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Quasilinear transport approach to equilibration of quark-gluon plasmas

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We derive the transport equations of quark-gluon plasma in the quasilinear approximation. The equations are either of the Balescu-Lenard or Fokker-Planck form. The plasma's dynamics is assumed to be governed by longitudinal chromoelectric fields. The isotropic plasma, which is stable, and the two-stream system, which is unstable, are considered in detail. A process of equilibration is briefly discussed in both cases. The peaks of the two-stream distribution are shown to rapidly dissolve in time.

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I. INTRODUCTION

The quark-gluon plasma (QGP), which is produced in relativistic heavy-ion collisions, is believed to be equilibrated within a time interval of order of 1 fm$/c$ or even shorter [1]. Such a fast equilibration is naturally explained assuming that the quark-gluon plasma is strongly coupled [2–4]. Then, scattering processes are very frequent and relaxation times are short. However, the theory of high-energy density QCD [5] suggests that due to the existence of a large momentum scale Q_s, at which the gluon density saturates, the plasma is rather weakly coupled at the early stage of the collision because of asymptotic freedom. Experimental data on jet quenching indicate that the coupling constant $\alpha_s \leq 0.3$ [6,7], even though the value assumes averaging over the whole evolution of the QCD medium created in the relativistic heavy-ion collision. Thus, the question arises how fast the weakly interacting plasma equilibrates. Because of anisotropic momentum distributions the early stage plasma is unstable with respect to the chromomagnetic plasma modes. The instabilities isotropize the system and thus speed up the process of its equilibration. The scenario of the instabilities-driven isotropization is reviewed in [8]. However, the complete evolution of the plasma momentum distribution is now accessible only by numerical simulations [9–11].

The transport theory of a weakly coupled quark-gluon plasma has been studied since the 1980s when the kinetic equations in the mean-field approximation were derived [12,13]. Although the mean-field dynamics is rather simplified, the equations are still difficult to solve due to their nonlinear structure. If one is interested in small deviations from equilibrium or any other homogeneous and stationary state, the equations can be linearized and then solved. The mean-field transport theory, which is linearized in small deviations from equilibrium, is now well understood, for a review see [14]. It is known to be equivalent to the effective QCD in the hard-thermal loop approximation. The linearized transport theory around any homogeneous and stationary but nonequilibrium plasma state was also worked out and the connection with the diagrammatic hard loop approximation was established [15,16]. Numerous problems of the theory of the quark-gluon plasma were successfully resolved within the hard loop approach. For example, a systematic method to eliminate infrared divergences, which plague perturbative calculations, was developed, see the reviews [17,18].

However, various questions cannot be addressed within the mean-field theory. For example, transport coefficients are then formally infinite. Thus, there were numerous efforts to derive transport equations of quark-gluon plasma which hold beyond the hard loop approximation [19–39]. These efforts were mostly concerned with the transport properties of an *equilibrium* quark-gluon plasma. Our motivation is rather different. We are interested in *equilibration* of quark-gluon plasma, in particular, in the equilibration of the system which is initially unstable. Thus, we intend to study how fluctuating deviations from a quasistationary nonequilibrium state influence the system's bulk or average momentum distribution. This effect of backreaction is particularly important in the case of unstable systems. The linear response theory describes how unstable modes initially grow in the presence of a nonequilibrium momentum distribution, but it says nothing on how the modes modify the plasma momentum distribution. Thus, the problem of equilibration cannot be addressed in such a theory.

Our objective here is to derive the transport equations where the bulk distribution function slowly evolves due to the interaction with fluctuating chromodynamic fields. We actually consider only a simplified problem of QGP in a self-consistently generated *longitudinal* chromoelectric field. This simplification is not much needed for isotropic plasma but it appears crucial to study anisotropic systems. Taking into account only the longitudinal chromoelectric field, we obtain the transport equations of the Fokker-Planck or Balescu-Lenard form which describe the effect

of backreaction. A similar, but incomplete effort was undertaken by Akkelin [39]. The derivation presented here closely follows the procedure developed for the electromagnetic plasma, where it is known as the quasilinear theory or the theory of a weakly turbulent plasma [40–42]. The theory assumes that the distribution function of plasma particles can be decomposed into a large but slowly varying regular part and a small fluctuating or turbulent one which oscillates fast. The average over the statistical ensemble of the turbulent part is assumed to vanish and thus the average of the distribution function equals its regular part. The turbulent contribution to the distribution function obeys the collisionless transport equation while the transport equation of the regular part, which is our main interest here, is determined by the fluctuation spectra. The fluctuations of chromodynamic fields, which are used to derive the quasilinear transport equations, were studied in [43] where stable and unstable plasma states were considered.

The Fokker-Planck equation derived here is somewhat similar to the equation obtained in [44,45]. It was used there to show that the chromomagnetized quark-gluon plasma exhibits an anomalous shear viscosity, as presence of the domains of chromomagnetic field leads to the momentum transport in the plasma.

Our paper is organized as follows. In Sec. II we present the QGP transport equations; the notation and conventions are introduced. The decomposition of the distribution functions into the regular and turbulent parts is discussed in Sec. III. The explicit expressions of the fluctuating distribution functions which obey collisionless transport equations are derived in Sec. IV. A general form of the equations of the regular distribution functions is found here as well. Further discussion splits into two parallel parts: Section V is devoted to the stable isotropic plasma while in Sec. VI the unstable two-stream system is discussed. Although we neglect transverse chromodynamic fields, the collision terms of transport equations, which are found here for an isotropic plasma, are very similar to those derived in [19,21,25,25,26,29–33]. As an application of the transport equations we derived, a process of equilibration of the isotropic plasma and of the two-stream system is discussed. The paper closes with a summary of our considerations and outlook.

II. PRELIMINARIES

The transport theory of a quark-gluon plasma, which forms the basis of our analysis, is formulated in terms of particles and classical fields. The particles—quarks, antiquarks and gluons—should be understood as sufficiently hard quasiparticle excitations of quantum fields of QCD while the classical fields are highly populated soft gluonic modes. An excitation is called ''hard'' when its momentum in the equilibrium rest frame is of order of the temperature T, and it is called ''soft'' when the momentum is of order gT with g being the coupling constant. Since we consider a weakly coupled quark-gluon plasma, the coupling constant is assumed to be small $g \ll 1$. In our further considerations the quasiparticles are treated as classical particles obeying Boltzmann statistics but the effect of quantum statistics can be easily taken into account.

The transport equations of quarks, antiquarks and gluons are assumed to be of the form

$$\begin{aligned} \mathcal{D}\, Q(t,\mathbf{r},\mathbf{p}) - \frac{1}{2}\{\mathbf{F}(t,\mathbf{r}), \nabla_p Q(t,\mathbf{r},\mathbf{p})\} &= 0, \\ \mathcal{D}\bar{Q}(t,\mathbf{r},\mathbf{p}) + \frac{1}{2}\{\mathbf{F}(t,\mathbf{r}), \nabla_p \bar{Q}(t,\mathbf{r},\mathbf{p})\} &= 0, \\ \mathcal{D}G(t,\mathbf{r},\mathbf{p}) - \frac{1}{2}\{\mathbf{F}(t,\mathbf{r}), \nabla_p G(t,\mathbf{r},\mathbf{p})\} &= 0. \end{aligned} \tag{1}$$

The (anti-)quark distribution functions $Q(t,\mathbf{r},\mathbf{p})$ and $\bar{Q}(t,\mathbf{r},\mathbf{p})$, which are $N_c \times N_c$ Hermitian matrices, belong to the fundamental representation of the SU(N_c) group, while the gluon distribution function $G(t,\mathbf{r},\mathbf{p})$, which is a $(N_c^2-1)\times(N_c^2-1)$ matrix, belongs to the adjoint representation. The distribution functions depend on the time (t), position ($\mathbf{r}$) and momentum ($\mathbf{p}$) variables. There is no explicit dependence on the timelike ($\mu = 0$) component of the four-vector p^μ as the distribution functions are assumed to be nonzero only for momenta obeying the mass-shell constraint $p^\mu p_\mu = 0$. Because the partons are assumed to be massless, the velocity $\mathbf{v}$ equals $\mathbf{p}/E_\mathbf{p}$ with $E_\mathbf{p} = |\mathbf{p}|$. $\mathcal{D} \equiv D^0 + \mathbf{v}\cdot\mathbf{D}$ is the covariant substantive derivative given by the covariant derivative which in the four-vector notation reads $D^\mu \equiv \partial^\mu - ig[A^\mu(x), \cdots]$ with $A^\mu(x)$ being the chromodynamic potential. The mean-filed terms of the transport Eqs. (1) are expressed through the color Lorentz force $\mathbf{F}(t,\mathbf{r}) \equiv g(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r}))$. The chromoelectric $\mathbf{E}(t,\mathbf{r})$ and chromomagnetic $\mathbf{B}(t,\mathbf{r})$ fields belong to either the fundamental or adjoint representation. To simplify the notation we use the same symbols $\mathcal{D}$, D^0, $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{B}$ for a given quantity in the fundamental or adjoint representation. The symbol $\{\ldots,\ldots\}$ denotes the anticommutator.

