

Jet quenching in glasma

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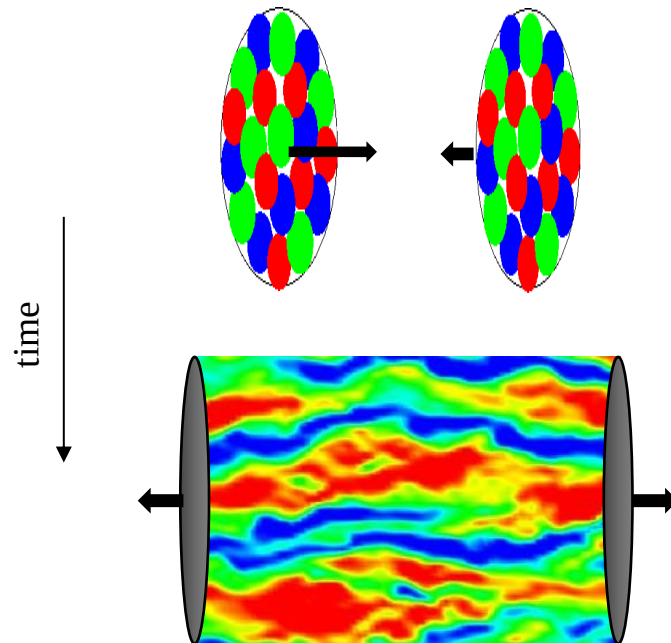
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in collaboration with **Margaret Carrington &**

**Alina Czajka, Wade Cowie, Bryce Friesen,
Jean-Yves Ollitrault and Doug Pickering**

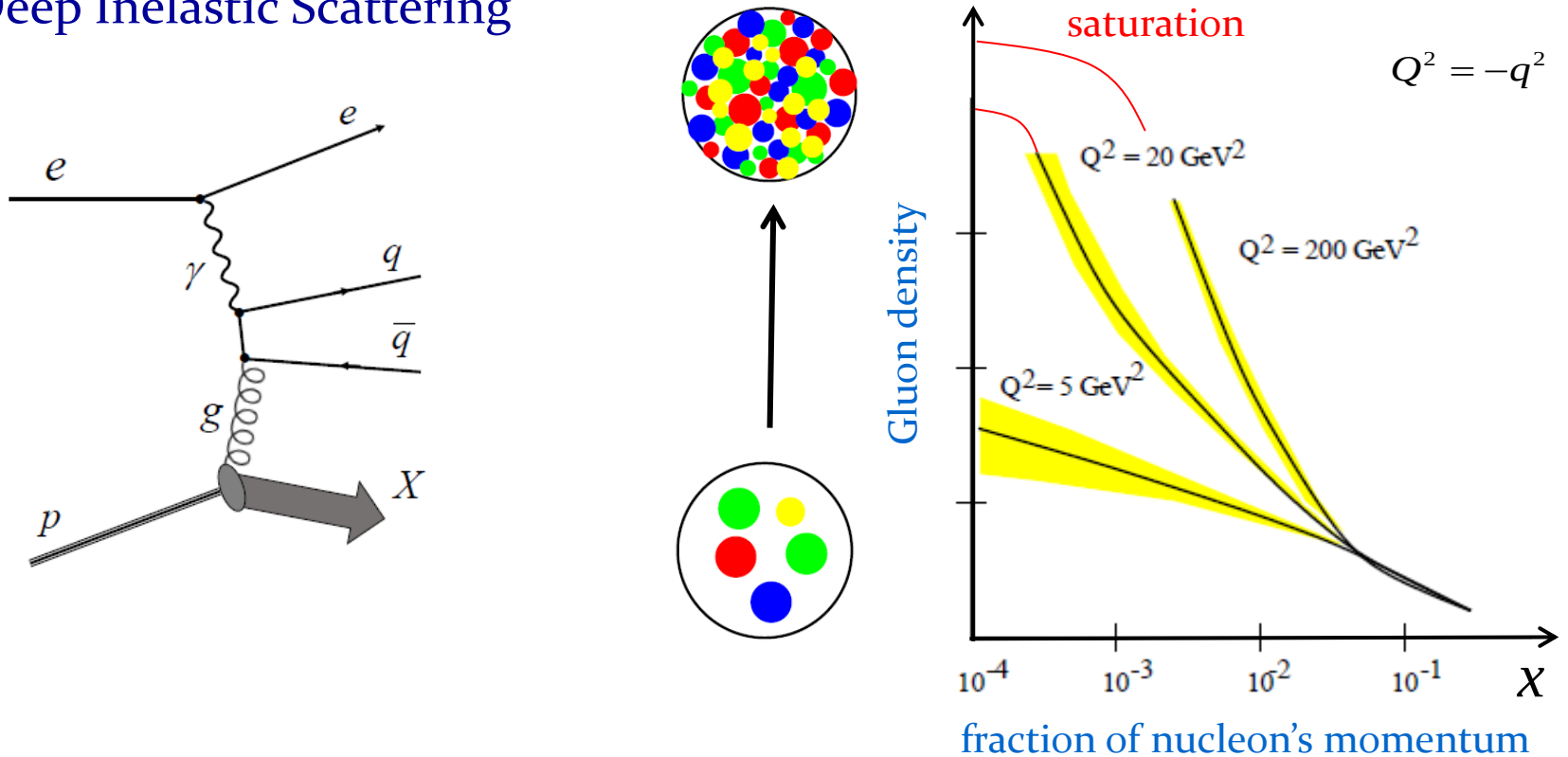
Glasma & Color Glass Condensate

Color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolve towards equilibrium.



