Z. Phys. C – Particles and Fields 27, 131–133 (1985)

Zeitschrift für Physik C Particles and Fields

On Multiplicity Distributions and a Pressure Ensemble

Stanisław Mrówczyński

JINR, Laboratory of High Energies, SU-141-980 Dubna, USSR

Received 9 April 1984; in revised form 16 July 1984

Abstract. Using the so called pressure ensemble, the multiplicity distribution in multi-fireball model is found. The distribution satisfies KNO scaling and fulfils other features of experimental multiplicity data of hadron–hadron and nucleus–nucleus interactions.

High multiplicities of particles produced in high energy collisions seem to be a good reason for applying statistical methods to study production processes. It is well-known [1] that fluctuations of the number, N, of particles in a grand canonical ensemble are (outside a phase-transition region) proportional to the root square of an average number of particles, $\langle N \rangle$. This leads to a disagreement with multiplicity data on hadron–hadron collisions which provides a linear dependence of dispersion, $D=\sqrt{\langle N^2 \rangle - \langle N \rangle^2}$, on average value—Wróblewski formula [2].

The volume of all systems, in particular those with different number of particles, belonging to the grand canonical ensemble is the same. However, the hot hadron gas which occurs in high energy collisions is not closed in any container with a fixed volume. Thus, in our opinion, a more appropriate basis for studying the problem is the so-called pressure ensemble [3]. In this ensemble one considers systems under fixed external pressure, while the volumes of the systems can be different. Up to our knowledge, Gorenstein was the first who applied the pressure ensemble to hadron physics [4]. In our considerations we use some results of Gorenstein's paper [4], where the distribution of the number of quarks in the MIT bag and possible implications for multiplicity distributions were discussed.

The grand canonical pressure partition function is defined as [5]

$$\Pi(\xi, T\mu) = \int_0^\infty dV \exp(-\xi V) \Xi(V, T, \mu),$$

where T is the temperature and μ is the chemical potential. ξ is a new intensive parameter related to the volume, V, of the system in the similar way as $\beta = T^{-1}$ is related to the energy. Ξ is the grand canonical partition function.

$$\Xi(V, T, \mu) = \sum_{N=0}^\infty z^N Q_N(V, T),$$

where $z = \exp \beta\mu$ and Q_N is the canonical partition function of N particles.

The probability of finding N particles, $\mathscr{P}_N$, in the grand canonical pressure ensemble is expressed as

$$\mathscr{P}_N = \frac{z^N}{\Pi} \int_0^\infty dV \exp(-\xi V) Q_N(V, T). \tag{1}$$

Let us consider the model of hadron gas extensively discussed in the literature [6], where attractive forces are represented by the mass spectrum of particles, $\rho(m)$, and repulsive forces by a Van der Waals correction to the volume. The canonical partition function of such gas looks like (in units where $c = \hbar = k = 1$)

$$\begin{aligned} Q_N(V, T) &= \frac{1}{N!} \int \prod_{i=1}^N \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3} dm_i \rho(m_i) \left(V - \sum_{i=1}^N v_i \right) \right] \\ &\quad \cdot \exp\left(-\sum_{i=1}^N \beta \sqrt{\bar{\mathbf{p}}_i^2 + m_i^2} \right) \Theta\left(V - \sum_{i=1}^N v_i \right). \end{aligned} \tag{2}$$

The integration is performed over the momenta and masses of particles. v_i is the volume of an i-th hadron. There are so many different types of particles that, except for high densities at low temperature, quantum

1 Permanent address: Institute for Nuclear Studies, High Energy Department, Hoza 69, PL-00-681 Warsaw, Poland

effects are negligible and Boltzmann statistics is sufficient.