The collision terms are neglected in the transport Eqs. (1). The collisionless equations are applicable in three physically different situations: when the distribution function is of (local) equilibrium form; when the timescale of processes of interest is much shorter than the average temporal separation of parton collisions; and when the system dynamics is dominated by the mean field. In our study we refer to all three situations. When the equilibration of isotropic plasma is discussed, it is crucial that the collision terms vanish in local equilibrium. In the case of an unstable two-stream system, the effects of collisions can be initially neglected, as the growth of unstable modes is very fast. Later on, the strong fields become mostly responsible for the system's evolution.

The transport equations are supplemented by the Yang-Mills equations describing a self-consistent generation of the chromoelectric and chromomagnetic fields. The equa-

tions read

$$\begin{aligned} \mathbf{D}\cdot\mathbf{E}(t,\mathbf{r}) &= \rho(t,\mathbf{r}) \\ \mathbf{D}\cdot\mathbf{B}(t,\mathbf{r}) &= 0, \\ \mathbf{D}\times\mathbf{E}(t,\mathbf{r}) &= -D_0\mathbf{B}(t,\mathbf{r}), \\ \mathbf{D}\times\mathbf{B}(t,\mathbf{r}) &= \mathbf{j}(t,\mathbf{r}) + D_0\mathbf{E}(t,\mathbf{r}), \end{aligned} \tag{2}$$

where the color four-current $j^\mu = (\rho, \mathbf{j})$ in the adjoint representation equals

$$j_a^\mu(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E_\mathbf{p}}\,\mathrm{Tr}[T^a G(t,\mathbf{r},\mathbf{p}) + \tau^a(Q(t,\mathbf{r},\mathbf{p}) - \bar{Q}(t,\mathbf{r},\mathbf{p}))], \tag{3}$$

where τ^a, T^a with $a = 1,\ldots,N_c^2-1$ are the SU(N_c) group generators in the fundamental and adjoint representations, normalized as $\mathrm{Tr}[\tau^a\tau^b] = \frac{1}{2}\delta^{ab}$ and $\mathrm{Tr}[T^aT^b] = N_c\delta^{ab}$. The set of transport Eqs. (1) and Yang-Mills Eqs. (2) is covariant with respect to SU(N_c) gauge transformations.

III. REGULAR AND FLUCTUATING QUANTITIES

We assume that the chromodynamic fields and distribution functions which enter the set of transport equations can be decomposed into the regular and fluctuating components. The quark distribution function is thus written down as

$$Q(t,\mathbf{r},\mathbf{p}) = \langle Q(t,\mathbf{r},\mathbf{p})\rangle + \delta Q(t,\mathbf{r},\mathbf{p}), \tag{4}$$

where $\langle\cdots\rangle$ denotes ensemble average; $\langle Q(t,\mathbf{r},\mathbf{p})\rangle$ is called the regular part while $\delta Q(t,\mathbf{r},\mathbf{p})$ is called the fluctuating or turbulent one. It directly follows from Eq. (4) that $\langle\delta Q\rangle = 0$. The regular contribution is assumed to be white, and it is expressed as

$$\langle Q(t,\mathbf{r},\mathbf{p})\rangle = n(t,\mathbf{r},\mathbf{p})I, \tag{5}$$

where I is the unit matrix in color space. Since the distribution function transforms under gauge transformations as $Q \to UQU^{-1}$, where U is the transformation matrix, the regular contribution of the form (5) is gauge independent. We also assume that

$$|\langle Q\rangle| \gg |\delta Q|, \qquad |\nabla_p\langle Q\rangle| \gg |\nabla_p\delta Q|, \tag{6}$$

but at the same time

$$\left|\frac{\partial\delta Q}{\partial t}\right| \gg \left|\frac{\partial\langle Q\rangle}{\partial t}\right|, \qquad |\nabla\delta Q| \gg |\nabla\langle Q\rangle|. \tag{7}$$

Analogous conditions are assumed for the antiquark and gluon distribution functions. What concerns the chromodynamic fields, we assume in accordance with (5) that their regular parts vanish and thus

$$\langle\mathbf{E}(t,\mathbf{r})\rangle = \langle\mathbf{B}(t,\mathbf{r})\rangle = 0. \tag{8}$$

We substitute the distribution functions (4) into the transport equations and the Yang-Mills equations and linearize the equations in the fluctuating contributions. The linearized transport and Yang-Mills equations remain rather complex. Therefore, we discuss here a simplified problem: we consider a QGP in the presence of turbulent *longitudinal* chromoelectric fields, but neglect the chromomagnetic and transverse chromoelectric fields. This simplification can be avoided for an isotropic plasma but it is needed, as explained below, to make progress on an analytical treatment for anisotropic systems which are our main interest here. The simplified transport equations then read

$$\begin{aligned} &\mathcal{D}\,\delta Q(t,\mathbf{r},\mathbf{p}) - g\mathbf{E}(t,\mathbf{r})\cdot\nabla_p n(t,\mathbf{r},\mathbf{p}) = 0, \\ &\mathcal{D}\delta\bar{Q}(t,\mathbf{r},\mathbf{p}) + g\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\bar{n}(t,\mathbf{r},\mathbf{p}) = 0, \\ &\mathcal{D}\delta G(t,\mathbf{r},\mathbf{p}) - g\mathbf{E}(t,\mathbf{r})\cdot\nabla_p n_g(t,\mathbf{r},\mathbf{p}) = 0, \end{aligned} \tag{9}$$

where $\mathcal{D} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$ denotes from now on the material (not covariant) derivative.

The equation describing the self-consistent generation of a longitudinal chromoelectric field is

$$\nabla\cdot\mathbf{E}_a(t,\mathbf{r}) = \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\delta N_a(t,\mathbf{r},\mathbf{p}), \tag{10}$$

where

$$\delta N_a(t,\mathbf{r},\mathbf{p}) \equiv \mathrm{Tr}[\tau^a(\delta Q(t,\mathbf{r},\mathbf{p}) - \delta\bar{Q}(t,\mathbf{r},\mathbf{p})) + T^a\delta G(t,\mathbf{r},\mathbf{p})]. \tag{11}$$

The linearized equations are formally Abelian but they include a fundamentally non-Abelian effect, i.e. the gluon contribution to the color current. Therefore, the gluon-gluon coupling is partly taken into account. The linearized Yang-Mills equation corresponds to the multicomponent electrodynamics of N_c charges (in the so-called Heaviside-Lorentz system of units). The equations, however, are no longer manifestly covariant with respect to SU(N_c) gauge transformations. Nevertheless, our final results are gauge independent.

