Instabilities Driven Equilibration in Nuclear Collisions

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Evidence of equilibration

- **success of thermal models in describing ratios of particle multiplicities**

  - chemical equilibrium of the final state

$$\frac{\langle n_a \rangle}{\langle n_b \rangle} \sim e^{-\frac{m_a - m_b}{T}}$$

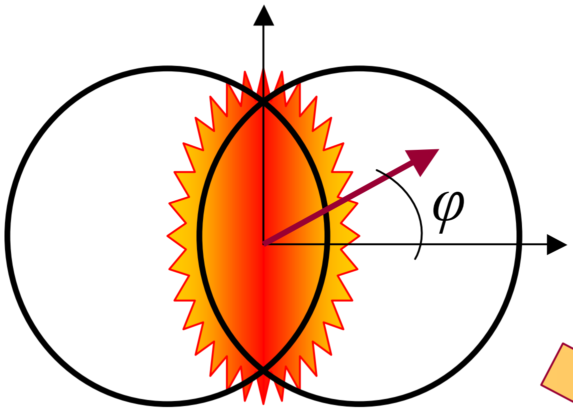
- **success of thermal models in describing particle p_T spectra**

  - thermal equilibrium of the final state

$$\frac{d^2 n}{dp_T^2} \sim e^{-\frac{\sqrt{m^2 + p_T^2}}{T}}$$

Evidence of equilibration at the early stage

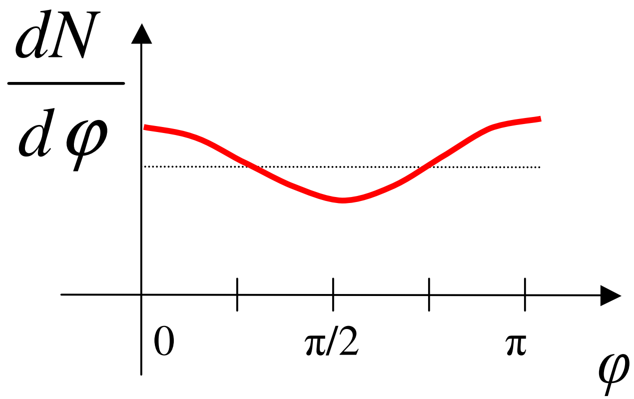
Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)\mathbf{v}=-\frac{\nabla p}{\rho}$$

Hydrodynamic requires local thermodynamical equilibrium!



Equilibration is fast

$$v_2 \sim \varepsilon = \left\langle \frac{x^2 - y^2}{x^2 + y^2} \right\rangle$$

Eccentricity decays due to the free streaming!

$\varepsilon \searrow \;\Rightarrow\; v_2 \searrow$ ➡ $t_{eq} \le 0.6\ \text{fm}/c$

time of equilibration

U. Heinz and P. Kolb, Nucl. Phys. **A702**, 269 (2002)

Collisions are too slow

Time scale of hard parton-parton scattering

hard scattering ~ momentum transfer of order of T

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

either single hard scattering or multiple soft scatterings

$$t_{\text{eq}} \approx t_{\text{hard}} \geq 2.6\ \text{fm}/c$$

R. Baier, A.H. Mueller, D. Schiff and D.T. Son, Phys. Lett. **B539**, 46 (2002)

Scenarios of fast equilibration

- **Production mechanism of particles obeys equilibrium momentum distributions – instantaneous equilibration**

Schwinger mechanism:

$$\frac{d^2 n}{dp_T^2} \sim e^{-\frac{2\pi m_T^2}{eE}}$$

A. Białas, Phys.Lett. **B466**, 301 (1999)
W. Florkowski, Acta Phys. Pol. **B35**, 799 (2004)
D. Kharzeev & K. Tuchin, hep-ph/0501234

- **Equilibration is fast because quark-qluon plasma is strongly coupled**

sQGP

E.V. Shuryak, J. Phys.**G30**, S1221 (2004)
E.V. Shuryak & I. Zahed, Phys. Rev. **C70**, 021901 (2004)

- **Instabilities drive equilibration - as in the EM plasma**

Instabilities driven equilibration

The most important contributions

St. Mrówczyński,
Color Collective Effects At The Early Stage Of Ultrarelativistic Heavy Ion Collisions,
Phys. Rev. C **49**, 2191 (1994).

St. Mrówczyński,
Color filamentation in ultrarelativistic heavy-ion collisions,
Phys. Lett. B **393**, 26 (1997).

P. Romatschke and M. Strickland,
Collective modes of an anisotropic quark gluon plasma,
Phys. Rev. D **68**, 036004 (2003)

P. Arnold, J. Lenaghan and G.D. Moore,
QCD plasma instabilities and bottom-up thermalization,
JHEP **0308**, 002 (2003)

} **Unstable Mode Analysis**

Numerical Simulations {

A. Rebhan, P. Romatschke and M. Strickland,
Hard-loop dynamics of non-Abelian plasma instabilities,
Phys. Rev. Lett. **94**, 102303 (2005)

A. Dumitru and Y. Nara,
QCD plasma instabilities and isotropization,
arXiv:hep-ph/0503121.

Instabilities driven equilibration

The most important contributions cont.

Effective Action

St. Mrówczyński and M. Thoma,
Hard Loop Approach to Anisotropic Systems,
Phys. Rev. D **62**, 036011 (2000)

P. Arnold and J. Lenaghan,
The abelianization of QCD plasma instabilities,
Phys. Rev. D **70**, 114007 (2004)

St. Mrówczyński, A. Rebhan and M. Strickland,
Hard-loop effective action for anisotropic plasmas,
Phys. Rev. D **70**, 025004 (2004)

Heavy-Ion Phenomenology

J. Randrup and St. Mrówczyński,
Chromodynamic Weibel instabilities in relativistic nuclear collisions,
Phys. Rev. C **68**, 034909 (2003)

P. Arnold, J. Lenaghan, G.D. Moore and L.G. Yaffe,
Apparent thermalization due to plasma instabilities in quark gluon plasma,
Phys. Rev. Lett. **94**, 072302 (2005)

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

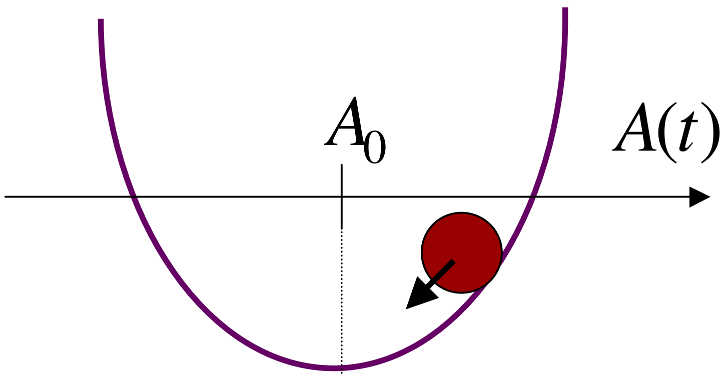
fluctuation

Instability

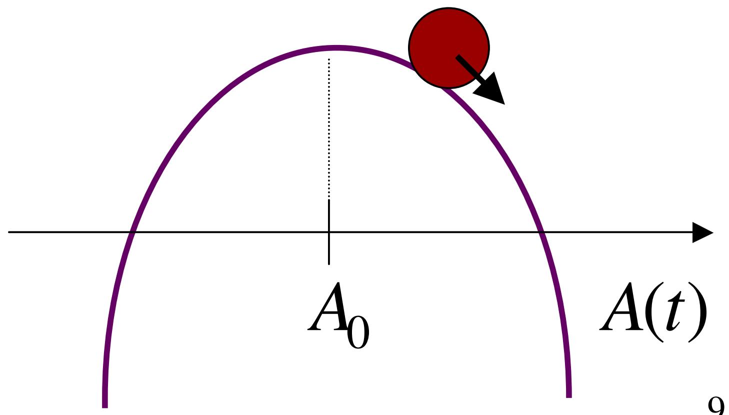
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**
- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