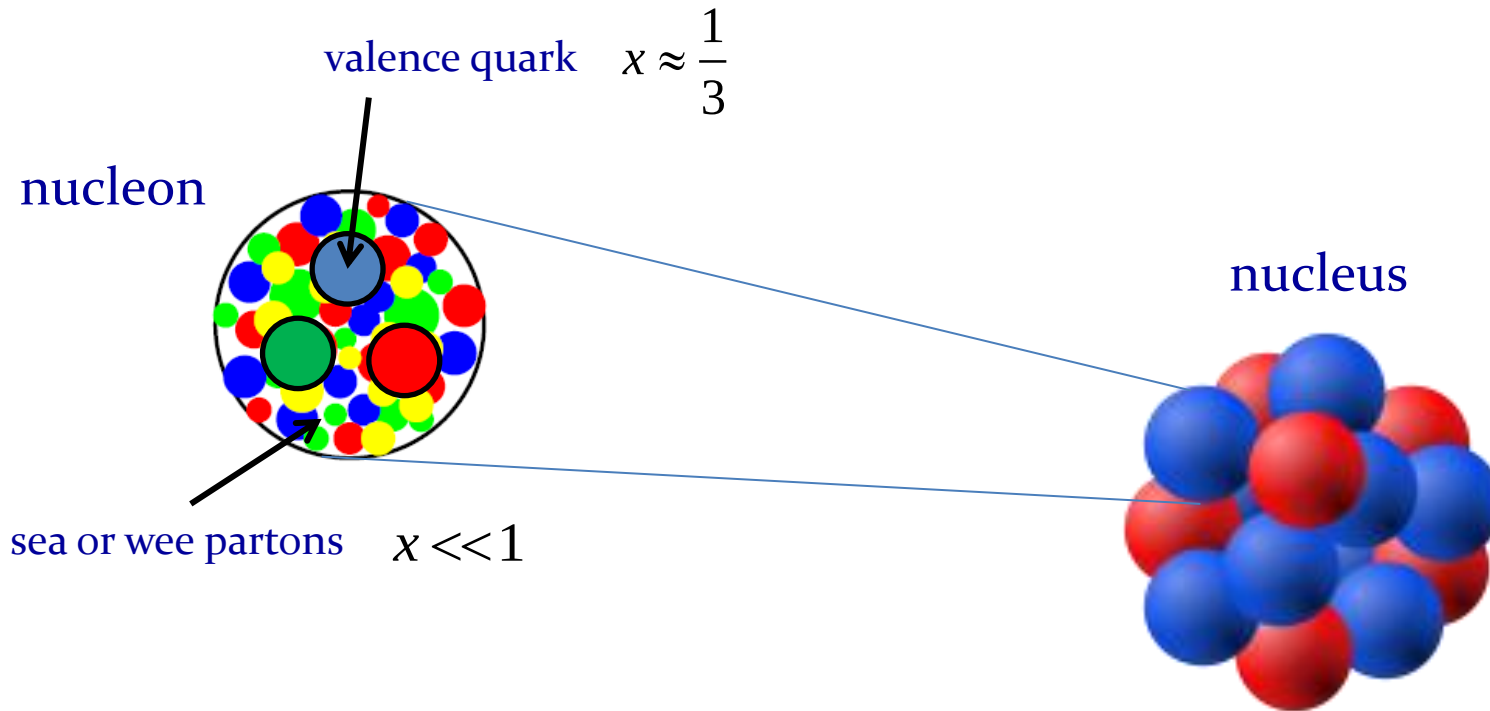
Saturation

Deep Inelastic Scattering



Saturated gluon system can be described in terms of classical chromodynamic fields.

Wee partons & valence quarks



In relativistic heavy-ion collisions

- ▶ Saturated wee partons – classical chromodynamic fields
- ▶ Valence quarks – classical sources of chromodynamic fields
- ▶ Saturation scale for A-A at LHC: $Q_s \approx 2 \text{ GeV} \gg \Lambda_{\text{QCD}} \approx 0.2 \text{ GeV} \Rightarrow \alpha_s(Q_s) \ll 1$

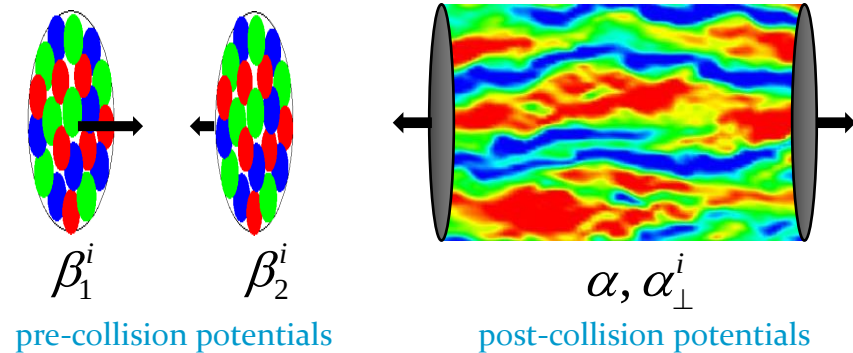
Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm \delta^{\mu\mp} \delta(x^\pm) \rho_{1,2}(\mathbf{x}_\perp)$$



Ansatz of gauge potentials

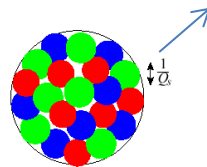
$$\begin{cases} A^+(x) = \Theta(x^+) \Theta(x^-) x^+ \alpha(\tau, \mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+) \Theta(x^-) x^- \alpha(\tau, \mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+) \Theta(x^-) \alpha_\perp^i(\tau, \mathbf{x}_\perp) \\ \quad + \Theta(-x^+) \Theta(x^-) \beta_1^i(\mathbf{x}_\perp) + \Theta(x^+) \Theta(-x^-) \beta_2^i(\mathbf{x}_\perp) \end{cases}$$

Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\frac{ig}{2} [\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

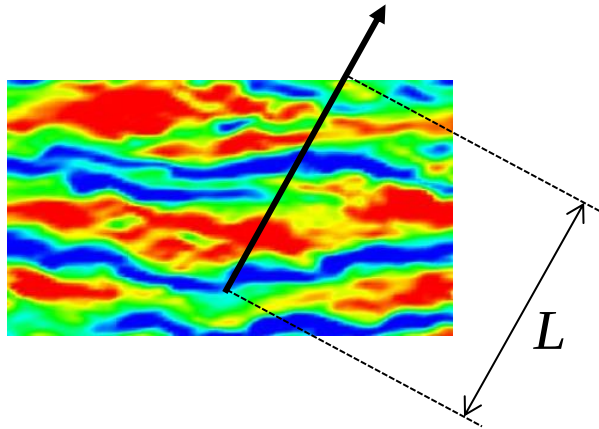
Gauge condition

$$x^+ A^- + x^- A^+ = 0$$



Jet quenching in glasma

How hard probes propagate through the glasma?



$$\frac{dE}{dx}, \hat{q} \text{ ?}$$

$$\frac{dE}{dx} \text{ - collisional energy loss}$$

$$\hat{q} \text{ - transverse momentum broadening}$$

$$\frac{dE^{\text{rad}}}{dx} = -\frac{1}{8} \alpha_s N_c \hat{q} L \text{ - radiative energy loss}$$

Fokker-Planck Equation

► Transport of hard probes can be described using the Fokker-Planck equation.

$$\overbrace{\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right)}^{\text{drift}} n(t, \mathbf{r}, \mathbf{p}) = \overbrace{\left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right)}^{\text{collisions}} n(t, \mathbf{r}, \mathbf{p})$$

$n(t, \mathbf{r}, \mathbf{p})$ - distribution function of hard probes

$$\mathbf{v} \equiv \frac{\mathbf{p}}{E_p}, \quad \nabla_p^i \equiv \frac{\partial}{\partial p_i}$$

$$X^{ij}(\mathbf{v}), Y^i(\mathbf{v}) \Rightarrow \begin{cases} \frac{dE}{dx} = -\frac{v^i}{v} Y^i(\mathbf{v}) & \text{collisional energy loss} \\ \hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