We now substitute the distribution functions (4) into the transport Eqs. (1). Instead of linearizing the equations in the fluctuating contributions, we take the ensemble average of the resulting equations and trace over the color indices. Thus we get

$$\begin{aligned} \mathcal{D}n - \frac{g}{N_c}\,\mathrm{Tr}\langle\mathbf{E}\cdot\nabla_p\delta Q\rangle = 0, \\ \mathcal{D}\bar{n} + \frac{g}{N_c}\,\mathrm{Tr}\langle\mathbf{E}\cdot\nabla_p\delta\bar{Q}\rangle = 0, \\ \mathcal{D}n_g - \frac{g}{N_c^2-1}\,\mathrm{Tr}\langle\mathbf{E}\cdot\nabla_p\delta G\rangle = 0. \end{aligned} \tag{12}$$

Since the regular part of distribution function is assumed to be color neutral, see Eq. (5), the terms of the form $\mathrm{Tr}[\langle\mathbf{E}\cdot\nabla_p n\rangle]$ vanish because the field $\mathbf{E}$ is traceless. The trace over color indices also cancels the terms originating from

covariant derivatives like $\mathrm{Tr}\langle[A^\mu, \delta Q]\rangle$. We finally note that the regular distribution function n is gauge independent and so is $\mathrm{Tr}\langle \mathbf{E}\cdot\nabla_p \delta Q\rangle$.

IV. SOLUTION OF THE LINEARIZED EQUATIONS

Because of the condition (7), the space-time dependence of the regular distribution functions is neglected in the linearized transport Eqs. (9) and the equations become easily solvable. We solve Eq. (9) with the initial conditions

$$\begin{aligned}\delta Q(t=0,\mathbf{r},\mathbf{p}) &= \delta Q_0(\mathbf{r},\mathbf{p}),\\ \delta\bar{Q}(t=0,\mathbf{r},\mathbf{p}) &= \delta\bar{Q}_0(\mathbf{r},\mathbf{p}),\\ \delta G(t=0,\mathbf{r},\mathbf{p}) &= \delta G_0(\mathbf{r},\mathbf{p}),\end{aligned} \tag{13}$$

using the one-sided Fourier transformation defined as

$$f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})} f(t,\mathbf{r}). \tag{14}$$

The inverse transformation is

$$f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3} e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})} f(\omega,\mathbf{k}), \tag{15}$$

where the real parameter $\sigma>0$ is chosen in such a way that the integral over ω is taken along a straight line in the complex ω-plane, parallel to the real axis, above all singularities of $f(\omega,\mathbf{k})$. We note that

$$-i\omega f(\omega,\mathbf{k}) = f(t=0,\mathbf{k}) + \int_0^\infty dt\int d^3r e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})} \times \frac{\partial f(t,\mathbf{r})}{\partial t}. \tag{16}$$

The linearized transport Eqs. (9), which are transformed by means of the one-sided Fourier transformation, are solved as

$$\begin{aligned}\delta Q(\omega,\mathbf{k},\mathbf{p}) &= i\frac{g\mathbf{E}\cdot\nabla_p n(\mathbf{p}) + \delta Q_0(\mathbf{k},\mathbf{p})}{\omega-\mathbf{k}\cdot\mathbf{v}},\\ \delta\bar{Q}(\omega,\mathbf{k},\mathbf{p}) &= i\frac{g\mathbf{E}\cdot\nabla_p \bar{n}(\mathbf{p}) - \delta\bar{Q}_0(\mathbf{k},\mathbf{p})}{\omega-\mathbf{k}\cdot\mathbf{v}},\\ \delta G(\omega,\mathbf{k},\mathbf{p}) &= i\frac{g\mathbf{E}\cdot\nabla_p n_g(\mathbf{p}) + \delta G_0(\mathbf{k},\mathbf{p})}{\omega-\mathbf{k}\cdot\mathbf{v}}.\end{aligned} \tag{17}$$

We note that the color electric field $\mathbf{E}(\omega,\mathbf{k})$ retains its full frequency and wave number dependence in these equations. Inverting the one-sided Fourier transformation, one finds the solutions of linearized transport equations as

$$\begin{aligned}\delta Q(t,\mathbf{r},\mathbf{p}) &= g\int_0^t dt'\mathbf{E}(t',\mathbf{r}-\mathbf{v}(t-t'))\cdot\nabla_p n(\mathbf{p}) + \delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p}),\\ \delta\bar{Q}(t,\mathbf{r},\mathbf{p}) &= -g\int_0^t dt'\mathbf{E}(t',\mathbf{r}-\mathbf{v}(t-t'))\cdot\nabla_p \bar{n}(\mathbf{p}) + \delta\bar{Q}_0(\mathbf{r}-\mathbf{v}t,\mathbf{p}),\\ \delta G(t,\mathbf{r},\mathbf{p}) &= g\int_0^t dt'\mathbf{E}(t',\mathbf{r}-\mathbf{v}(t-t'))\cdot\nabla_p n_g(\mathbf{p}) + \delta G_0(\mathbf{r}-\mathbf{v}t,\mathbf{p}),\end{aligned} \tag{18}$$

where we assumed that $\mathbf{E}(\omega,\mathbf{k})$ is an analytic function of ω. With the help of solutions (18), the force terms in the transport Eqs. (12) become

$$\begin{aligned}\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\rangle &= g\int_0^t dt'\nabla_p^i\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}-\mathbf{v}(t-t'))\rangle\nabla_p^j n(\mathbf{p}) + \nabla_p^i\langle E^i(t,\mathbf{r})\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle,\\ \langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta\bar{Q}(t,\mathbf{r},\mathbf{p})\rangle &= -g\int_0^t dt'\nabla_p^i\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}-\mathbf{v}(t-t'))\rangle\nabla_p^j \bar{n}(\mathbf{p}) + \nabla_p^i\langle E^i(t,\mathbf{r})\delta\bar{Q}_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle,\\ \langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta G(t,\mathbf{r},\mathbf{p})\rangle &= g\int_0^t dt'\nabla_p^i\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}-\mathbf{v}(t-t'))\rangle\nabla_p^j n_g(\mathbf{p}) + \nabla_p^i\langle E^i(t,\mathbf{r})\delta G_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle.\end{aligned} \tag{19}$$

We conclude that the transport Eqs. (12) are determined by the correlation functions $\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}')\rangle$, $\langle E^i(t,\mathbf{r})\delta Q_0(\mathbf{r}',\mathbf{p})\rangle$, $\langle E^i(t,\mathbf{r})\delta\bar{Q}_0(\mathbf{r}',\mathbf{p})\rangle$, and $\langle E^i(t,\mathbf{r})\delta G_0(\mathbf{r}',\mathbf{p})\rangle$. To compute these functions, the state of the plasma must be specified. Although we are mainly interested in an anisotropic plasma, we start with the isotropic case. Thereafter, we consider the two-stream system.

V. ISOTROPIC PLASMA

For the case of isotropic plasma, the correlation functions of both longitudinal and transverse fields are well known [43]. Here we limit our considerations to longitudinal chromoelectric fields, whose correlation function is [43]:

$$\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\rangle = \frac{g^2}{2}\delta^{ab}\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\int_{-\infty+i\sigma'}^{\infty+i\sigma'}\frac{d\omega'}{2\pi}\int\frac{d^3k'}{(2\pi)^3}e^{-i(\omega t+\omega' t'-\mathbf{k}\cdot\mathbf{r}-\mathbf{k}'\cdot\mathbf{r}')}\frac{k^i k'^j}{\mathbf{k}^2\mathbf{k}'^2}\frac{(2\pi)^3\delta^{(3)}(\mathbf{k}'+\mathbf{k})}{\varepsilon_L(\omega,\mathbf{k})\varepsilon_L(\omega',\mathbf{k}')} \times\int\frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{(\omega-\mathbf{k}\cdot\mathbf{v})(\omega'-\mathbf{k}'\cdot\mathbf{v})}, \tag{20}$$

where $f(\mathbf{p})\equiv n(\mathbf{p})+\bar{n}(\mathbf{p})+2N_c n_g(\mathbf{p})$ and $\varepsilon_L(\omega,\mathbf{k})$ is the longitudinal chromodielectric function discussed in the

Appendix. Note that we do not assume that $n(\mathbf{p})$, $\bar{n}(\mathbf{p})$, and $n_g(\mathbf{p})$ are given by the thermal equilibrium distributions, only that they are isotropic functions of $\mathbf{p}$. Zeroes of $\varepsilon_L(\omega, \mathbf{k})$ and $\varepsilon_L(\omega', \mathbf{k}')$ as well as those of the denominators $(\omega - \mathbf{k}\cdot\mathbf{v})$ and $(\omega' - \mathbf{k}'\cdot\mathbf{v})$ contribute to the integrals over ω and ω'. However, when the plasma system under consideration is stable with respect to longitudinal modes, all zeroes of ε_L lie in the lower half-plane of complex ω. Consequently, the contributions associated with these zeroes exponentially decay in time, and they vanish in the long-time limit of both t and t'.