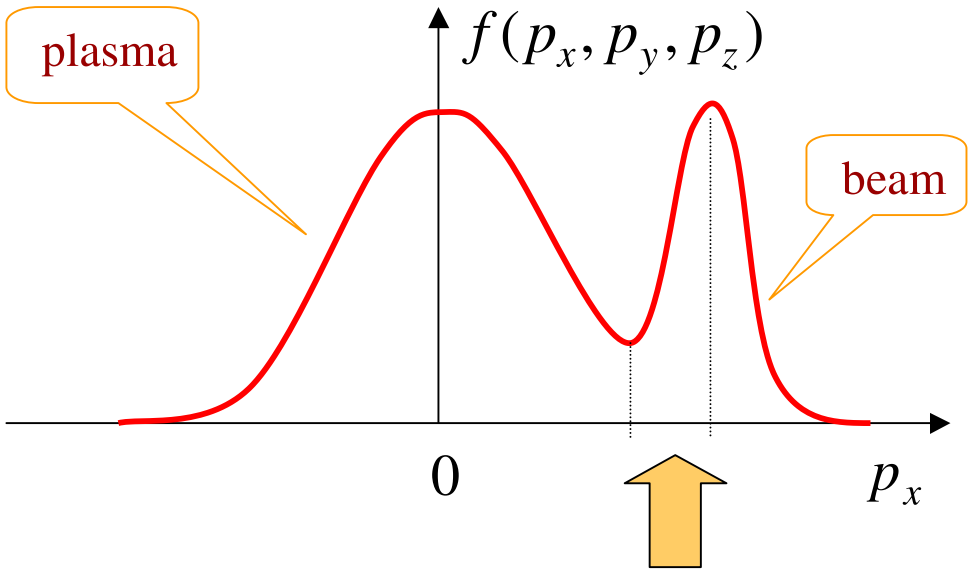
- **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}, \ \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$

- **transverse modes** – $\mathbf{k} \perp \mathbf{E}, \ \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

Logitudinal modes

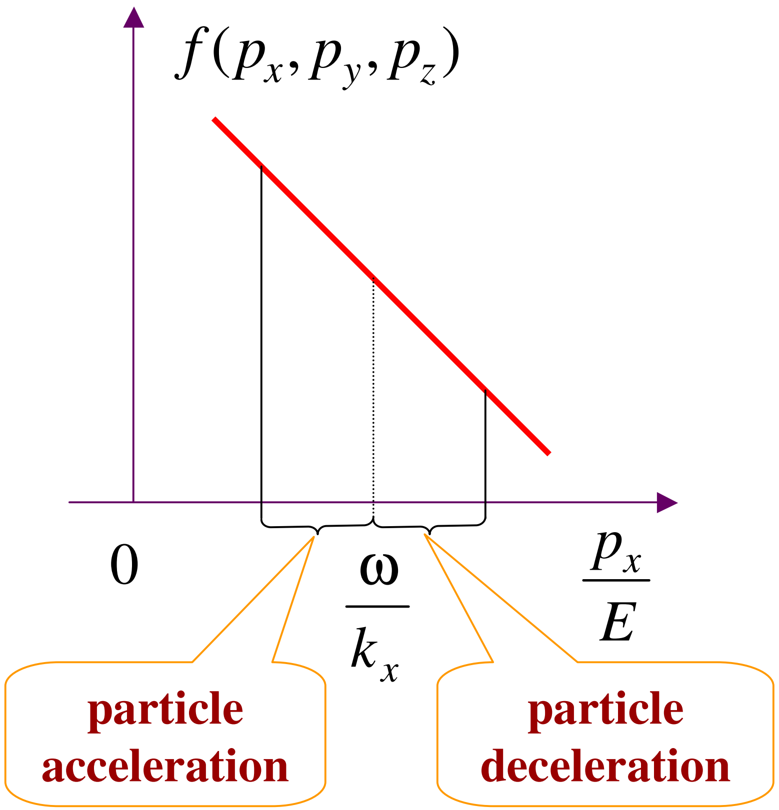
unstable configuration



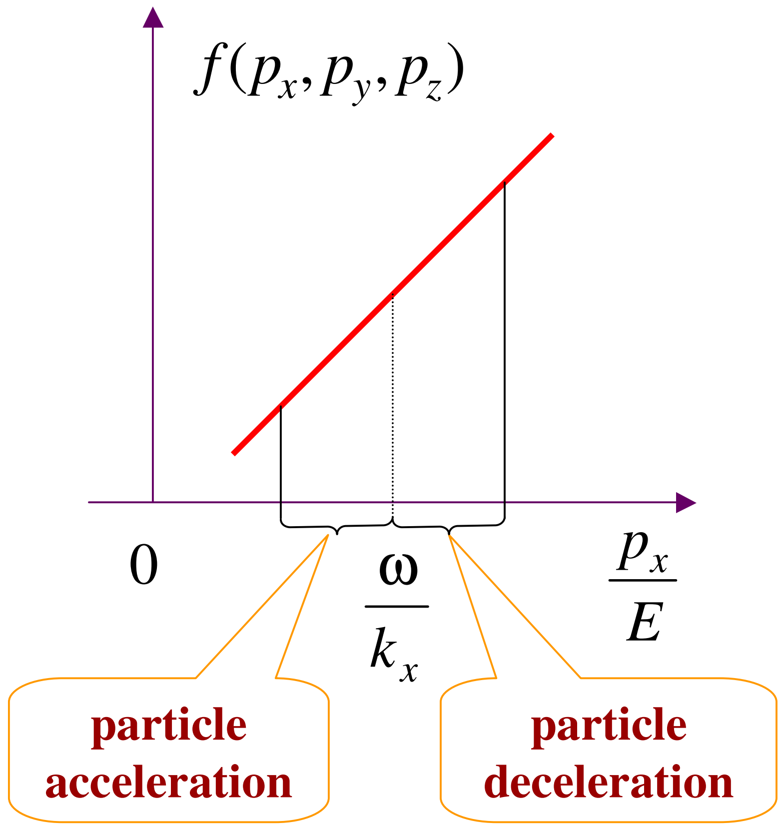
Energy is transferred from particles to fields