Substituting (2) and (1), one finds the geometrical distribution

$$\mathscr{P}_N = (1-q)q^N, \tag{3}$$

where

$$q \equiv \frac{z}{\xi}\int \frac{d^3\mathbf{p}}{(2\pi)^3} dm\,\rho(m)\exp(-\beta\sqrt{\bar{\mathbf{p}}^2+m^2}-\xi v).$$

It is seen that the form of (3) is independent of details of the hadron gas model. In particular, the distribution (3) is valid for ideal gas.

The geometrical distribution fulfils approximately KNO scaling [7] for $\langle N\rangle \gg 1$. Thus, the Wrøblewski formula is also approximately satisfied. Although, the shape of the geometrical distribution is far from the experimental one.

As pointed out many years ago, experimental data cannot be described within the model where the existence of one fireball is assumed [8]. Only a few-fireball model can be realistic [9]. In the case of e^+e^- annihilation into hadrons, jets can be interpreted as a result of the existence of two or more fireballs [10].

Let us consider a model of k-fireballs. To simplify our discussion we transform the discrete distribution (3) into continuous one (4)

$$\mathscr{P}(n) = a\exp(-an), \tag{4}$$

where n is a continuous variable. When $\langle N\rangle \gg 1$,

$$\mathscr{P}_N \cong \mathscr{P}(n), \quad q = \exp(-a) \cong 1-a.$$

Because the statistical methods are applicable to $\langle N\rangle \gg 1$, it is sufficient for us to use the continuous distribution (4).

We restrict ourselves to the simplest situation where the thermodynamic characteristics of all fireballs are the same. Assuming that the fireballs independently contribute to the resulting multiplicity, the distribution of particles emitted by these fireballs is the convolution of k distributions (4):

$$\mathscr{P}^k(n) = \int_0^\infty \prod_{i=1}^k [dn_i \mathscr{P}(n_i)]\,\delta\left(n-\sum_{i=1}^k n_i\right) = \frac{a^k}{(k-1)!}\exp(-an)n^{k-1}. \tag{5}$$

The above distribution has been also found by other authors [4, 11] whose argumentation, however, has been different than ours.

The average value of n is expressed by the formula

$$\langle n\rangle = \frac{k}{a} \tag{6}$$

and the second moment of (5) is the following

$$D = \sqrt{\langle n^2\rangle - \langle n\rangle^2} = \frac{\sqrt{k}}{a} = \frac{\langle n\rangle}{\sqrt{k}}. \tag{7}$$

The ratio $D/\langle n\rangle$ is simply related to a number of fireballs.

Multiplying $\mathscr{P}^k(n)$ by $\langle n\rangle$, we get

$$\langle n\rangle \mathscr{P}^k(n) = \frac{k^k}{(k-1)!}\left[\frac{n}{\langle n\rangle}\right]^{k-1}\exp\left(-k\frac{n}{\langle n\rangle}\right).$$

Thus, the distribution (5) exactly fulfils KNO scaling. An analogous discrete distribution satisfies KNO approximately.

In the case of hadron–hadron or lepton–lepton collisions we expect that an increase of multiplicity with incident energy is mainly due to decreasing of the thermodynamical factor "a" present in (6). The assumption of constancy of the number of fireballs leads to KNO scaling and, in particular, to the linear function $D(\langle n\rangle)$. In order to compare the distribution (5) with experimental data on hadron–hadron collisions, we assume that the number of fireballs is the same in each event. This assumption is quite realistic in the case of e^+e^- or $\bar{p}p$ annihilation. Because in hadron–hadron interactions there are collisions with different inelasticity coefficient, the above assumption is only approximately valid. Comparing (7) with the experimental value of $D/\langle n\rangle$ ratio [2, 12], we find that k is about 3 up to the collider energy region while at $\sqrt{S} = 540$ GeV $k \cong 4$. Thus, the number of fireballs seems to slowly increase with incident energy. In Fig. 1 we compare the experimental multiplicity distribution in $\bar{p}p$ interactions at collider [12] with the predictions of (5) for $k=4$. Keeping in mind the simplicity of the model, the agreement is quite satisfactory.