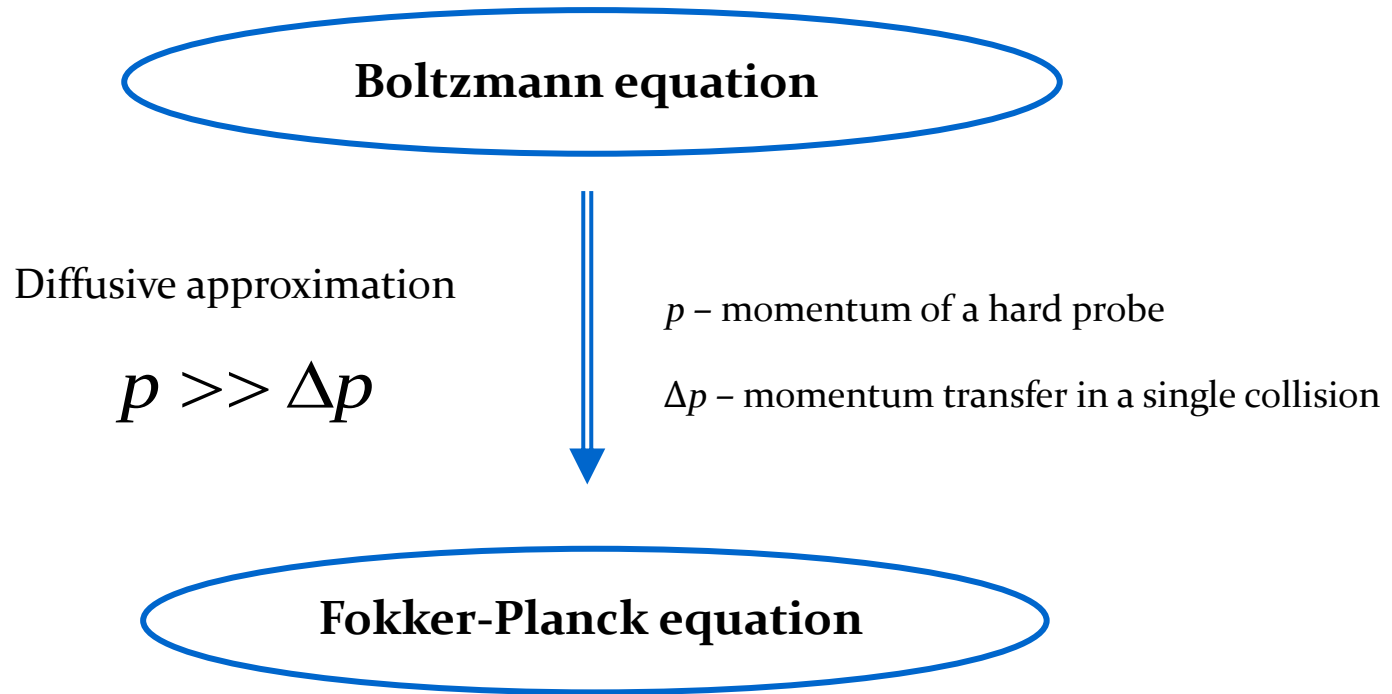
$$n(t, \mathbf{r}, \mathbf{p}) = n_{\text{eq}}(\mathbf{p}) \sim e^{-\frac{E_p}{T}}$$

solves FK equation

$\Leftrightarrow$

$$Y^j(\mathbf{v}) = \frac{v^i}{T} X^{ij}(\mathbf{v})$$

Origin of Fokker-Planck Equation



How to obtain a Fokker-Planck equation for glasma?

Apply the *quasilinear* method known in plasma physics.

Derivation of Fokker-Planck Equation

The dynamics is assumed to be dominated by strong classical fields.

Liouville (Vlasov) equation

$$p_\mu D^\mu Q(t, \mathbf{r}, \mathbf{p}) - \frac{g}{2} p^\mu \{F_{\mu\nu}(t, \mathbf{r}), \partial_p^\nu Q(t, \mathbf{r}, \mathbf{p})\} = 0$$

free streaming

mean-field force

$Q(t, \mathbf{r}, \mathbf{p})$ - exact distribution function of hard probes which is the $N_c \times N_c$ matrix

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \dots]$$

chromodynamic strength tensor

$$\{A, B\} \equiv AB + BA,$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

Derivation of Fokker-Planck Equation

Regular and fluctuating quantities

fluctuating part

$$Q(t, \mathbf{r}, \mathbf{p}) = \langle Q(t, \mathbf{r}, \mathbf{p}) \rangle + \delta Q(t, \mathbf{r}, \mathbf{p})$$

regular colorless part

$$\langle Q(t, \mathbf{r}, \mathbf{p}) \rangle = n(t, \mathbf{r}, \mathbf{p})I$$

$n(t, \mathbf{r}, \mathbf{p})$ - averaged distribution function

► $|n| \gg |\delta Q|, \quad |\nabla_p n| \gg |\nabla_p \delta Q|$

► $|\frac{\partial n}{\partial t}| \ll |\frac{\partial \delta Q}{\partial t}|, \quad |\nabla n| \ll |\nabla \delta Q|$

► $\langle \mathbf{E} \rangle = 0, \quad \langle \mathbf{B} \rangle = 0, \quad \mathbf{E}, \mathbf{B}, A^\mu \sim \delta Q$

Derivation of Fokker-Planck Equation

$$Q(t, \mathbf{r}, \mathbf{p}) = n(t, \mathbf{r}, \mathbf{p})I + \delta Q(t, \mathbf{r}, \mathbf{p})$$

Liouville (Vlasov) equation

Lorentz force

$$\mathbf{F} \equiv g(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$(D^0 + \mathbf{v} \cdot \mathbf{D})Q - \mathbf{F} \cdot \nabla_p Q = 0$$

ensemble averaging

$$\Downarrow \text{Tr}\langle \dots \rangle$$

collision term

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) n(t, \mathbf{r}, \mathbf{p}) = \frac{1}{N_c} \text{Tr} \langle \mathbf{F}(t, \mathbf{r}) \cdot \nabla_p \delta Q(t, \mathbf{r}, \mathbf{p}) \rangle$$

Fluctuations provide a collision term.

Derivation of Fokker-Planck Equation

How to compute the collision term?

$$C \equiv \frac{1}{N_c} \text{Tr} \langle \mathbf{F} \cdot \nabla_p \delta Q \rangle = ?$$

$$Q(t, \mathbf{r}, \mathbf{p}) = n(t, \mathbf{r}, \mathbf{p}) I + \delta Q(t, \mathbf{r}, \mathbf{p})$$

Liouville (Vlasov) equation

$$(D^0 + \mathbf{v} \cdot \mathbf{D}) Q - \mathbf{F} \cdot \nabla_p Q = 0$$

$$\begin{aligned} n &\gg |\delta Q| \\ |\nabla_p n| &\gg |\nabla_p \delta Q| \end{aligned}$$

linearization

$$\begin{aligned} \left| \frac{\partial n}{\partial t} \right| &\ll \left| \frac{\partial \delta Q}{\partial t} \right| \\ |\nabla n| &\ll |\nabla \delta Q| \end{aligned}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \delta Q = \mathbf{F} \cdot \nabla_p n$$