Logitudinal modes

Electric field decays - damping



Electric field grows - instability

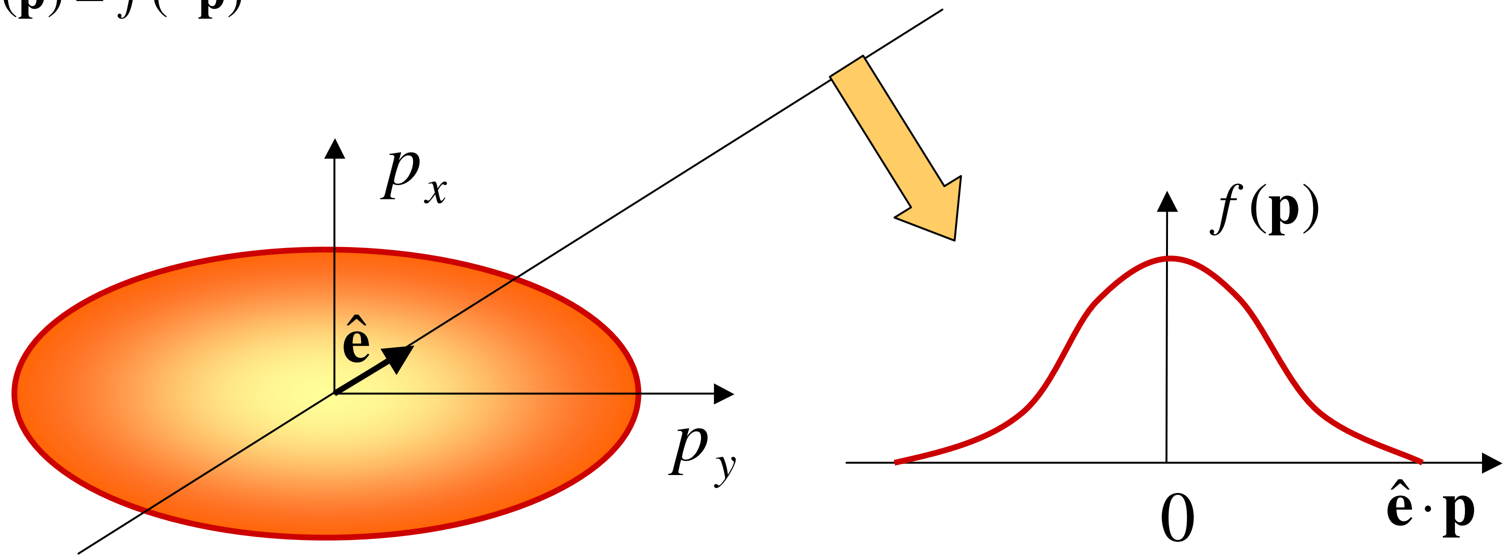


$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

Transverse modes

Unstable transverse modes occur due anisotropic momentum distribution

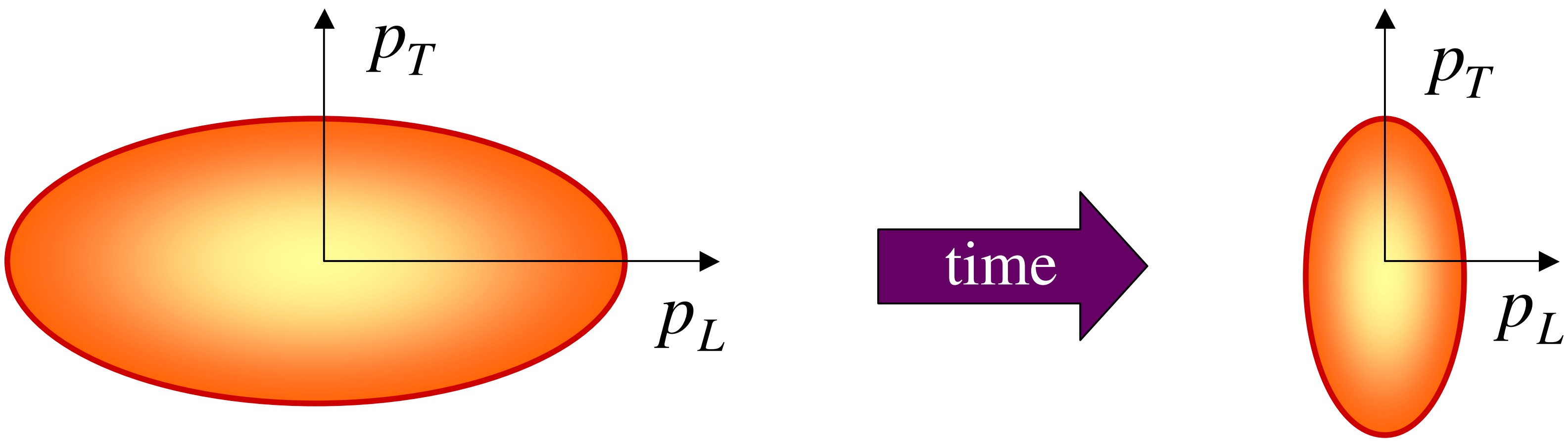
$f(\mathbf{p}) = f(-\mathbf{p})$



Momentum distribution distribution can monotonously decrease in every direction

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic



CM after 1-st collisions

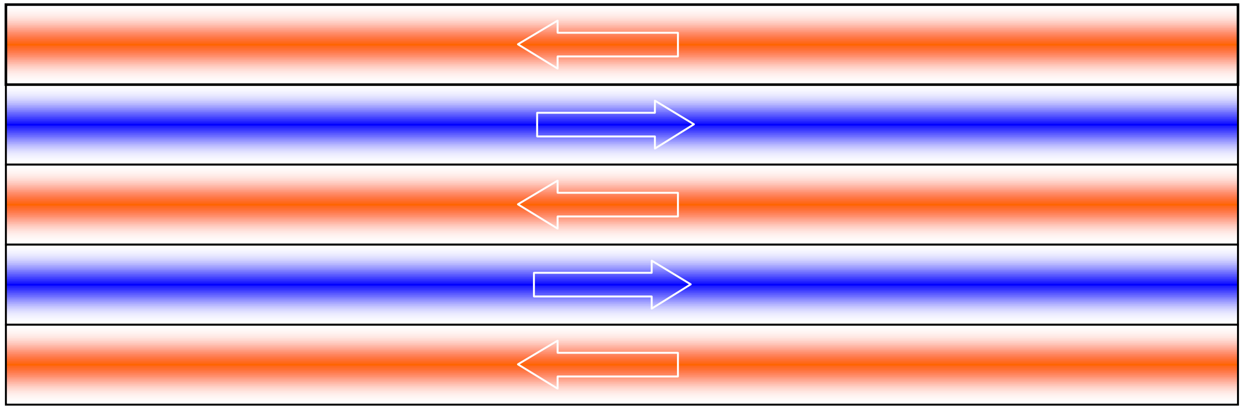
local rest frame

Seeds of instability

Current fluctuations

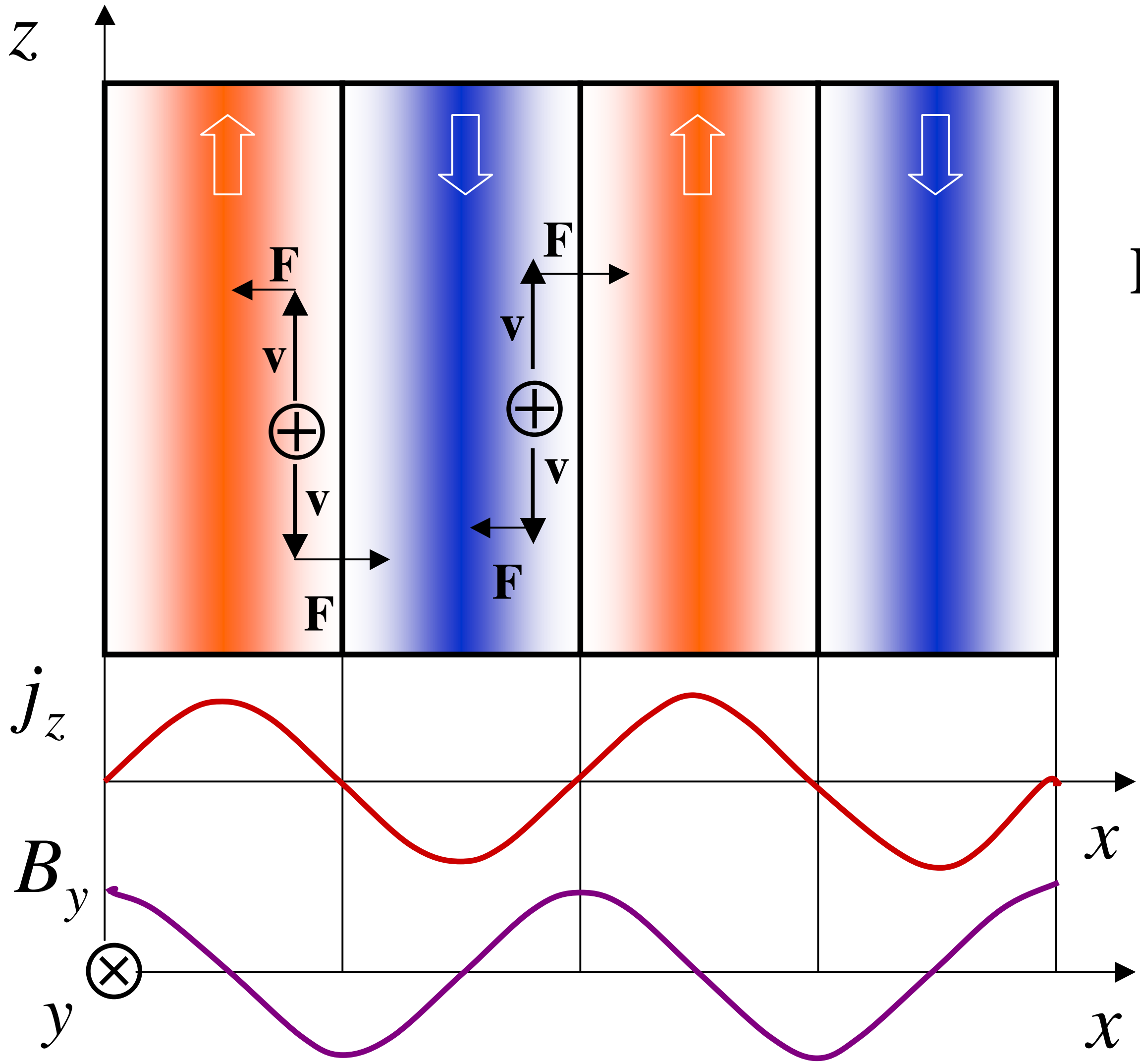
$$\left\langle j_a^\mu(x_1)\, j_b^\nu(x_2)\right\rangle = \frac{1}{2}\delta^{ab}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p})\delta^{(3)}(\mathbf{x}-\mathbf{v}t)$$

$$x_1 = (t_1, \mathbf{x}_1)\,, \quad x_2 = (t_2, \mathbf{x}_2)\,, \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



$$\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$$

$$\nabla\times\mathbf{B} = \mathbf{j}$$

Dispersion equation

Equation of motion of chromodynamic field A^μ in momentum space

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0$$

gluon self-energy

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with Im$\omega > 0$ $\Rightarrow$ $A^\mu(x) \sim \mathrm{e}^{\mathrm{Im}\,\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. **How to get it?**

Transport theory – distribution functions

Distribution functions of

quarks	$Q(p,x)$	3 x 3 matrix (quarks and antiquarks)
antiquarks	$\overline{Q}(p,x)$	
gluons	$G(p,x)$	8 x 8 matrix

Distribution functions are gauge dependent

$$Q(p,x) \rightarrow U(x)Q(p,x)U^{-1}(x) \quad \Rightarrow \quad \mathrm{Tr}[Q(p,x)]$$

