)$
$l=2$	x			Not found
$m=1$	z			0

$$K_{100}(\mathbf{v},\mathbf{q})=(v_x{}^2+v_y{}^2)A+v_z{}^2\left[\frac{8}{(4+Q^2)^2}-A\right],$$

where $Q\equiv|\mathbf{q}|$ and

$$A\equiv\frac{4}{Q^3}\left[\arctan(Q/2)-\frac{2Q}{4+Q^2}\right].$$

One also finds

$$K_{100}(\mathbf{v},\mathbf{q}=0)=\tfrac{1}{3}(v_x{}^2+v_y{}^2)+\tfrac{1}{6}v_z{}^2 .$$

For $\mathbf{v}\perp\mathbf{q}$ and $\mathbf{v}^2=1$, $K_{100}(\mathbf{v},\mathbf{q})=A$.

To make the formulas given in the appendix of correct dimension one has to replace Q by aQ where a is the Bohr atom radius. Additionally the form factors $\mathbf{D}^{nlm}$ should be divided by a while K_{100} by a^2.

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