Derivation of Fokker-Planck Equation

Solution of the linearized transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \delta Q(t, \mathbf{r}, \mathbf{p}) = \mathbf{F}(t, \mathbf{r}) \cdot \nabla_p n(\mathbf{p})$$

initial value

$$\delta Q(t, \mathbf{r}, \mathbf{p}) = \int_0^t dt' \mathbf{F}(t', \mathbf{r} - \mathbf{v}(t - t')) \cdot \nabla_p n(\mathbf{p}) + \delta Q_0(\mathbf{r} - \mathbf{v}t, \mathbf{p})$$



$$C \equiv \frac{1}{N_c} \text{Tr} \langle \mathbf{F}(t, \mathbf{r}) \cdot \nabla_p \delta Q(t, \mathbf{r}, \mathbf{p}) \rangle = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right) n(\mathbf{p})$$

$$\left\{ \begin{array}{l} X^{ij}(\mathbf{v}) = \frac{1}{N_c} \int_0^t dt' \langle F^i(t, \mathbf{r}) F^j(t', \mathbf{r} - \mathbf{v}(t - t')) \rangle \\ Y^i(\mathbf{v}) = \frac{1}{N_c} \langle F^i(t, \mathbf{r}) \delta Q_0(\mathbf{r} - \mathbf{v}t, \mathbf{p}) \rangle \frac{1}{n(\mathbf{p})} \end{array} \right.$$

Fokker-Planck Equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) n(t, \mathbf{r}, \mathbf{p}) = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right) n(t, \mathbf{r}, \mathbf{p})$$

$$\begin{cases} X^{ij}(\mathbf{v}) = \frac{1}{N_c} \int_0^t dt' \langle F^i(t, \mathbf{r}) F^j(t', \mathbf{r} - \mathbf{v}(t-t')) \rangle \\ Y^j(\mathbf{v}) = \frac{\mathbf{v}^i}{T} X^{ij}(\mathbf{v}) \end{cases} \quad \mathbf{F} \equiv g(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

► The collision term is given by field correlators $\langle E^i E^j \rangle, \langle B^i E^j \rangle, \langle B^i B^j \rangle$

► Gauge covariance is lost due to the linearization!

To restore gauge covariance: $\langle E_a^i(t, \mathbf{r}) E_a^j(t', \mathbf{r}') \rangle \rightarrow \langle E_a^i(t, \mathbf{r}) \Omega_{ab}(t, \mathbf{r} | t', \mathbf{r}') E_b^j(t', \mathbf{r}') \rangle$

$$\Omega(t, \mathbf{r} | t', \mathbf{r}') \equiv P \exp \left[ig \int_{(t', \mathbf{r}')}^{(t, \mathbf{r})} ds_\mu A^\mu(s) \right]$$

Field correlators in Equilibrium QGP

space-time translational invariance

fluctuation spectrum

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega(t-t') - \mathbf{k}(\mathbf{r}-\mathbf{r}'))} \overbrace{\langle E_a^i E_b^j \rangle_{\omega, \mathbf{k}}}$$

$$\langle E_a^i E_b^j \rangle_{\omega, \mathbf{k}} = 2\delta^{ab} \frac{\omega^4}{e^{\beta|\omega|} - 1} \left[\frac{k^i k^j}{\mathbf{k}^2} \frac{\text{Im} \varepsilon_L(\omega, \mathbf{k})}{|\omega^2 \varepsilon_L(\omega, \mathbf{k})|^2} + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\text{Im} \varepsilon_T(\omega, \mathbf{k})}{|\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2|^2} \right]$$

$$\langle B_a^i B_b^j \rangle_{\omega, \mathbf{k}} = 2\delta^{ab} \frac{\omega^2 \mathbf{k}^2}{e^{\beta|\omega|} - 1} \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\text{Im} \varepsilon_T(\omega, \mathbf{k})}{|\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2|^2}$$

$$\langle B_a^i E_b^j \rangle_{\omega, \mathbf{k}} = \langle E_a^j B_b^i \rangle_{\omega, \mathbf{k}} = 2\delta^{ab} \frac{\omega^3}{e^{\beta|\omega|} - 1} \varepsilon^{imj} k^m \frac{\text{Im} \varepsilon_T(\omega, \mathbf{k})}{|\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2|^2}$$

$\varepsilon_{L,T}(\omega, \mathbf{k})$ - chromodielectric functions

Fokker-Planck Equation of Equilibrium QGP

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) n(t, \mathbf{r}, \mathbf{p}) = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right) n(t, \mathbf{r}, \mathbf{p})$$

Isotropy

$$X^{ij}(\mathbf{v}) \equiv X_L(\mathbf{v}) \frac{v^i v^j}{\mathbf{v}^2} + X_T(\mathbf{v}) \left(\delta^{ij} - \frac{v^i v^j}{\mathbf{v}^2} \right), \quad Y^j(\mathbf{v}) = \frac{v^j}{T} X^{ij}(\mathbf{v}) = \frac{v^j}{T} X_L(\mathbf{v})$$

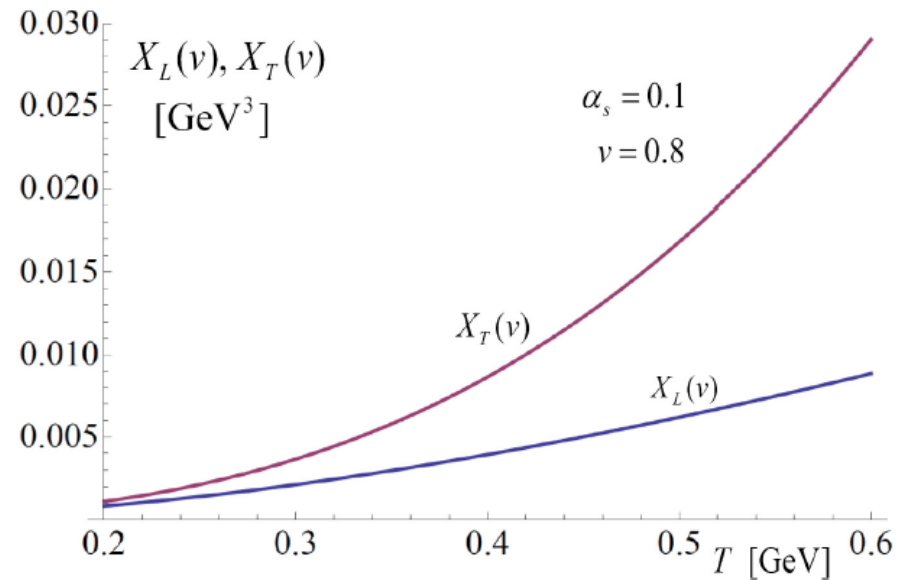
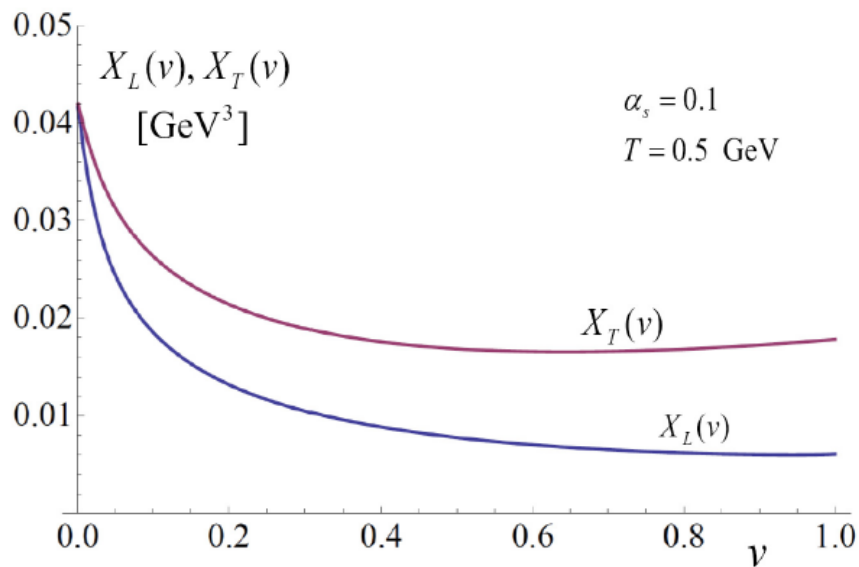
$$\begin{cases} X_L(\mathbf{v}) = \dots \\ X_T(\mathbf{v}) = \dots \end{cases}$$