gauge independent

Baryon current

$$b^{\mu}(x) = \frac{1}{3}\mathrm{Tr}\int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}}{E}[Q(p,x) - \overline{Q}(p,x)]$$

Color current

$$j^{\mu}(x) = -\frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}}{E}\{Q(p,x) - \frac{1}{3}\mathrm{Tr}[Q(p,x)] + ...\}$$

antiquarks & gluons

Transport theory – transport equations

fundamental

$$\left(p_\mu D^\mu - g p^\mu F_{\mu\nu}(x)\partial_p^\nu\right) Q(p,x) = C$$

$$\left(p_\mu D^\mu + g p^\mu F_{\mu\nu}(x)\partial_p^\nu\right) \overline{Q}(p,x) = \overline{C}$$

adjoint

$$\left(p_\mu \mathcal{D}^\mu - g p^\mu \mathcal{F}_{\mu\nu}(x)\partial_p^\nu\right) G(p,x) = C_g$$

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$

mean-field generation

collisionless limit: $C = \overline{C} = C_g = 0$

Transport theory - linearization

fluctuation

$$Q(p,x) = Q_0(p) + \delta Q(p,x)$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| >> |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p)| >> |\partial_p^\mu \delta Q(p,x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$

$$p_\mu D^\mu \delta \overline{Q}(p,x) + g p^\mu F_{\mu\nu}(x) \partial_p^\nu \overline{Q}_0(p) = 0$$

$$p_\mu \mathcal{D}^\mu \delta G(p,x) - g p^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu G_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p,x) = g\int d^4x'\,\Delta_p(x-x')\,p^\mu F_{\mu\nu}(x)\partial_p^\nu Q_0(p)$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$j^\mu[\delta Q, \delta\overline{Q}, \delta G]$$

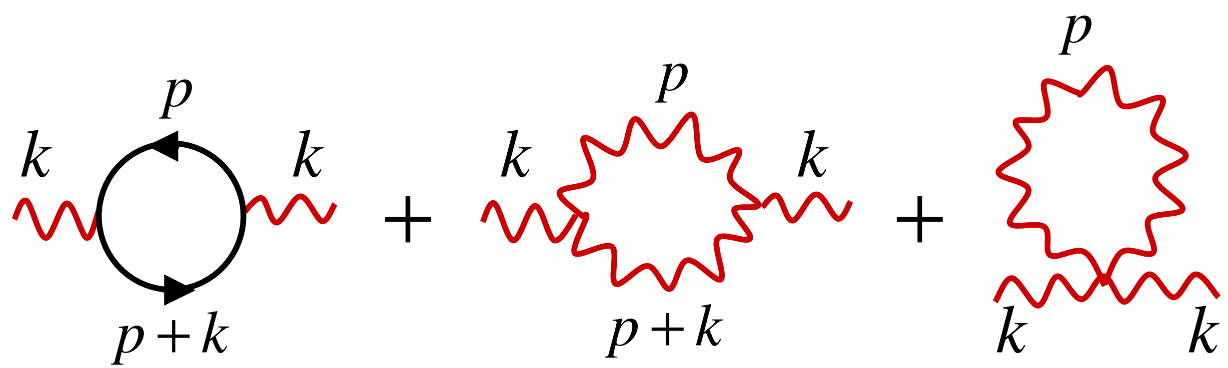
$$j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\right]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$\Pi^{\mu\nu}(k) = \Big[$  $\Big]$