We are further interested in the long-time limit of $\langle E^i(t, \mathbf{r})E^i(t', \mathbf{r}')\rangle$. The only nonvanishing contribution corresponds to the poles at $\omega = \mathbf{k}\cdot\mathbf{v}$ and $\omega' = \mathbf{k}'\cdot\mathbf{v}$. This contribution reads

$$\langle E_a^i(t, \mathbf{r})E_b^j(t', \mathbf{r}')\rangle = \frac{g^2}{2}\delta^{ab}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot[\mathbf{v}(t-t')-(\mathbf{r}-\mathbf{r}')]}\frac{k^ik^j}{\mathbf{k}^4}\frac{f(\mathbf{p})}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v}, \mathbf{k})|^2}. \tag{21}$$

The correlation functions like $\langle E_a^i(t, \mathbf{r})\delta Q_0(\mathbf{r}', \mathbf{p}')\rangle$ are not computed in Ref. [43], but they can be readily inferred from the formulas given there. One finds

$$\begin{aligned}\langle E_a^i(t, \mathbf{r})\delta Q_0(\mathbf{r}', \mathbf{p}')\rangle &= -g\tau^a\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r}-\mathbf{k}'\cdot\mathbf{r}')}\frac{k^i}{\mathbf{k}^2}\frac{(2\pi)^3\delta^{(3)}(\mathbf{k}'+\mathbf{k})}{\varepsilon_L(\omega, \mathbf{k})}\frac{n(\mathbf{p}')}{\omega-\mathbf{k}\cdot\mathbf{v}'}\\ &= ig\tau^a\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot(\mathbf{v}'t-\mathbf{r}+\mathbf{r}'))}\frac{k^i}{\mathbf{k}^2}\frac{n(\mathbf{p}')}{\varepsilon_L(\mathbf{k}\cdot\mathbf{v}', \mathbf{k})},\end{aligned} \tag{22}$$

where the last equality holds in the long-time limit which is carried by the contribution corresponding to the pole $\omega = \mathbf{k}\cdot\mathbf{v}'$. Similarly, one finds

$$\begin{aligned}\langle E_a^i(t, \mathbf{r})\delta\bar{Q}_0(\mathbf{r}', \mathbf{p}')\rangle &= g\tau^a\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r}-\mathbf{k}'\cdot\mathbf{r}')}\frac{k^i}{\mathbf{k}^2}\frac{(2\pi)^3\delta^{(3)}(\mathbf{k}'+\mathbf{k})}{\varepsilon_L(\omega, \mathbf{k})}\frac{\bar{n}(\mathbf{p}')}{\omega-\mathbf{k}\cdot\mathbf{v}'}\\ &= -ig\tau^a\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot(\mathbf{v}'t-\mathbf{r}+\mathbf{r}'))}\frac{k^i}{\mathbf{k}^2}\frac{\bar{n}(\mathbf{p}')}{\varepsilon_L(\mathbf{k}\cdot\mathbf{v}', \mathbf{k})},\end{aligned} \tag{23}$$

$$\begin{aligned}\langle E_a^i(t, \mathbf{r})\delta G_0(\mathbf{r}', \mathbf{p}')\rangle &= -gT^a\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r}-\mathbf{k}'\cdot\mathbf{r}')}\frac{k^i}{\mathbf{k}^2}\frac{(2\pi)^3\delta^{(3)}(\mathbf{k}'+\mathbf{k})}{\varepsilon_L(\omega, \mathbf{k})}\frac{n_g(\mathbf{p}')}{\omega-\mathbf{k}\cdot\mathbf{v}'}\\ &= igT^a\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot(\mathbf{v}'t-\mathbf{r}+\mathbf{r}'))}\frac{k^i}{\mathbf{k}^2}\frac{n_g(\mathbf{p}')}{\varepsilon_L(\mathbf{k}\cdot\mathbf{v}', \mathbf{k})}.\end{aligned} \tag{24}$$

Substituting the correlation functions (21) and (22) into (19), one finds

$$\begin{aligned}\mathrm{Tr}\langle\mathbf{E}(t, \mathbf{r})\cdot\nabla_p\delta Q(t, \mathbf{r}, \mathbf{p})\rangle &= \frac{g^3}{4}(N_c^2-1)\int_0^t dt'\nabla_p^i\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')(t-t')}\frac{k^ik^j}{\mathbf{k}^4}\frac{f(\mathbf{p}')}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v}', \mathbf{k})|^2}\nabla_p^j n(\mathbf{p})\\ &\quad + i\frac{g}{2}(N_c^2-1)\nabla_p^i\int\frac{d^3k}{(2\pi)^3}\frac{k^i}{\mathbf{k}^2}\frac{n(\mathbf{p})}{\varepsilon_L(\mathbf{k}\cdot\mathbf{v}, \mathbf{k})},\end{aligned} \tag{25}$$

and analogous expressions for $\mathrm{Tr}\langle\mathbf{E}(t, \mathbf{r})\cdot\nabla_p\delta\bar{Q}(t, \mathbf{r}, \mathbf{p})\rangle$ and $\mathrm{Tr}\langle\mathbf{E}(t, \mathbf{r})\cdot\nabla_p\delta G(t, \mathbf{r}, \mathbf{p})\rangle$. As shown in [43], $\mathrm{Tr}\langle E^i(t, \mathbf{r})E^j(t', \mathbf{r}')\rangle$ is gauge independent within the linear response approach. The same arguments used to show this apply to $\mathrm{Tr}\langle E^i(t, \mathbf{r})\delta Q_0(\mathbf{r}', \mathbf{p}')\rangle$. Thus, we conclude that the collision term of the transport Eq. (12), $\mathrm{Tr}\langle\mathbf{E}(t, \mathbf{r})\cdot\nabla_p\delta Q(t, \mathbf{r}, \mathbf{p})\rangle$, is gauge independent.

Let us now discuss the first term on the right-hand side of (25). Computing the integral over t' we get

$$\begin{aligned}\mathrm{Tr}\langle\mathbf{E}(t, \mathbf{r})\cdot\nabla_p\delta Q(t, \mathbf{r}, \mathbf{p})\rangle_{(1)} &= \frac{g^3}{4}(N_c^2-1)\nabla_p^i\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4}\frac{f(\mathbf{p}')}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v}', \mathbf{k})|^2}\\ &\quad\times\left(-i\frac{\cos(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')t)-1}{\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')}+\frac{\sin(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')t)}{\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')}\right)\nabla_p^j n(\mathbf{p}).\end{aligned} \tag{26}$$

The first term does not contribute to the integral because it is an odd function of $\mathbf{k}$. Since in the limit $t\to\infty$ we have

$$\lim_{t\to\infty}\frac{\sin(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')t)}{\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')} = \pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')), \tag{27}$$

one finally finds

$$\mathrm{Tr}\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})\rangle_{(1)} = \frac{g^3}{4}\pi(N_c^2-1)\nabla_p^i \int \frac{d^3p'}{(2\pi)^3}\int \frac{d^3k}{(2\pi)^3}\frac{k^i k^j}{\mathbf{k}^4}\frac{f(\mathbf{p}')}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v}',\mathbf{k})|^2}\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))\nabla_p^j n(\mathbf{p}). \quad (28)$$

Analogously, one computes $\mathrm{Tr}\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta\bar{Q}(t,\mathbf{r},\mathbf{p})\rangle_{(1)}$ and $\mathrm{Tr}\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta G(t,\mathbf{r},\mathbf{p})\rangle_{(1)}$.