For heavy quarks

$$v \ll 1, \quad g \ll 1$$

$$X_L(\mathbf{v}) = X_T(\mathbf{v}) \approx \frac{g^2 C_F}{12\pi} m_D^2 T \log \left(\frac{T}{m_D} \right) \quad C_F \equiv \frac{N_c^2 - 1}{2N_c}$$

Fokker-Planck Equation of Equilibrium QGP



Quantitative agreement with $X_L(v)$ & $X_T(v)$ obtained from the Boltzmann collision term by means of the diffusive approximation.

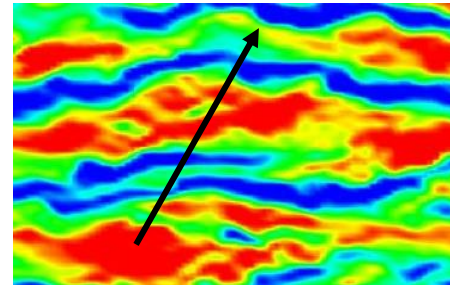
The standard FP equation is reproduced!

Transport of hard probes in glasma

$$X^{ij}(\mathbf{v}) = \frac{g}{N_c} \int_0^t dt' \left\{ \left\langle E^i(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{jkl} v^k \left\langle E^i(t, \mathbf{r}) B^l(t', \mathbf{r}') \right\rangle \right. \\ \left. + \varepsilon^{ikl} v^k \left\langle B^l(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{ikl} \varepsilon^{jmn} v^k v^m \left\langle B^l(t, \mathbf{r}) B^n(t', \mathbf{r}') \right\rangle \right\}$$

$$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t - t')$$

$$\left\{ \begin{array}{l} \hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) \\ \frac{dE}{dx} = - \frac{v^i v^j}{vT} X^{ij}(\mathbf{v}) \end{array} \right.$$

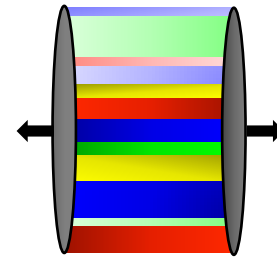


Rough estimate

Density of energy accumulated in the fields

►
$$\mathcal{E}_{\text{field}} = \frac{1}{2} \left(\langle E_a^i E_a^i \rangle + \langle B_a^i B_a^i \rangle \right)$$

E & B fields along the axis z



Density of energy released in a central collision

►
$$\mathcal{E}_{\text{coll}} = \frac{c_{\text{inel}} A \sqrt{s}}{\pi R_A^2 l}$$

$$\mathcal{E}_{\text{coll}} = \mathcal{E}_{\text{field}} \implies \left\{ \begin{array}{l} -\frac{dE}{dx} \sim (0 \div 10) \left[\frac{\text{GeV}}{\text{fm}} \right] \\ \hat{q} \sim 10 \left[\frac{\text{GeV}^2}{\text{fm}} \right] \end{array} \right.$$

$c_{\text{inel}} = 0.5, \quad A = 200, \quad \sqrt{s} = 5 \text{ TeV}, \quad l = 1 \text{ fm}$

Realistic calculations in proper time expansion

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time τ is treated as a small parameter $\tau \ll Q_s^{-1}$

Fully analytic approach

M. Carrington, A. Czajka & St. Mrówczyński, Nuclear Physics A **1001**, 121914 (2020)

M. Carrington, A. Czajka & St. Mrówczyński, European Physical Journal A **58**, 5 (2022)

M. Carrington, A. Czajka & St. Mrówczyński, Physical Review C **106**, 034904 (2022)

M. Carrington, A. Czajka & St. Mrówczyński, Physics Letters B **834**, 137464 (2022)

M. Carrington, A. Czajka & St. Mrówczyński, Physical Review C **106**, 034904 (2022)

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

M. Carrington & St. Mrówczyński, Acta Physica Polonica B **55**, 4-A3 (2024) [review article](#)

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Physical Review C **110**, 054903 (2024)

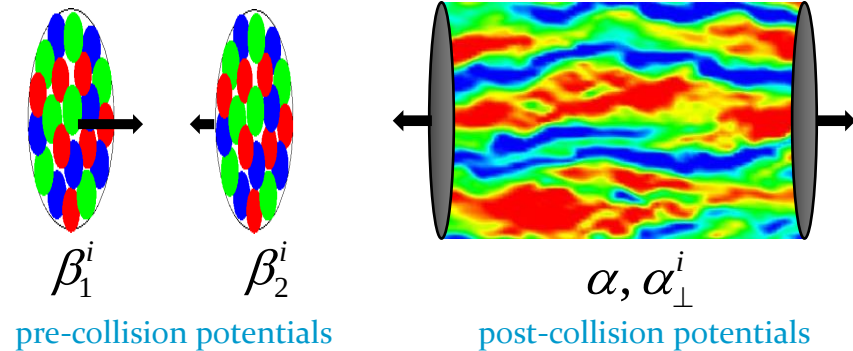
Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm \delta^{\mu\mp} \delta(x^\pm) \rho_{1,2}(\mathbf{x}_\perp)$$



Ansatz of gauge potentials

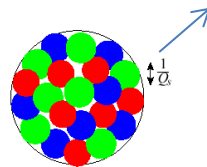
$$\begin{cases} A^+(x) = \Theta(x^+) \Theta(x^-) x^+ \alpha(\tau, \mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+) \Theta(x^-) x^- \alpha(\tau, \mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+) \Theta(x^-) \alpha_\perp^i(\tau, \mathbf{x}_\perp) \\ \quad + \Theta(-x^+) \Theta(x^-) \beta_1^i(\mathbf{x}_\perp) + \Theta(x^+) \Theta(-x^-) \beta_2^i(\mathbf{x}_\perp) \end{cases}$$

Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\frac{ig}{2} [\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+ A^- + x^- A^+ = 0$$



Proper time expansion

$$\alpha(\tau, \mathbf{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{(n)}(\mathbf{x}_\perp), \quad \alpha_\perp^i(\tau, \mathbf{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau \ll Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)} = \alpha_{\perp(n)}^i = 0 \quad \text{for } n=1, 3, 5, \dots$$

0th order - oboundary conditions

$$\begin{cases} \alpha_{(0)} = -\frac{ig}{2} [\beta_1^i, \beta_2^i] \\ \alpha_{\perp(0)}^i = \beta_1^i + \beta_2^i \end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases} \alpha_{(2)} = -\frac{ig}{16} [D^j, [D^j, [\beta_1^i, \beta_2^i]]] \\ \alpha_{\perp(2)}^i = \frac{ig}{4} \varepsilon^{zij} \varepsilon^{zkl} [D^j, [\beta_1^k, \beta_2^l]] \end{cases}$$

$$D^i \equiv \partial^i - ig(\beta_1^i + \beta_2^i)$$

Fully analytic approach!