Hard loop approximation: $k^{\mu} << p^{\mu}$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}}{E}\Big[g^{\nu\lambda} - \frac{p^{\nu}k^{\lambda}}{p^{\sigma}k_{\sigma} + i0^{+}}\Big]\frac{\partial f(\mathbf{p})}{\partial p^{\lambda}}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_{\mu}\Pi^{\mu\nu}(k) = 0$$

Dispersion equation

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(k)$$ **chromodielectric tensor**

$$k^\mu \equiv (\omega, \mathbf{k})$$

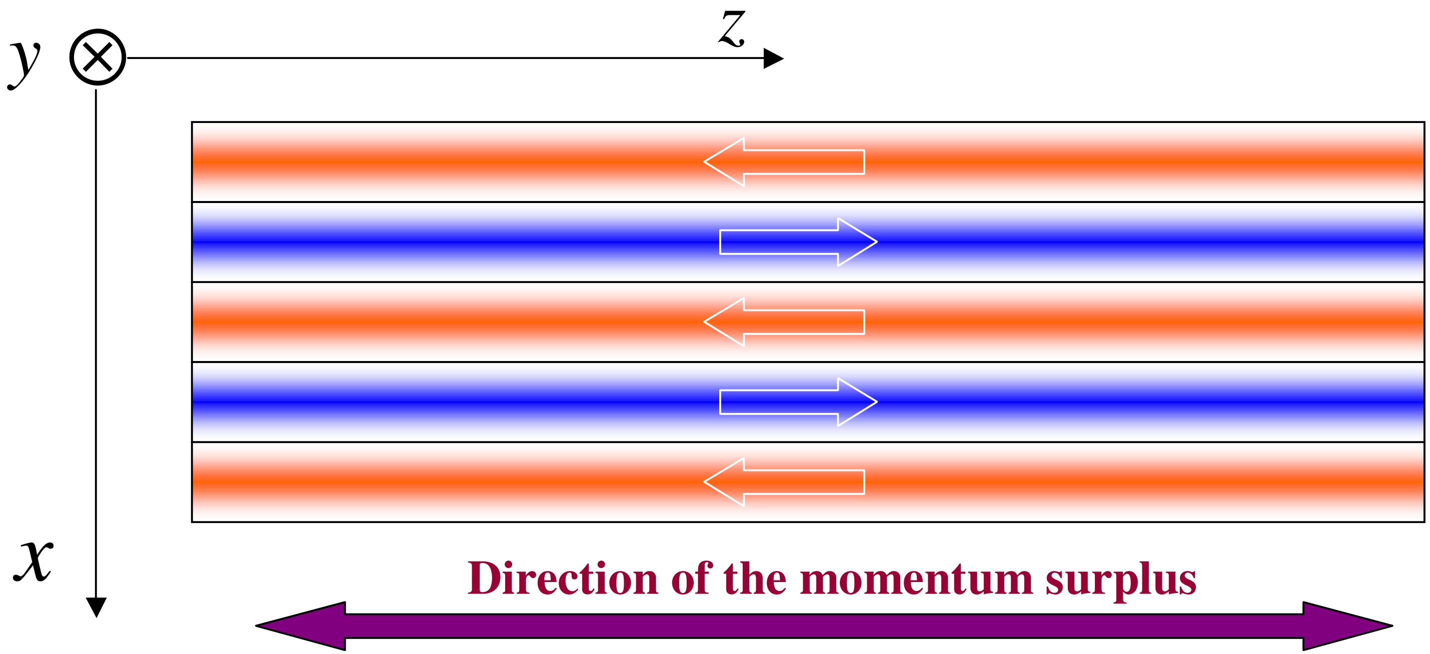
Dispersion equation

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

$$\mathbf{v} \equiv \mathbf{p}/E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0,0,j), \quad \mathbf{E} = (0,0,E), \quad \mathbf{k} = (k,0,0)$$

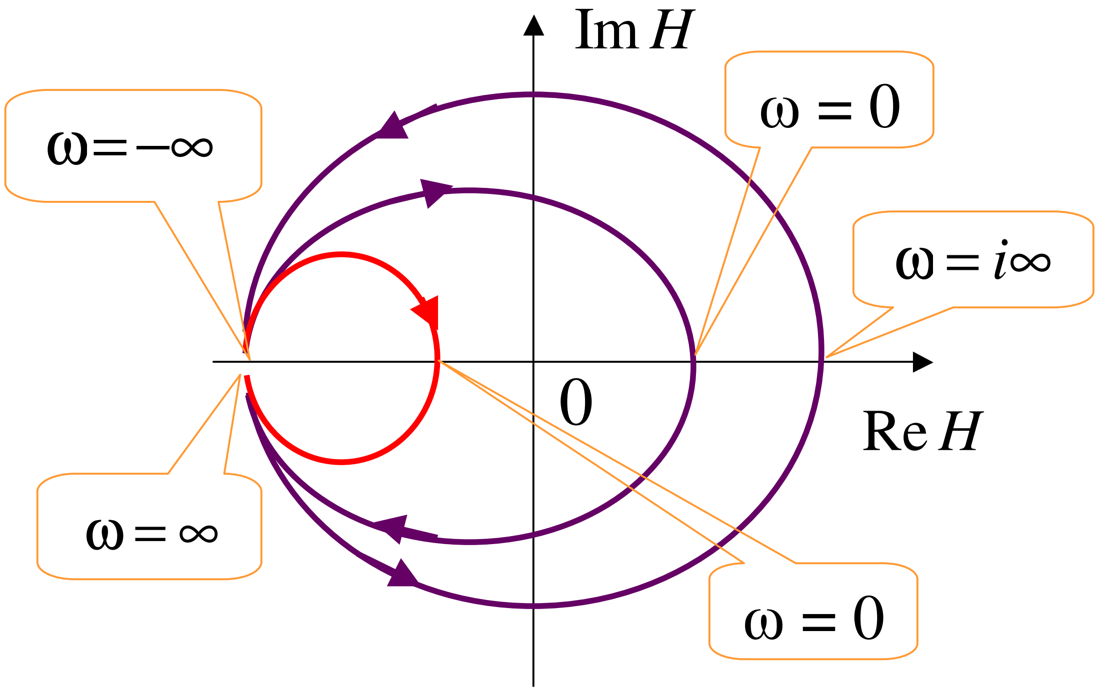
Dispersion equation

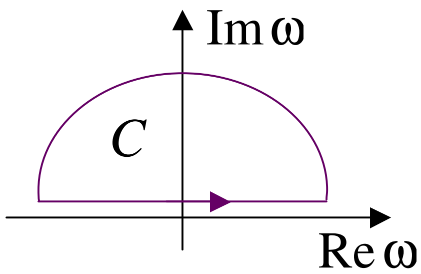
$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