The second term on the right-hand side of (25) can be written as

$$\mathrm{Tr}\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\rangle_{(2)} = \frac{g}{2}(N_c^2-1)\nabla_p^i\int\frac{d^3k}{(2\pi)^3}\times\frac{k^i}{\mathbf{k}^2}\frac{\Im\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}n(\mathbf{p}), \quad (29)$$

because the term with $\Re\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})$ is an odd function of $\mathbf{k}$ (see the Appendix). Alternatively, one can argue that the right-hand side of (29) has to be real as the left-hand side is real. In the same way one finds $\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta\bar{Q}(t,\mathbf{r},\mathbf{p})\rangle_{(2)}$ and $\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta G(t,\mathbf{r},\mathbf{p})\rangle_{(2)}$.

With the formulas derived above, the transport Eqs. (12) can now be written either in the Balescu-Lenard form or the Fokker-Planck form.

A. Balescu-Lenard equations

Using the formula (A2) to express $\Im\varepsilon_L$ through the distribution function, the transport Eqs. (12) get the Balescu-Lenard form [42]

$$\begin{aligned}\mathcal{D}n(t,\mathbf{r},\mathbf{p}) &= \nabla_p\cdot\mathbf{S}[n,\bar{n},n_g],\\ \mathcal{D}\bar{n}(t,\mathbf{r},\mathbf{p}) &= \nabla_p\cdot\bar{\mathbf{S}}[n,\bar{n},n_g],\\ \mathcal{D}n_g(t,\mathbf{r},\mathbf{p}) &= \nabla_p\cdot\mathbf{S}_g[n,\bar{n},n_g],\end{aligned} \quad (30)$$

where, as previously, $\mathcal{D}$ is the material derivative, and

$$\begin{aligned}S^i[n,\bar{n},n_g] &\equiv \int\frac{d^3p'}{(2\pi)^3}B^{ij}(\mathbf{v},\mathbf{v}')[\nabla_p^j n(\mathbf{p})f(\mathbf{p}') - n(\mathbf{p})\nabla_{p'}^j f(\mathbf{p}')],\\ \bar{S}^i[n,\bar{n},n_g] &\equiv \int\frac{d^3p'}{(2\pi)^3}B^{ij}(\mathbf{v},\mathbf{v}')[\nabla_p^j \bar{n}(\mathbf{p})f(\mathbf{p}') - \bar{n}(\mathbf{p})\nabla_{p'}^j f(\mathbf{p}')],\\ S_g^i[n,\bar{n},n_g] &\equiv \int\frac{d^3p'}{(2\pi)^3}B_g^{ij}(\mathbf{v},\mathbf{v}')[\nabla_p^j n_g(\mathbf{p})f(\mathbf{p}') - n_g(\mathbf{p})\nabla_{p'}^j f(\mathbf{p}')],\end{aligned} \quad (31)$$

with

$$B^{ij}(\mathbf{v},\mathbf{v}') \equiv \frac{g^4}{8}\frac{N_c^2-1}{N_c}\int\frac{d^3k}{(2\pi)^3}\frac{k^i k^j}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}, \quad (32)$$

and

$$B_g^{ij}(\mathbf{v},\mathbf{v}') = \frac{2N_c^2}{N_c^2-1}B^{ij}(\mathbf{v},\mathbf{v}'). \quad (33)$$

Since the interaction processes that are taken into account conserve the numbers of particles of every species $(q,\bar{q},g)$, the transport equations in the Balescu-Lenard form (30) can be seen as continuity equations in momentum space with $\mathbf{S}$, $\bar{\mathbf{S}}$, $\mathbf{S}_g$ playing a role of currents. One observes that for classical equilibrium functions

$$f^{\mathrm{eq}}(\mathbf{p}),\ n^{\mathrm{eq}}(\mathbf{p}),\ \bar{n}^{\mathrm{eq}}(\mathbf{p}),\ n_g^{\mathrm{eq}}(\mathbf{p}) \sim e^{-E_p/T}, \quad (34)$$

the collision terms (31) vanish, as expected, because

$$(v^i - v'^i)B^{ij}(\mathbf{v},\mathbf{v}') = 0. \quad (35)$$

If $\varepsilon_L(\omega,\mathbf{k})$ is replaced by unity, i.e. if one ignores the chromodielectric properties of the plasma, the tensor $B^{ij}(\mathbf{v},\mathbf{v}')$ is easily found to be

$$B^{ij}(\mathbf{v},\mathbf{v}') = \frac{g^4}{32\pi}\frac{N_c^2-1}{N_c}\frac{L}{|\mathbf{v}-\mathbf{v}'|}\times\left(\delta^{ij}-\frac{(v^i-v'^i)(v^j-v'^j)}{(\mathbf{v}-\mathbf{v}')^2}\right), \quad (36)$$

with

$$L \equiv \int dk/k = \ln(k_{\max}/k_{\min}). \quad (37)$$

The parameter L is called the Coulomb logarithm and the collision term with the tensor $B^{ij}(\mathbf{v},\mathbf{v}')$ of the form (36) is called the Landau collision term [42]. Estimating $k_{\max}$ as the system temperature T and $k_{\min}$ as the Debye mass $m_D \sim gT$, one finds $L \sim \ln(1/g)$.

It may appear strange that we start with the collisionless transport Eqs. (1) to derive the collision terms. This procedure, which is commonly used in the plasma literature, is well justified, however, see e.g. Ref. [42]. The collision terms, which are derived above, represent the effect of fluctuating soft fields on the hard quasiparticles. It is important to note that the collision terms are dominated, as they should be, by the soft wave vectors. Consequently, the collisions of quasiparticles involving the exchange of hard momenta, which are neglected in Eqs. (1), do not need to be taken into account at lowest order.

B. Fokker-Planck equations

Sometimes it is more convenient to use the transport equations in the Fokker-Planck form. Following Ref. [42], one rewrites Eqs. (30) as

$$(\mathcal{D} - \nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y^i(\mathbf{v}))n(t,\mathbf{r},\mathbf{p}) = 0,$$
$$(\mathcal{D} - \nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y^i(\mathbf{v}))\bar{n}(t,\mathbf{r},\mathbf{p}) = 0, \quad (38)$$
$$(\mathcal{D} - \nabla_p^i X_g^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y_g^i(\mathbf{v}))n_g(t,\mathbf{r},\mathbf{p}) = 0,$$

where

$$X^{ij}(\mathbf{v}) \equiv \frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4}\frac{f(\mathbf{p}')}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v}',\mathbf{k})|^2} \times 2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')) = \int\frac{d^3p'}{(2\pi)^3}f(\mathbf{p}')B^{ij}(\mathbf{v},\mathbf{v}'), \quad (39)$$

$$Y^i(\mathbf{v}) \equiv \frac{g^2}{2}(N_c^2-1)\int\frac{d^3k}{(2\pi)^3}\frac{k^i}{\mathbf{k}^2}\frac{\Im\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2} = -\frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^i}{\mathbf{k}^4}\frac{\mathbf{k}\cdot\nabla_{p'}f(\mathbf{p}')}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2} \times 2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')) = -\int\frac{d^3p'}{(2\pi)^3}\nabla_{p'}^j f(\mathbf{p}')B^{ij}(\mathbf{v},\mathbf{v}'), \quad (40)$$

and

$$X_g^{ij}(\mathbf{v}) = \frac{2N_c^2}{N_c^2-1}X^{ij}(\mathbf{v}), \quad (41)$$

$$Y_g^i(\mathbf{v}) = \frac{2N_c^2}{N_c^2-1}Y^i(\mathbf{v}). \quad (42)$$

The Eqs. (38) appear to be linear but actually they are not: the coefficients $X^{ij}(\mathbf{v})$, $Y^i(\mathbf{v})$, $X_g^{ij}(\mathbf{v})$ and $Y_g^i(\mathbf{v})$ depend on the distribution functions. When the distribution functions are of the classical equilibrium form ($f^{\rm eq}(\mathbf{p})$, $n^{\rm eq}(\mathbf{p})$, $\bar{n}^{\rm eq}(\mathbf{p})$, $n_g^{\rm eq}(\mathbf{p}) \sim e^{-E_p/T}$), we have the relation

$$Y^i(\mathbf{v}) = \frac{v^i}{T}X^{ij}(\mathbf{v}). \quad (43)$$

Consequently, the Fokker-Planck collision terms vanish in equilibrium, as do the Balescu-Lenard collision terms.