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

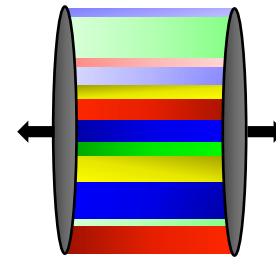
$$E^i = F^{i0}, \quad B^i = \frac{1}{2} \varepsilon^{ijk} F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0, 0, E), \quad \mathbf{B}_{(0)} = (0, 0, B)$$

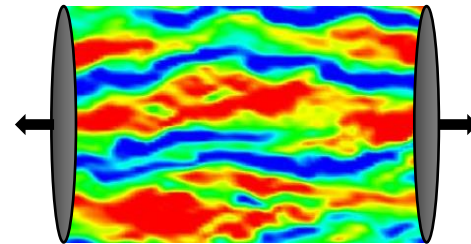
$$E_{(0)}^z(\mathbf{x}_\perp) = -ig[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)]$$

$$B_{(0)}^z(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta_1^i(\mathbf{x}_\perp), \beta_2^j(\mathbf{x}_\perp)]$$



E & B fields along the axis z

At higher orders transverse fields show up



Field correlators

The correlators

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle, \quad \langle E_a^i(t, \mathbf{r}) B_b^j(t', \mathbf{r}') \rangle, \quad \langle B_a^i(t, \mathbf{r}) B_b^j(t', \mathbf{r}') \rangle$$

are expressed through

$$\sum \partial^i \partial^j \langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \dots \beta_c^m(\mathbf{z}_\perp) \rangle$$

IR regulator $m = \Lambda_{\text{QCD}}$

In covariant gauge $\partial_\mu \beta^\mu = 0$

$$-\nabla^2 \beta^+(\mathbf{x}_\perp) = \rho(\mathbf{x}_\perp) \Rightarrow \beta^+(\mathbf{x}_\perp) = \frac{1}{2\pi} \int d^2 x'_\perp K_0(m |\mathbf{x}_\perp - \mathbf{x}'_\perp|) \rho(\mathbf{x}'_\perp)$$

The potentials are transformed from the covariant to light-cone gauge

Wick theorem

$$\langle \rho_a^k(\mathbf{x}_\perp) \rho_b^l(\mathbf{y}_\perp) \dots \rho_c^m(\mathbf{z}_\perp) \rangle = \sum \Pi \langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \rangle$$

Glasma graph approximation

$$\langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \dots \beta_c^m(\mathbf{z}_\perp) \rangle = \sum \Pi \langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \rangle = \sum \Pi B_{ab}^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

Basic two-point correlator

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv \langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \rangle = \int d^2 x'_\perp d^2 y'_\perp \cdots \langle \rho_a^i(\mathbf{x}'_\perp) \rho_b^j(\mathbf{y}'_\perp) \rangle$$

$$\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \rangle = g^2 \mu \delta^{ab} \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4} Q_s^2$$

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv \delta^{ab} \left(\delta^{ij} C_1(r) - \hat{r}^i \hat{r}^j C_2(r) \right)$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

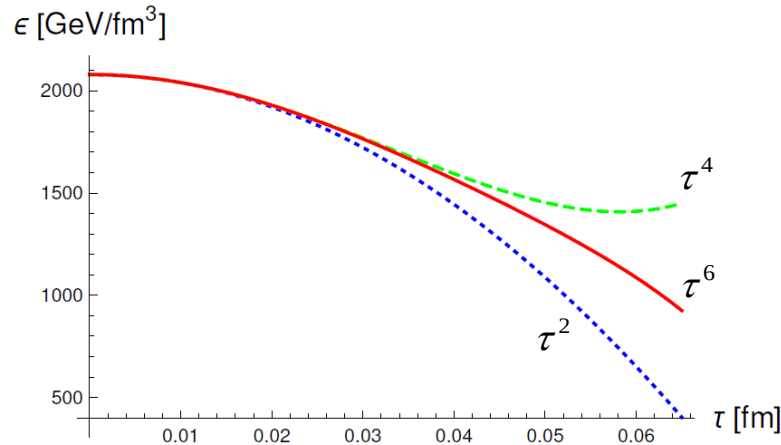
$$\left\{ \begin{array}{l} C_1(r) \equiv \frac{m^2 K_0(mr)}{g^2 N_c (mr K_1(mr) - 1)} \left\{ \exp \left[\frac{g^4 N_c \mu (mr K_1(mr) - 1)}{4\pi m^2} \right] - 1 \right\} \\ C_2(r) \equiv \frac{m^3 r K_1(mr)}{g^2 N_c (mr K_1(mr) - 1)} \left\{ \exp \left[\frac{g^4 N_c \mu (mr K_1(mr) - 1)}{4\pi m^2} \right] - 1 \right\} \end{array} \right. \quad \begin{array}{l} \approx \# \log(mr) \\ r \ll m^{-1} \end{array}$$

UV regularization required

$$r > Q_s^{-1}$$

Digression: $T^{\mu\nu}$ & one-point correlator

energy density



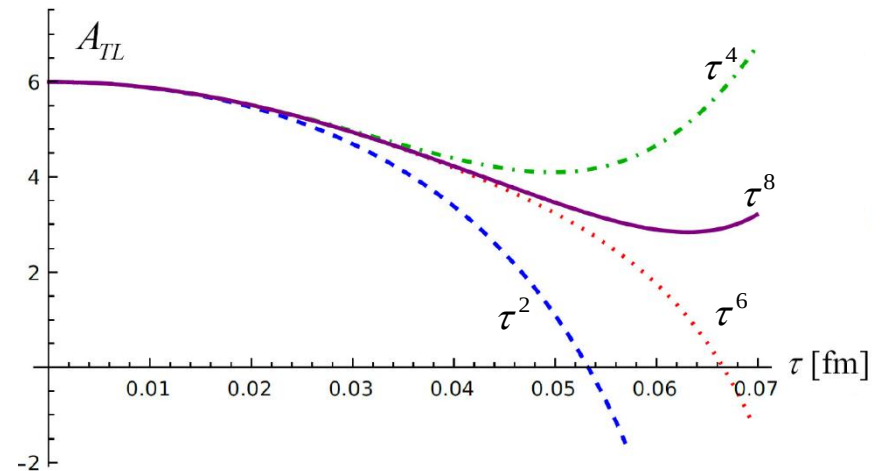
$\langle T^{\mu\nu}(\mathbf{x}_\perp) \rangle$ energy-momentum tensor