Existence of unstable modes – Penrose criterion

$$H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)$$

$$\oint_C \frac{d\omega}{2\pi i} \frac{1}{H(\omega)} \frac{dH(\omega)}{d\omega} = \begin{cases} \oint_C \frac{d\omega}{2\pi i} \frac{d \ln H(\omega)}{d\omega} = \ln H(\omega)\Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{cases}$$





There are unstable modes if

$$H(\omega = 0) < 0$$

Unstable solution

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho\sigma_\perp^4}{\sigma_\parallel} \frac{1}{(p_\perp^2 + \sigma_\perp^2)^3} e^{-\frac{p_\parallel^2}{2\sigma_\parallel^2}}$$

$$\rho = 6\ \mathrm{fm}^{-3}$$

$$\alpha_s = g^2/4\pi = 0.3$$

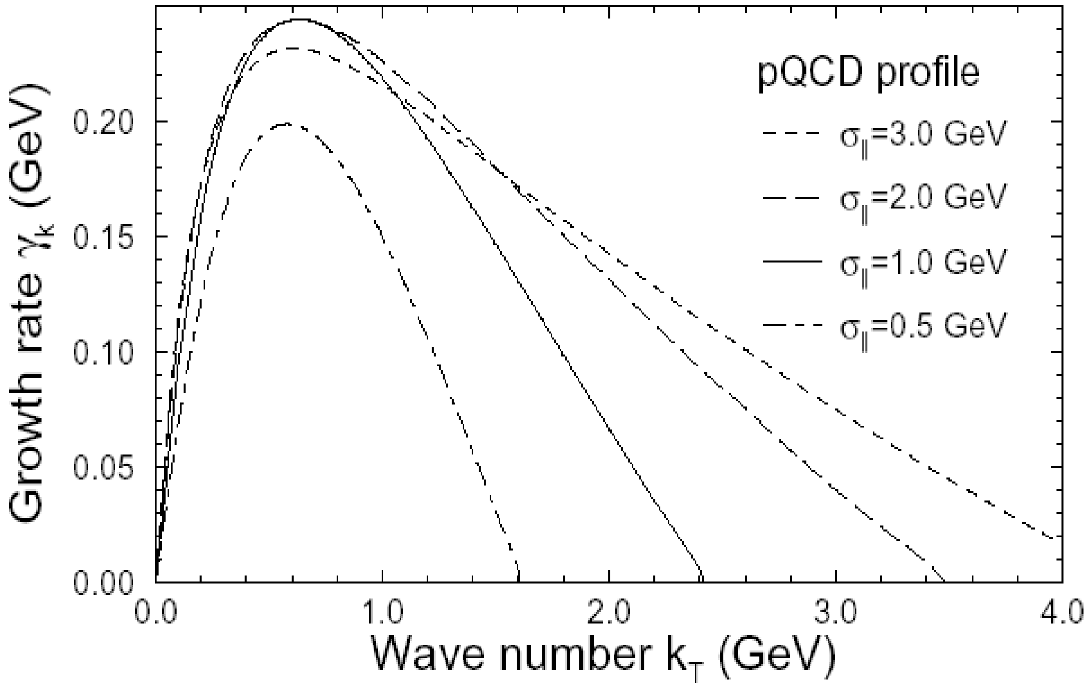
$$\sigma_\perp = 0.3\ \mathrm{GeV}$$

$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

solution

$$\omega(k) = \pm i\,\gamma_k$$

$$0 < \gamma_k \in \Re$$



J. Randrup and St. M., Phys. Rev. C **68**, 034909 (2003)

Growth of instabilities – numerical simulation

Classical system of colored particles & fields

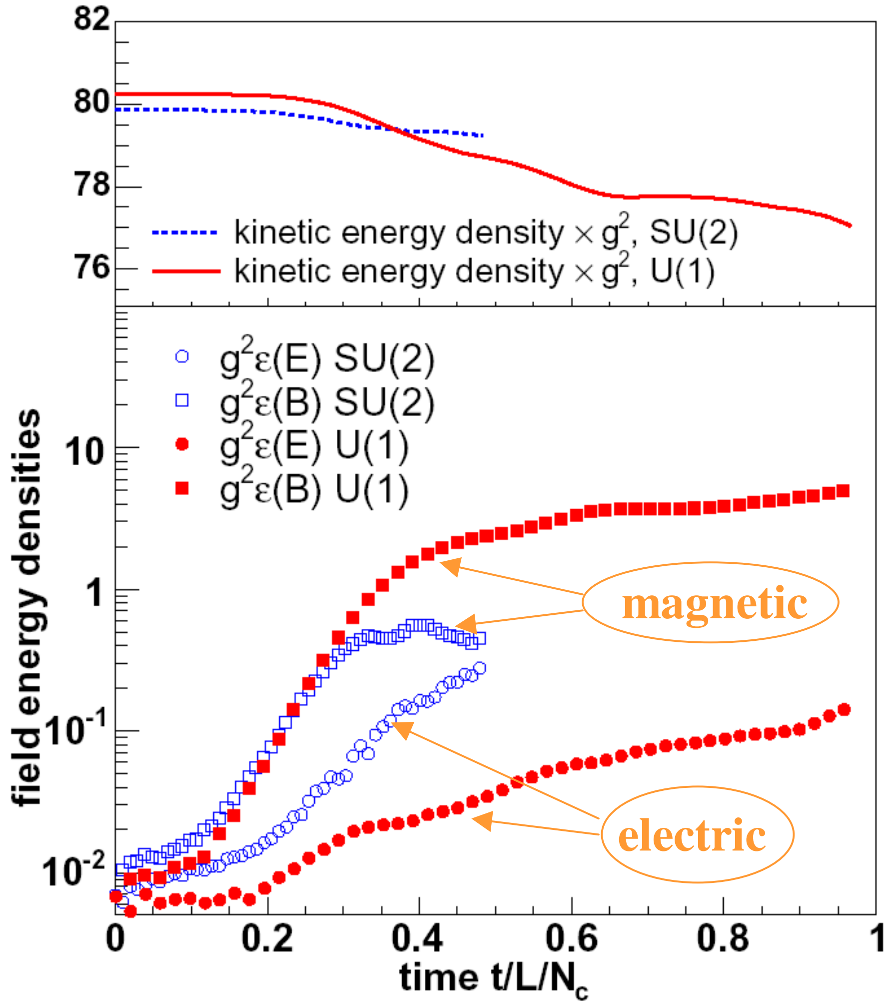
initial fields: **Gaussian noise as in Color Glass Condensate**

initial particle distribution:

$$f_0(\mathbf{p},\mathbf{x}) \sim \delta(p_x)\, e^{-\frac{\sqrt{p_y^2+p_z^2}}{p_{\mathrm{hard}}}}$$