Since the system is assumed to be isotropic, $X^{ij}(\mathbf{v})$ and $Y^i(\mathbf{v})$ can be expressed as follows:

$$X^{ij}(\mathbf{v}) = a\delta^{ij} + bv^iv^j, \quad (44)$$

$$Y^i(\mathbf{v}) = cv^i, \quad (45)$$

with

$$a = \frac{1}{2}\int\frac{d^3p'}{(2\pi)^3}f(\mathbf{p}')[\delta^{ji} - v^jv^i]B^{ij}(\mathbf{v},\mathbf{v}') \quad (46)$$

$$b = \frac{1}{2}\int\frac{d^3p'}{(2\pi)^3}f(\mathbf{p}')[3v^jv^i - \delta^{ji}]B^{ij}(\mathbf{v},\mathbf{v}') \quad (47)$$

$$c = -\int\frac{d^3p'}{(2\pi)^3}v^i\nabla_{p'}^j f(\mathbf{p}')B^{ij}(\mathbf{v},\mathbf{v}'). \quad (48)$$

Because of the system's isotropy, the coefficients a, b, c can depend only on $\mathbf{v}^2$. In the ultrarelativistic limit, which is adopted here, $\mathbf{v}^2 = 1$, and consequently a, b, c are independent of $\mathbf{v}$. We also note that in equilibrium the coefficients are related as

$$Tc = a + b, \quad (49)$$

which follows from Eq. (43).

When $\varepsilon_L(\omega,\mathbf{k})$ is replaced, as previously, by unity one finds that $b = 0$ and

$$a \equiv \frac{g^4}{96\pi^3}(N_c^2-1)L\int_0^\infty dpp^2f(\mathbf{p}), \quad (50)$$

$$c \equiv -\frac{g^4}{96\pi^3}(N_c^2-1)L\int_0^\infty dpp^2\frac{df(\mathbf{p})}{dp}. \quad (51)$$

Using the relations (A5), the coefficient c can be expressed in terms of the Debye mass as

$$c = \frac{g^2}{24\pi^2}(N_c^2-1)Lm_D^2. \quad (52)$$

Furthermore, in equilibrium, $a = cT$.

We note that in spite of our neglect of transverse chromodynamic fields, the collision terms for the isotropic plasma derived here are very similar to those derived in [19,21,25,26,29–33].

C. Equilibration of an isotropic plasma

As an application of the Fokker-Planck Eqs. (38) we discuss the problem of plasma equilibration. In this section we limit our considerations to quarks, as the analysis for antiquarks and gluons is very similar. We consider the system which is homogenous and mostly equilibrated but a small fraction ($\lambda \ll 1$) of the particles, denoted by $\delta n(t,\mathbf{p})$, is out of equilibrium. One asks on what time scale the system reaches the equilibrium. The distribution function is assumed to be of the form

$$n(t,\mathbf{p}) = (1-\lambda)n^{\rm eq}(\mathbf{p}) + \lambda\delta n(t,\mathbf{p}). \quad (53)$$

In the course of equilibration $n(t,\mathbf{p})$ tends to $n^{\rm eq}(\mathbf{p})$. Since the particle number is conserved within the transport theory approach developed here, $\delta n(t,\mathbf{p})$ is not reduced to zero in the equilibration process but it tends to $n^{\rm eq}(\mathbf{p})$.

We define the rate of equilibration Γ through the relation

$$\frac{\partial n}{\partial t} = \Gamma\delta n. \quad (54)$$

We note that Γ is either positive, when δn grows going to $n^{\rm eq}$, and it is negative, when δn decreases going to $n^{\rm eq}$. Using the Fokker-Planck Eq. (38), the definition (54) gives

$$\Gamma = \frac{1}{\delta n}(\nabla_p^i X^{ij} \nabla_p^j + \nabla_p^i Y^i)\delta n. \tag{55}$$

Since the fraction of particles with nonequilibrium distribution is assumed to be small, the coefficients a, b, c from the formulas (44) and (45) are given by the equilibrium function $n^{\mathrm{eq}} \sim e^{-E_p/T}$. Using the approximate expression of a (50) with $b = 0$ and $c = a/T$, Eq. (55) is rewritten as

$$\Gamma = \frac{a}{\delta n}\left(\nabla_p^2 + \frac{1}{T}\mathbf{v}\cdot\nabla_p + \frac{2}{TE_p}\right)\delta n. \tag{56}$$

The equilibration rate obviously depends of the form of δn. Here we consider the case where the small fraction of partons has an equilibrium distribution of temperature T_0 which differs from the temperature T of the bulk of the partons. Thus, $\delta n \sim e^{-E_p/T_0}$. Then, the equilibration rate (56) equals

$$\Gamma = a\frac{T - T_0}{T_0^2 T E_p}(E_p - 2T_0). \tag{57}$$

For $T = T_0$, the whole system is in equilibrium and, as expected, $\Gamma = 0$. When $T > T_0$, the distribution e^{-E_p/T_0} is steeper than $e^{-E_p/T}$. Equation (57) tells us that δn decreases for $E_p < 2T_0$ and grows for $E_p > 2T_0$ during the equilibration process. When $T < T_0$, we have the opposite situation. In both cases, the slope of the distribution function δn tends to the slope of n^{eq}. With the coefficient a given by (50), the formula (57) quantitatively predicts how fast the equilibrium is approached.

VI. TWO-STREAM SYSTEM

The two-stream configuration provides an interesting case of an unstable plasma. The correlation function of longitudinal chromoelectric fields, which is needed to derive the transport equations, was computed in [43]. Unfortunately the correlation function for transverse fields is not known. This limits our considerations to longitudinal fields.

The distribution function of the two-stream system is chosen as

$$f(\mathbf{p}) = (2\pi)^3 n[\delta^{(3)}(\mathbf{p} - \mathbf{q}) + \delta^{(3)}(\mathbf{p} + \mathbf{q})], \tag{58}$$

where n is the effective parton density in a single stream. The distribution function (58) should be treated as an idealization of the two-peak distribution where the particles have momenta close to $\mathbf{q}$ or $-\mathbf{q}$.

To compute $\varepsilon_L(\omega, \mathbf{k})$ we first perform an integration by parts in (A1) and then substitute the distribution function (58) into the resulting formula. We obtain

$$\begin{aligned}\varepsilon_L(\omega, \mathbf{k}) &= 1 - \mu^2\frac{\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2}{\mathbf{k}^2}\left[\frac{1}{(\omega - \mathbf{k}\cdot\mathbf{u})^2} + \frac{1}{(\omega + \mathbf{k}\cdot\mathbf{u})^2}\right] \\ &= \frac{(\omega - \omega_+(\mathbf{k}))(\omega + \omega_+(\mathbf{k}))(\omega - \omega_-(\mathbf{k}))(\omega + \omega_-(\mathbf{k}))}{(\omega^2 - (\mathbf{k}\cdot\mathbf{u})^2)^2},\end{aligned} \tag{59}$$

where $\mathbf{u} \equiv \mathbf{q}/E_{\mathbf{q}}$ is the stream velocity, $\mu^2 \equiv g^2 n/2E_{\mathbf{q}}$ and $\pm\omega_\pm(\mathbf{k})$ are the four roots of the dispersion equation $\varepsilon_L(\omega, \mathbf{k}) = 0$ which are explicitly given by

$$\omega_\pm^2(\mathbf{k}) = \frac{1}{\mathbf{k}^2}[\mathbf{k}^2(\mathbf{k}\cdot\mathbf{u})^2 + \mu^2(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2) \pm \mu\sqrt{(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2)(4\mathbf{k}^2(\mathbf{k}\cdot\mathbf{u})^2 + \mu^2(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2))}]. \tag{60}$$

One can show that $0 < \omega_+(\mathbf{k}) \in \mathbb{R}$ for any $\mathbf{k}$, while $\omega_-(\mathbf{k})$ is imaginary for $\mathbf{k}\cdot\mathbf{u} \neq 0$ and $\mathbf{k}^2(\mathbf{k}\cdot\mathbf{u})^2 < 2\mu^2(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2)$. ω_- represents the well-known two-stream electrostatic instability generated by a mechanism analogous to the Landau damping. For $\mathbf{k}^2(\mathbf{k}\cdot\mathbf{u})^2 \geq 2\mu^2(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2)$, the ω_- mode is stable: $0 < \omega_-(\mathbf{k}) \in \mathbb{R}$.