$$N_c = 3, \quad g = 1, \quad Q_s = 2 \text{ GeV}, \quad m = 0.2 \text{ GeV}$$

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L} \quad p_T \equiv \langle T^{xx} \rangle, \quad p_L \equiv \langle T^{zz} \rangle$$

$$\tau = 0 \Rightarrow p_T = -p_L = \varepsilon \Rightarrow A_{TL} = 6$$

anisotropy

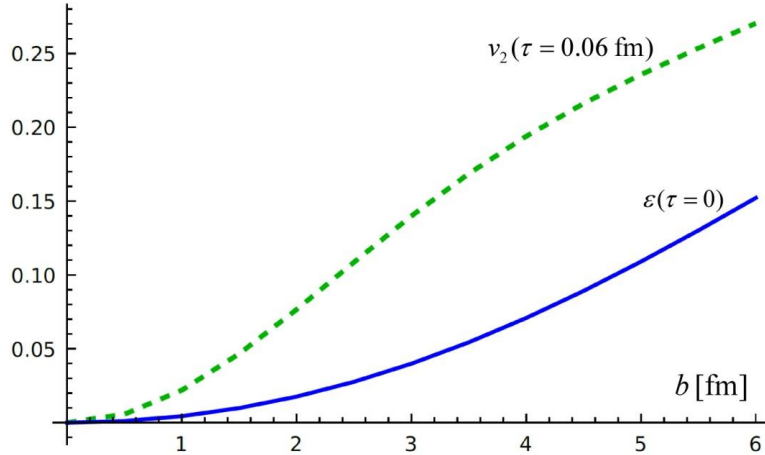


M. Carrington, A. Czajka & St. Mrówczyński, European Physical Journal A **58**, 5 (2022)

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Digression cont.

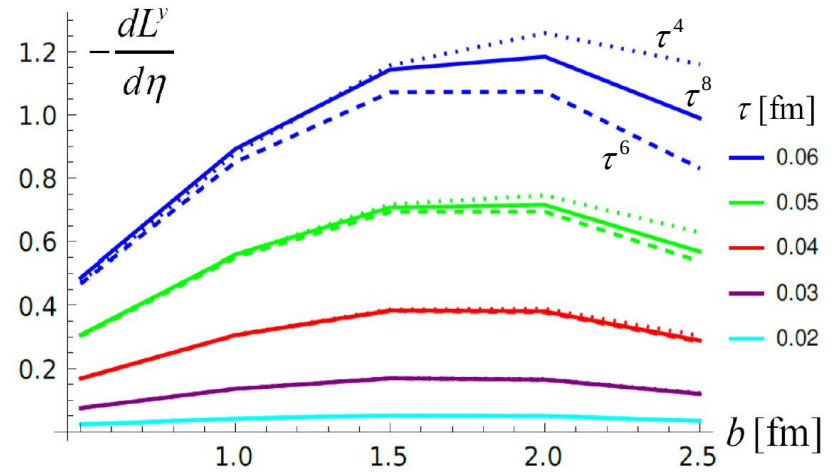
elliptic flow vs. eccentricity



$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}, \quad \epsilon = \frac{\int d^2R \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} T^{00}}$$

$$\frac{dL^\eta}{d\eta} = -\tau^2 \int d^2R R^x T^{\tau\eta}$$

angular momentum



Fokker-Planck Equation

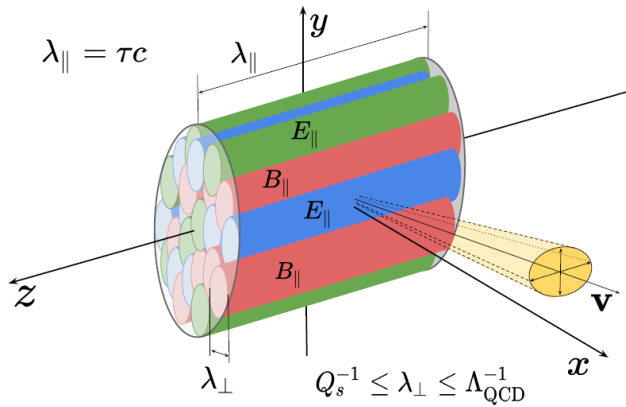
$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) n(t, \mathbf{r}, \mathbf{p}) = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v}) \right) n(t, \mathbf{r}, \mathbf{p})$$

$$\left\{ \begin{aligned} X^{ij}(\mathbf{v}) &= \frac{g}{N_c} \int_0^t dt' \left\{ \left\langle E^i(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{jkl} v^k \left\langle E^i(t, \mathbf{r}) B^l(t', \mathbf{r}') \right\rangle \right. \\ &\quad \left. + \varepsilon^{ikl} v^k \left\langle B^l(t, \mathbf{r}) E^j(t', \mathbf{r}') \right\rangle + \varepsilon^{ikl} \varepsilon^{jmn} v^k v^m \left\langle B^l(t, \mathbf{r}) B^n(t', \mathbf{r}') \right\rangle \right\} \\ Y^j(\mathbf{v}) &= \frac{v^j}{T} X^{ij}(\mathbf{v}) \end{aligned} \right.$$

The correlators are computed order by order.

$$\left\{ \begin{aligned} \hat{q} &= \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) \\ \frac{dE}{d\chi} &= - \frac{v^i}{v} Y^i(\mathbf{v}) \end{aligned} \right.$$