$$p_{\mathrm{hard}} = 10\ \mathrm{GeV}$$

$$L = 40\ \mathrm{fm} \qquad \rho = 10\ \mathrm{fm}^{-3}$$



A. Dumitru and Y. Nara, arXiv:hep-ph/0503121

Abelanization

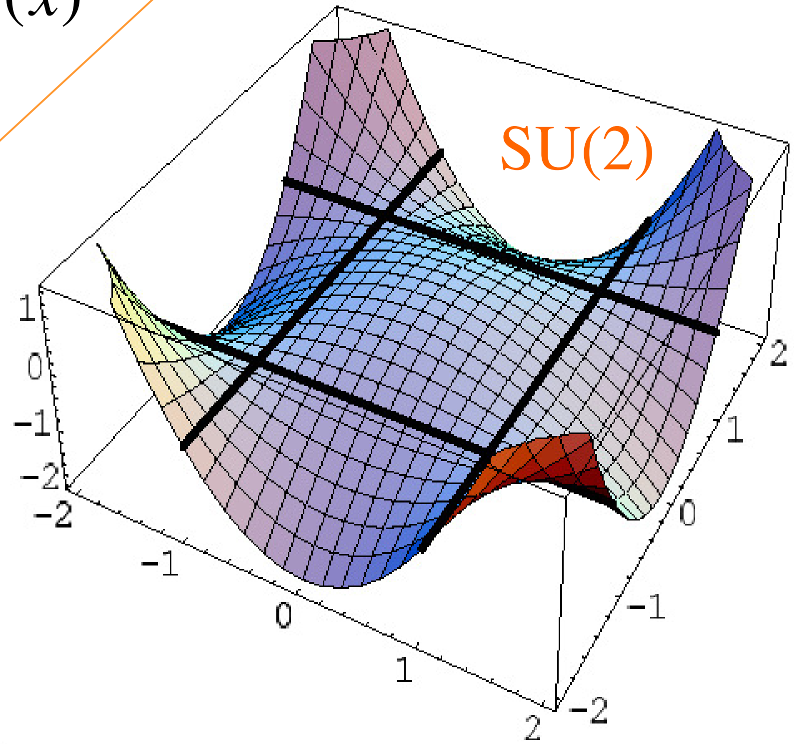
$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2 \mathbf{A}^a \cdot \mathbf{A}^a + \frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

the gauge $A_0^a = 0, \quad A_i^a(t, x, y, z) = A_i^a(x)$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} \mathbf{B}^a \mathbf{B}^a$$

$$= -\frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a + \frac{g}{2} f_{abc} \mathbf{A}^b \times \mathbf{A}^c$$



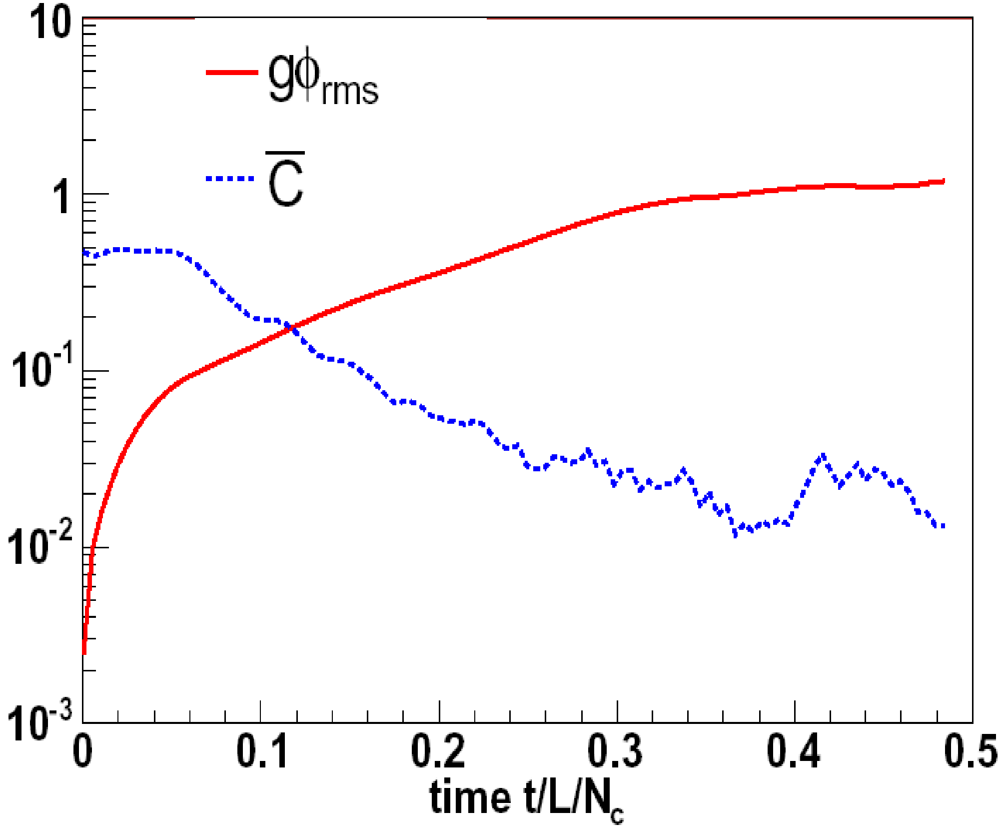
P. Arnold and J. Lenaghan,Phys. Rev. D **70**, 114007 (2004)

Abelanization – numerical simulation

Classical system of colored particles & fields

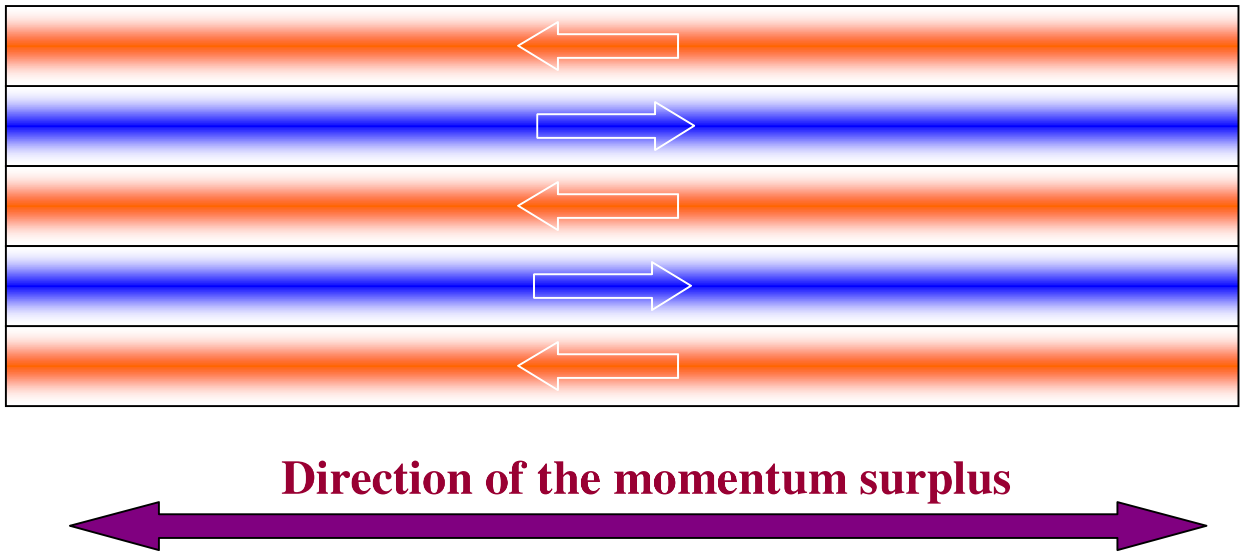
$$\phi_{\mathrm{rms}} = \sqrt{\int_0^L \frac{dx}{2L} \mathrm{Tr}[\mathbf{A}^2]}$$