The terms like $\langle \mathbf{E}(t, \mathbf{r})\cdot\nabla_p \delta Q(t, \mathbf{r}, \mathbf{p})\rangle$, which enter the transport Eqs. (12), are given by Eqs. (19). As for the isotropic plasma one needs to specify the correlation functions $\langle E_a^i(\omega, \mathbf{k})E_b^i(\omega', \mathbf{k}')\rangle$, $\langle \mathbf{E}(t, \mathbf{r})\delta Q_0(\mathbf{r}', \mathbf{p}')\rangle$, etc. The correlation function of the longitudinal fields $\langle E_a^i(\omega, \mathbf{k})E_b^j(\omega', \mathbf{k}')\rangle$ was found in [43]:

$$\begin{aligned}\langle E_a^i(\omega, \mathbf{k})E_b^j(\omega', \mathbf{k}')\rangle = &-g^2\delta^{ab}n\frac{(2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}')}{\mathbf{k}^2}\frac{k^i k^j}{\mathbf{k}^2}[\omega\omega' + (\mathbf{k}\cdot\mathbf{u})(\mathbf{k}'\cdot\mathbf{u})] \\ &\times \frac{\omega^2 - (\mathbf{k}\cdot\mathbf{u})^2}{(\omega - \omega_-(\mathbf{k}))(\omega + \omega_-(\mathbf{k}))(\omega - \omega_+(\mathbf{k}))(\omega + \omega_+(\mathbf{k}))} \\ &\times \frac{\omega'^2 - (\mathbf{k}'\cdot\mathbf{u})^2}{(\omega' - \omega_-(\mathbf{k}'))(\omega' + \omega_-(\mathbf{k}'))(\omega' - \omega_+(\mathbf{k}'))(\omega' + \omega_+(\mathbf{k}'))}.\end{aligned} \tag{61}$$

We are particularly interested in the contributions of the unstable modes to the correlation function. For this reason we consider the domain of wave vectors obeying $\mathbf{k}\cdot\mathbf{u} \neq 0$ and $\mathbf{k}^2(\mathbf{k}\cdot\mathbf{u})^2 < 2\mu^2(\mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{u})^2)$ when $\omega_-(\mathbf{k})$ is imaginary

and the mode is unstable. We write $\omega_-(\mathbf{k}) = i\gamma_{\mathbf{k}}$ with $0 < \gamma_{\mathbf{k}} \in \mathbb{R}$. The contribution coming from the modes $\pm\omega_-(\mathbf{k})$ then equals [43]

$$\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\rangle_{\text{unstable}} = \frac{g^2}{2}\delta^{ab}n\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^4}\frac{k^ik^j}{(\omega_+^2-\omega_-^2)^2}\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^2}{\gamma_{\mathbf{k}}^2} \times[(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)\cosh(\gamma_{\mathbf{k}}(t+t'))+(\gamma_{\mathbf{k}}^2-(\mathbf{k}\cdot\mathbf{u})^2)\cosh(\gamma_{\mathbf{k}}(t-t'))]. \tag{62}$$

As Eq. (62) shows, the contribution of the unstable modes to the field-field correlation function is space translation invariant—it depends only on the difference $(\mathbf{r}-\mathbf{r}')$. If the initial plasma is on average homogeneous, it remains so over the course of its evolution. The time dependence of the correlation function (62), however, is very different from the spatial dependence. The electric field grows exponentially and so does the correlation function, both in $(t+t')$ and $(t-t')$. The fluctuation spectrum also evolves in time as the growth rate of the unstable modes is wave-vector dependent. After a sufficiently long time the fluctuation spectrum will be dominated by the fastest growing modes.

The correlation function $\langle\mathbf{E}(t,\mathbf{r})\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle$ is, as previously, given by Eqs. (29). Since the dielectric function (59) is real, the correlation functions $\langle\mathbf{E}(t,\mathbf{r})\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle$, $\langle\mathbf{E}(t,\mathbf{r})\delta\bar{Q}_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle$ and $\langle\mathbf{E}(t,\mathbf{r})\delta G_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})\rangle$ all vanish. Therefore,

$$\begin{aligned}\mathrm{Tr}\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\rangle &= g\int_0^t dt'\nabla_p^i\langle E^i(t,\mathbf{r})E^j(t',\mathbf{r}-\mathbf{v}(t-t'))\rangle\nabla_p^j n(\mathbf{p})\\ &= \frac{g^3}{4}(N_c^2-1)n\int_0^t dt'\nabla_p^i\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\mathbf{v}(t-t')}}{\mathbf{k}^4}\frac{k^ik^j}{(\omega_+^2-\omega_-^2)^2}\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^2}{\gamma_{\mathbf{k}}^2}[(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)\\ &\quad\times\cosh(\gamma_{\mathbf{k}}(t+t'))+(\gamma_{\mathbf{k}}^2-(\mathbf{k}\cdot\mathbf{u})^2)\cosh(\gamma_{\mathbf{k}}(t-t'))]\nabla_p^j n(\mathbf{p}).\end{aligned} \tag{63}$$

Performing the integration over t' and keeping only the real part, one finds

$$\begin{aligned}\mathrm{Tr}\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\rangle &= \frac{g^3}{4}(N_c^2-1)n\nabla_p^i\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4(\omega_+^2-\omega_-^2)^2}\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^2}{\gamma_{\mathbf{k}}^2(\gamma_{\mathbf{k}}^2+(\mathbf{k}\mathbf{v})^2)}[\gamma_{\mathbf{k}}^3\sinh(2\gamma_{\mathbf{k}}t)+(\mathbf{k}\cdot\mathbf{u})^2\\ &\quad\times(\gamma_{\mathbf{k}}\sinh(2\gamma_{\mathbf{k}}t)+(\mathbf{k}\cdot\mathbf{v})\sin((\mathbf{k}\cdot\mathbf{v})t)\cosh(\gamma_{\mathbf{k}}t)-\gamma_{\mathbf{k}}\cos((\mathbf{k}\cdot\mathbf{v})t)\sinh(\gamma_{\mathbf{k}}t))]\nabla_p^j n(\mathbf{p}).\end{aligned} \tag{64}$$

Neglecting the oscillating terms, we finally get

$$\mathrm{Tr}\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\rangle = \frac{g^3}{2}(N_c^2-1)n\nabla_p^i\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4(\omega_+^2-\omega_-^2)^2}\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^3}{\gamma_{\mathbf{k}}(\gamma_{\mathbf{k}}^2+(\mathbf{k}\mathbf{v})^2)}\sinh(2\gamma_{\mathbf{k}}t)\nabla_p^j n(\mathbf{p}). \tag{65}$$

In an analogous way one can obtain explicit expressions for $\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta\bar{Q}(t,\mathbf{r},\mathbf{p})\rangle$ and $\langle\mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta G(t,\mathbf{r},\mathbf{p})\rangle$. We do not present these here, because they do not provide any new insight.