In the case of collisions with nuclei the number of fireballs can strongly depend on impact parameter. Thus, the multiplicity distribution in inelastic nucleus–nucleus collisions is the sum of distributions (5) with different k. Such summation gives the distribution which is "wider" than that in pp interactions. If we consider central nucleus–nucleus collisions where an

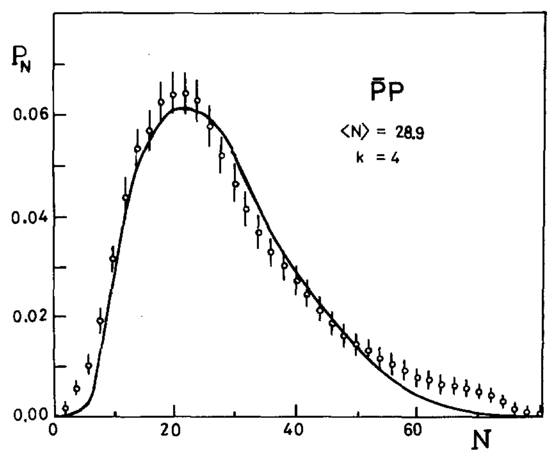

Fig. 1. The multiplicity distribution of charged particles produced in antiproton–proton collisions at $\sqrt{S} = 540$ GeV. The data are taken from [12]. The solid line is found according to the formula (5) with $k=4$

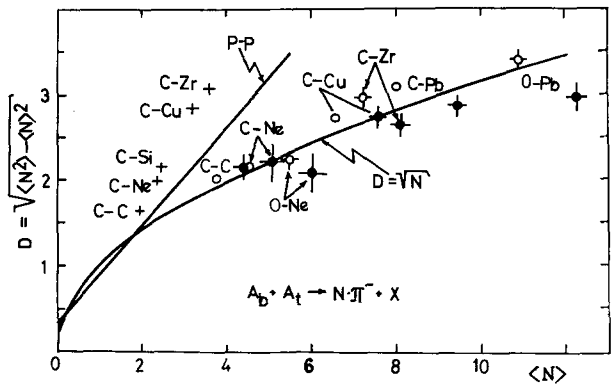

Fig. 2. The dispersion versus the average number of negative pions produced in nucleus-nucleus collisions at a momentum 4.5 GeV/c per nucleon. Crosses—inelastic collisions, circles—central collisions. The full and open circles correspond to different central triggers, for details see [13]. The straight line corresponding to multiplicity distributions for proton–proton inelastic collisions is found according to Wrøblewski formula [2]

impact parameter is restricted to small values, we expect that the number k does not change significantly from event to event. Because the number of fireballs in central collisions with nuclei is greater than in pp interactions, the multiplicity distribution should be "narrower" than in pp, see formula (7). In Fig. 2 taken from [13] it is shown the dispersion versus an average value for π^- produced in nucleus–nucleus, inelastic and central, collisions for fixed incident energy of projectile and different targets. Both features of the multiplicity distributions discussed above are seen. If we assume that at fixed projectile energy the thermodynamical factor "a" is independent of target mass and the increase of multiplicity is due to increase of the number of fireballs, we find, comparing (6) and (7), $D=\sqrt{a\langle n\rangle}$. As shown in Fig. 2 such a Poisson type relation has been found in the experiment [13]. Because "a" decreases with incident energy we expect that at future higher energies experiments the multiplicity distributions in central nucleus–nucleus collisions will be "narrower" than the Poisson one.

If it were possible to choose such a phase-space region of secondaries where only one source contributes, the geometrical distribution (3) would be found in this region. A good candidate for such a region is the backward hemisphere in LAB for nucleus–nucleus collisions. Because the emission of backward particles is kinematically unfavorable, we expect that only fireball with the smallest value of rapidity significantly contributes to this region. Consequently, the multiplicity distribution of backward particles should be geometrical. It occurs that such a distribution has been experimentally found [14, 15], see Fig. 3.