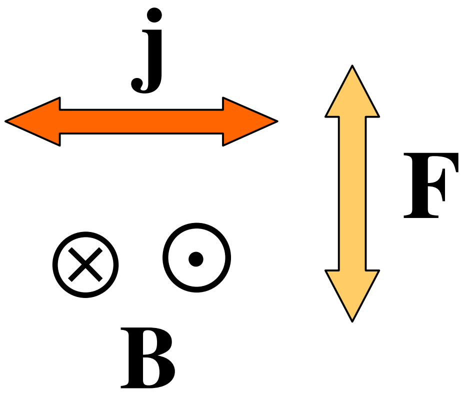
$$\overline{C} = \int_0^L \frac{dx}{L} \frac{\sqrt{\mathrm{Tr}((i[A_y, A_z])^2)}}{\mathrm{Tr}[\mathbf{A}^2]}$$



A. Dumitru and Y. Nara, arXiv:hep-ph/0503121

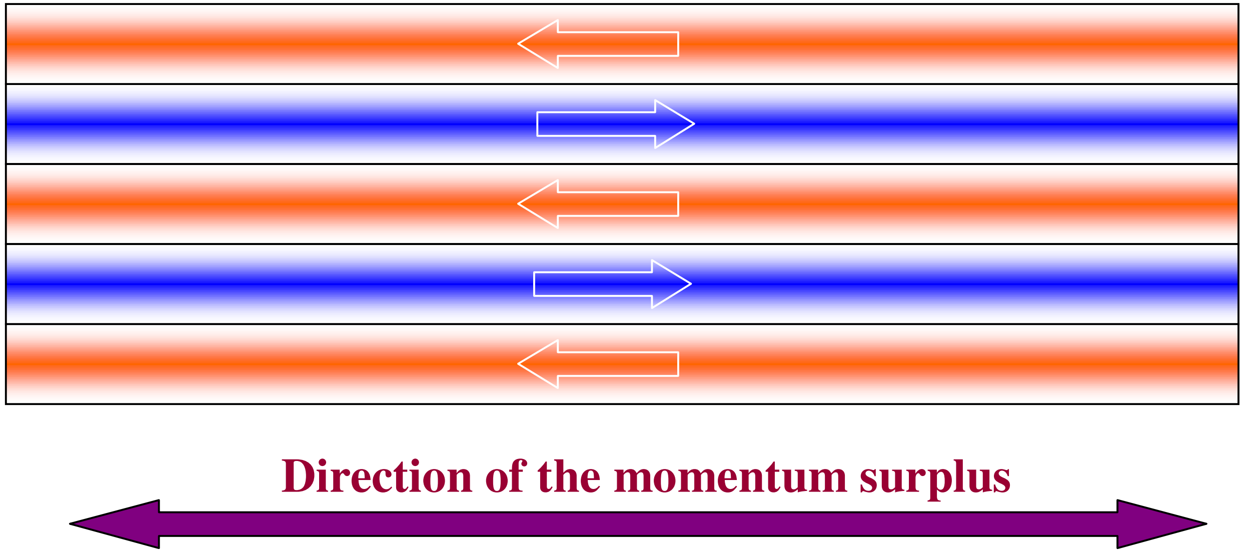
Isotropization - particles

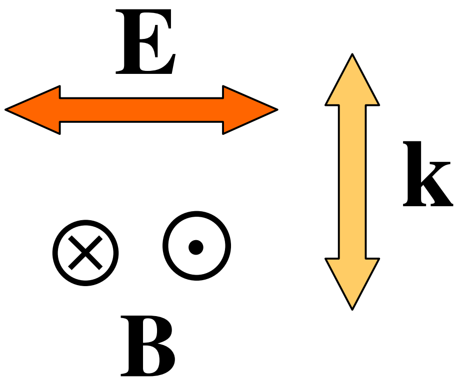




$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields





$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

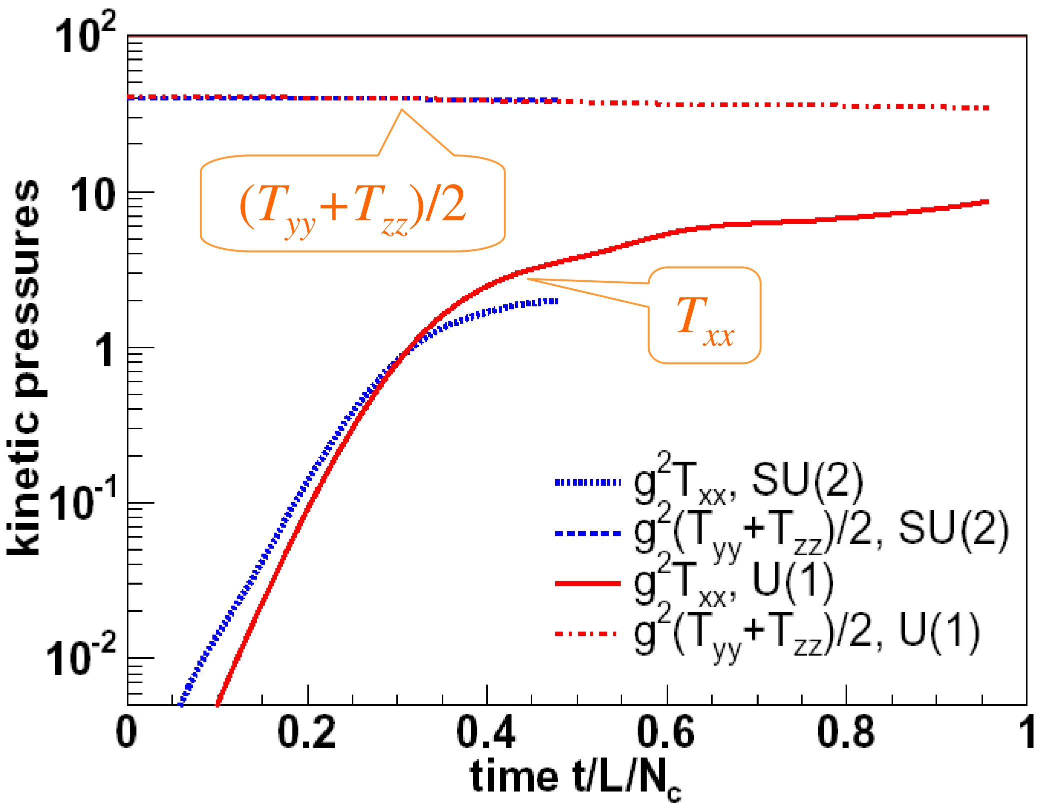
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

$$T_{xx} = (T_{yy} + T_{zz})/2$$



A. Dumitru and Y. Nara, arXiv:hep-ph/0503121

Conclusion

The scenario of instabilities driven equilibration seems to be a plausible solution of the fast equilibration problem of relativistic heavy-ion collisions