Since we explicitly integrated over the distribution function (58) in deriving these results, we only give the transport Eqs. (12) for the two-stream system in the Fokker-Planck form:

$$\begin{aligned}(\mathcal{D}-\nabla_p^iX^{ij}(t,\mathbf{v})\nabla_p^j)\begin{Bmatrix}n(t,\mathbf{r},\mathbf{p})\\ \bar{n}(t,\mathbf{r},\mathbf{p})\end{Bmatrix} &= 0,\\ (\mathcal{D}-\nabla_p^iX_g^{ij}(t,\mathbf{v})\nabla_p^j)n_g(t,\mathbf{r},\mathbf{p}) &= 0,\end{aligned} \tag{66}$$

where

$$X^{ij}(t,\mathbf{v}) \equiv \frac{g^4}{4}\frac{N_c^2-1}{N_c}n\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4(\omega_+^2+\gamma_{\mathbf{k}}^2)^2} \times\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^3}{\gamma_{\mathbf{k}}(\gamma_{\mathbf{k}}^2+(\mathbf{k}\mathbf{v})^2)}\sinh(2\gamma_{\mathbf{k}}t), \tag{67}$$

and $X_g^{ij}(t,\mathbf{v}) \equiv 2N_c^2X^{ij}(t,\mathbf{p})/(N_c^2-1)$.

To get an idea how the two-stream system evolves according to the Fokker-Planck Eqs. (66), we take into account in the integral (67) only those wave vectors which are parallel to the stream velocity $\mathbf{u}$, with the latter being chosen along the axis x. The only nonvanishing component of $X^{ij}(t,\mathbf{v})$ is then $X^{xx}(t,\mathbf{v})$. Neglecting the dependence of $X^{xx}(t,\mathbf{v})$ on $\mathbf{p}$ and assuming that the system is homogenous, the Fokker-Planck Eq. (66) for quarks becomes a one-dimensional diffusion equation

$$\frac{\partial n(t,\mathbf{p})}{\partial t} = D(t)\frac{\partial^2 n(t,\mathbf{p})}{\partial p_x^2}, \tag{68}$$

with the diffusion coefficient $D(t) \equiv X^{xx}(t)$ depending on time approximately as

$$D(t) = de^{2\gamma t}, \tag{69}$$

where d and γ are constants.

If the distribution function is initially of the form

$$n(t=0,\mathbf{p}) = 2\pi\tilde{n}\delta(p_x - q), \tag{70}$$

where $\tilde{n}$ is independent of p_x, the solution of the diffusion Eq. (68) is found as

$$n(t,\mathbf{p}) = \tilde{n}\sqrt{\frac{2\pi\gamma}{d(e^{2\gamma t}-1)}}\exp\left[-\frac{\gamma(p_x-q)^2}{2d(e^{2\gamma t}-1)}\right]. \tag{71}$$

The distribution function (71) is normalized in such a way that

$$\int\frac{dp_x}{2\pi}n(t,\mathbf{p}) = \tilde{n}.$$

According to the solution (71), the electric field growing due to the electrostatic instability rapidly washes out the peaklike structures of the two-stream distribution function (58). It should be understood, however, that the solution (71) is valid only for time intervals which are sufficiently short that the distribution function used to compute the coefficient $X^{ij}(t,\mathbf{v})$ is not much different from the function (58). Nevertheless, the solution (71) shows how the equilibration process commences.

VII. SUMMARY AND OUTLOOK

We have developed here the quasilinear transport theory of a weakly coupled quark-gluon plasma. Our main motivation was to study the equilibration of plasmas that are initially unstable. The field fluctuation spectrum, which is found within the linear response approach, determines the evolution of the regular distribution functions. More specifically, the fluctuations of chromodynamic fields provide collision terms to the transport equations of the regular distribution functions. We have limited our considerations to longitudinal chromoelectric fields, as then the field correlation functions are known for both the isotropic and two-stream systems. The collision terms were found in either the Balescu-Lenard or Fokker-Planck form. In the case of an isotropic plasma we showed how the system equilibrates when a small fraction of particles has a different temperature than the bulk.

The case of the two-stream system is more interesting. The Fokker-Planck equation could be approximately written as an equation of diffusion in momentum space. The diffusion coefficient, which is given by the chromoelectric fields for the two-stream instability, exponentially grows in time. We found the exact solution of the diffusion equation, which showed that the peaklike structures in the parton momentum distribution dissolve rapidly.

In nonrelativistic plasmas it is often a well-justified approximation to keep only longitudinal electric fields and to neglect magnetic and transverse electric fields [40,41]. In the case of ultrarelativistic plasmas, this is no longer true. If initially the fields are purely longitudinal, the transverse fields are automatically generated, and they are dynamically important. Therefore, the ultrarelativistic plasma considered here, where the transverse fields are neglected, should be rather treated as a toy model which we have studied mostly for the sake of analytical tractability. With this simplified example we have been able to elucidate some general features of the problem. Physically better motivated situations will require substantial numerical work, which is less conducive to general insights.

The considerations presented here clearly demonstrate the usefulness of the quasilinear transport theory for the study of equilibration processes of quark-gluon plasmas. As mentioned in the Introduction, numerical studies indicate that the unstable chromomagnetic plasma modes play an important role at the early stage of the quark-gluon plasma produced in relativistic heavy-ion collisions. Therefore, it would be of considerable interest to compute the correlation functions of transverse fields in arbitrary anisotropic plasmas in order to derive the relevant transport equations. As explained in [43], there is no conceptual difficulty in such a computation, but one has to invert the matrix $\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$. This is easily done for isotropic plasmas but for anisotropic plasmas one obtains a rather complex expression which is very cumbersome for further analytic calculations [46]. Except for some special cases, numerical methods seem to be unavoidable. Such computational studies are beyond the scope of the present work but progress in this direction will be hopefully reported soon.

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APPENDIX

We discuss here the longitudinal chromodielectric permeability $\varepsilon_L(\omega,\mathbf{k})$ which is known to be

$$\varepsilon_L(\omega,\mathbf{k}) = 1 + \frac{g^2}{2\mathbf{k}^2}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{k}\cdot\nabla_p f(\mathbf{p})}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+}. \tag{A1}$$

Applying the identity

$$\frac{1}{x \pm i0^+} = \mathcal{P}\frac{1}{x} \mp i\pi\delta(x)$$

to Eq. (A1), one immediately finds $\Im\varepsilon_L(\omega,\mathbf{k})$

$$\Im\varepsilon_L(\omega,\mathbf{k}) = -\frac{g^2}{4\mathbf{k}^2}\int\frac{d^3p}{(2\pi)^3}2\pi\delta(\omega-\mathbf{k}\cdot\mathbf{v})\mathbf{k}\cdot\nabla_p f(\mathbf{p}). \tag{A2}$$

If the plasma is isotropic $\nabla_p f(\mathbf{p})$ can be expressed as

$$\nabla_p f(\mathbf{p}) = \frac{df(\mathbf{p})}{dE_p}\mathbf{v}. \tag{A3}$$

And if the partons are additionally masslees, the integral in (A1) factorizes into the angular integral and the integral over $p \equiv |\mathbf{p}|$. Then, one finds the real and imaginary parts of the longitudinal chromodielectric permeability $\varepsilon_L(\omega,\mathbf{k})$ as

$$\begin{aligned}\Re\varepsilon_L(\omega,\mathbf{k}) &= 1+\frac{m_D^2}{\mathbf{k}^2}\left[1-\frac{\omega}{2|\mathbf{k}|}\ln\left|\frac{\omega+|\mathbf{k}|}{\omega-|\mathbf{k}|}\right|\right],\\ \Im\varepsilon_L(\omega,\mathbf{k}) &= \frac{\pi}{2}\Theta(\mathbf{k}^2-\omega^2)\frac{m_D^2\omega}{|\mathbf{k}|^3},\end{aligned} \tag{A4}$$

where the Debye mass m_D is

$$m_D^2 \equiv -\frac{g^2}{4\pi}\int_0^\infty dp p^2\frac{df(\mathbf{p})}{dp}. \tag{A5}$$

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