We summarize our considerations as follows. In the pressure ensemble one finds the geometrical distribution of the number of particles emitted from one source. Assuming the existence of a few fireballs in high energy collisions, we are led to the multiplicity distribution being the convolution of a few geometrical ones. This resulting distribution satisfies KNO scaling and fulfils other features of experimental multiplicity data of hadron–hadron and nucleus–nucleus interactions.

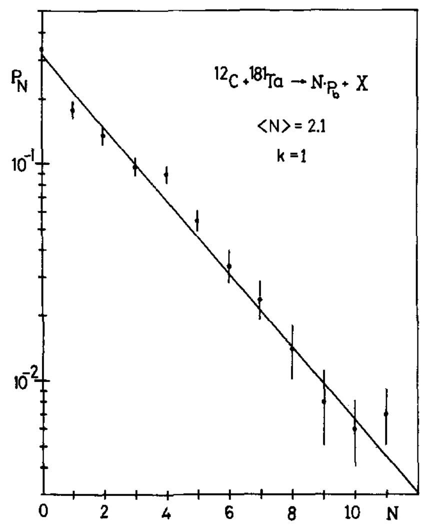

Fig. 3. The multiplicity distribution of protons with momentum greater than 240 MeV/c (non-evaporation protons) emitted in backward (in LAB) hemisphere for $^{12}C-^{181}Ta$ collisions at a momentum 4.2 GeV/c per nucleon. The data are taken from [14]. The solid line corresponds to the geometrical distribution (3)

Acknowledgements. I am grateful to M. Gaździcki, A.I. Golokhvastov, S.A. Khorozov and M. Šumbera for fruitful discussions.

I would like to express my thanks to R. Hagedorn for instructive and extensive comments on the manuscript.

References

1. See e.g. K. Huang: Statistical mechanics. New York: John Wiley and Sons, Inc., 1963
2. A. Wrøblewski: Acta Phys. Polon. **B4**, 857 (1973)
3. D. Ter Haar, H. Wergeland: Phys. Rep. **C1**, 31 (1971); A. Münster: Statistical physics, Berlin, Heidelberg, New York: Springer, 1969
4. M.I. Gorenstein: Yad. Fiz. **34**, 1630 (1980)
5. R. Hagedorn: Z. f. Phys. C—Particles and Fields **17**, 265 (1983)
6. J. Baacke: Acta Phys. Polon. **B8**, 625 (1977); R. Hagedorn, J. Rafelski: Phys. Lett. **B97**, 136 (1980); M.I. Gorenstein, V.K. Petrov, G.M. Zinoviev: Phys. Lett. **B106**, 327 (1982); M.I. Gorenstein, G.M. Zinoviev, V.K. Petrov, V.P. Shelest: Teor. Mat. Fiz. **52**, 346 (1982); J.I. Kapusta: Phys. Rev. **D23**, 2453 (1981)
7. Z. Koba, H.B. Nielsen, P. Olesen: Nucl. Phys. **B40**, 317 (1972)
8. P. Ciok et al.: Nuovo Cimento **8**, 166 (1958)
9. For review of fireball models see E.L. Feinberg: Usp. Fiz. Nauk. **139**, 1 (1983)
10. T.F. Hoang: Z. Phys. C—Particles and Fields **18**, 19 (1983)
11. N. Suzuki: Prog. Theor. Phys. **51**, 1629 (1974); A. Giovannini: Nuovo Cimento **A34**, 647 (1976); P. Carruthers, C.C. Shih: Phys. Lett. **B127**, 242 (1983)
12. K. Alpgård et al.: Phys. Lett. **B121**, 209 (1983)
13. M.Kh. Anikina et al.: Z. Phys. C—Particles and Fields **9**, 105 (1981)
14. N. Angelov et al.: Yad. Fiz. **32**, 1582 (1980); **33**, 190 (1981)
15. G.S. Shabratova et al.: Z. Phys. A—Atoms and Nuclei **318**, 75 (1984)