Hard probes in glasma - $\hat{q}$



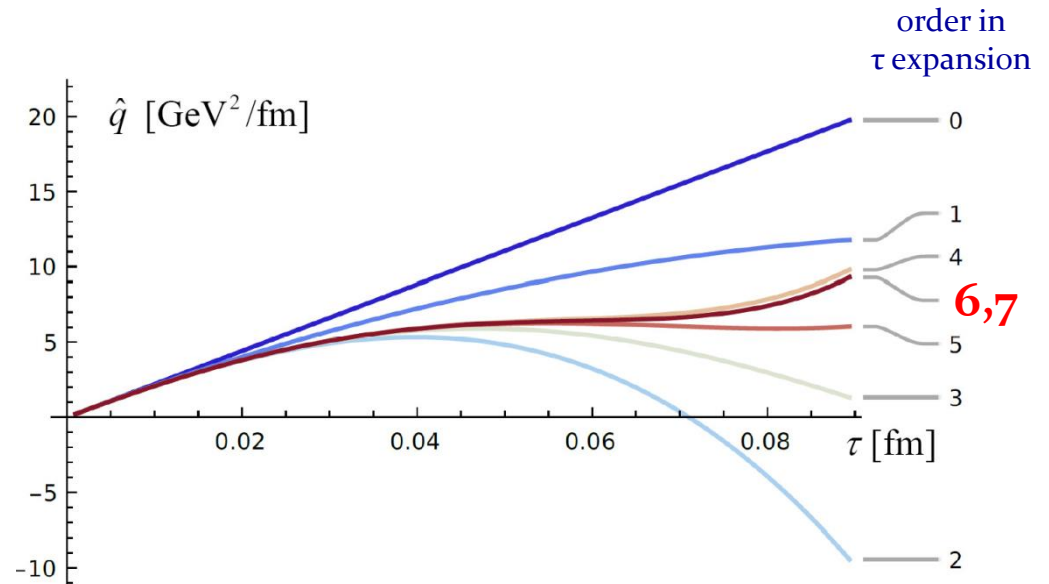
$$N_c = 3, \quad g = 1$$

$$Q_s = 2 \text{ GeV}$$

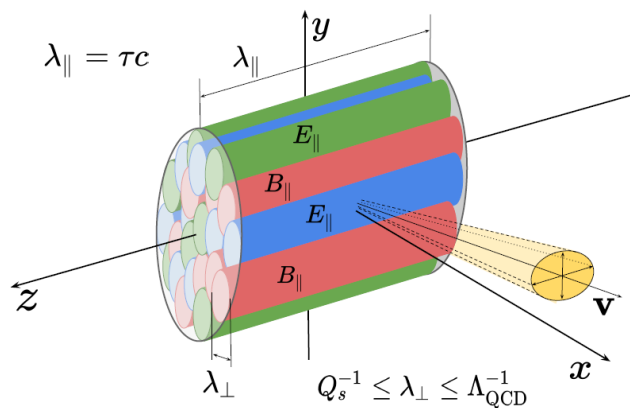
$$m = 0.2 \text{ GeV}$$

$$v = v_{\perp} = 1$$

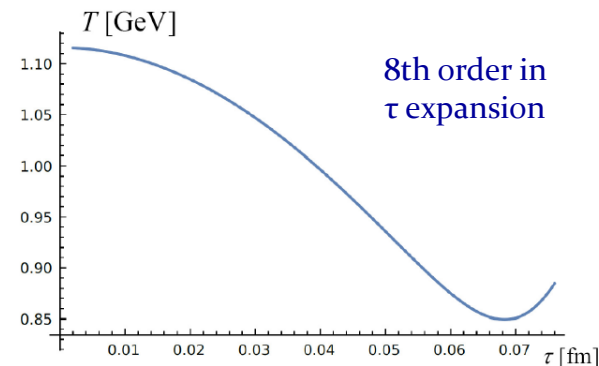
$$\hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v})$$



Hard probes in glasma - $\frac{dE}{dx}$



$$\frac{dE}{dx} = -\frac{1}{T} \frac{\mathbf{v}^i \mathbf{v}^i}{\mathbf{v}} X^{ij}(\mathbf{v})$$

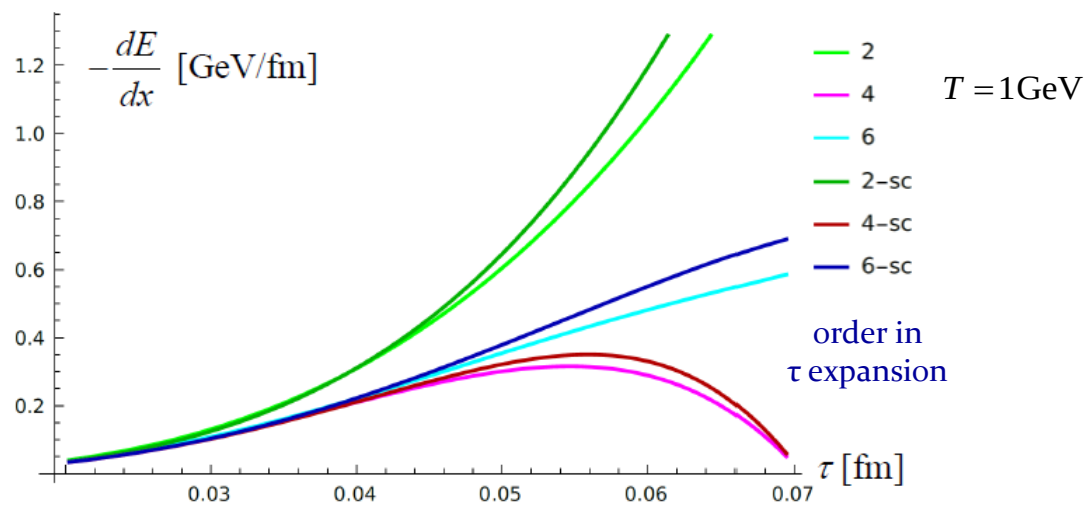


$$N_c = 3, \quad g = 1$$

$$Q_s = 2 \text{ GeV}$$

$$m = 0.2 \text{ GeV}$$

$$\mathbf{v} = \mathbf{v}_{\perp} = 1$$



Gauge dependence

$$\triangleright \langle E_a^i(t, \mathbf{r}) E_a^j(t', \mathbf{r}') \rangle \rightarrow \langle E_a^i(t, \mathbf{r}) \Omega_{ab}(t, \mathbf{r} | t', \mathbf{r}') E_b^j(t', \mathbf{r}') \rangle$$

$$\triangleright \Omega(t, \mathbf{r} | t', \mathbf{r}') \equiv P \exp \left[ig \int_{(t', \mathbf{r}')}^{(t, \mathbf{r})} ds_\mu A^\mu(s) \right]$$

$$\triangleright \frac{1}{N_c^2 - 1} \langle \Omega(t, \mathbf{r} | t - t', \mathbf{r}' - \mathbf{v} t') \rangle \approx 1 - \frac{g^2 N_c}{N_c^2 - 1} \langle A^\mu(x) A^\nu(x) \rangle \Delta s_\mu \Delta s_\nu = 1 - \frac{N_c Q_s^2}{8\pi} \left[\ln \left(\frac{Q_s^2}{m^2} + 1 \right) + \frac{Q_s^2}{Q_s^2 + m^2} \right]$$

$$\triangleright \frac{1}{N_c^2 - 1} \langle \Omega(t, \mathbf{r} | t - t', \mathbf{r}' - \mathbf{v} t') \rangle = 1 - \frac{N_c Q_s^2}{8\pi} \left[\ln \left(\frac{Q_s^2}{m^2} + 1 \right) + \frac{Q_s^2}{Q_s^2 + m^2} \right] \mathbf{v}^2 t'^2 \approx 0.84$$

$$N_c = 3, \quad g = 1, \quad Q_s = 2 \text{ GeV}, \quad m = 0.2 \text{ GeV}, \quad \mathbf{v} = 1, \quad t' = 0.06 \text{ fm}$$

Glasma impact on jet quenching

Glasma

$$\hat{q}_{\max} = 6 \text{ GeV}^2 / \text{fm}$$

$$t_{\max} = 0.06 \text{ fm}$$

Equilibrium QGP

$$\hat{q} = 3T^3$$

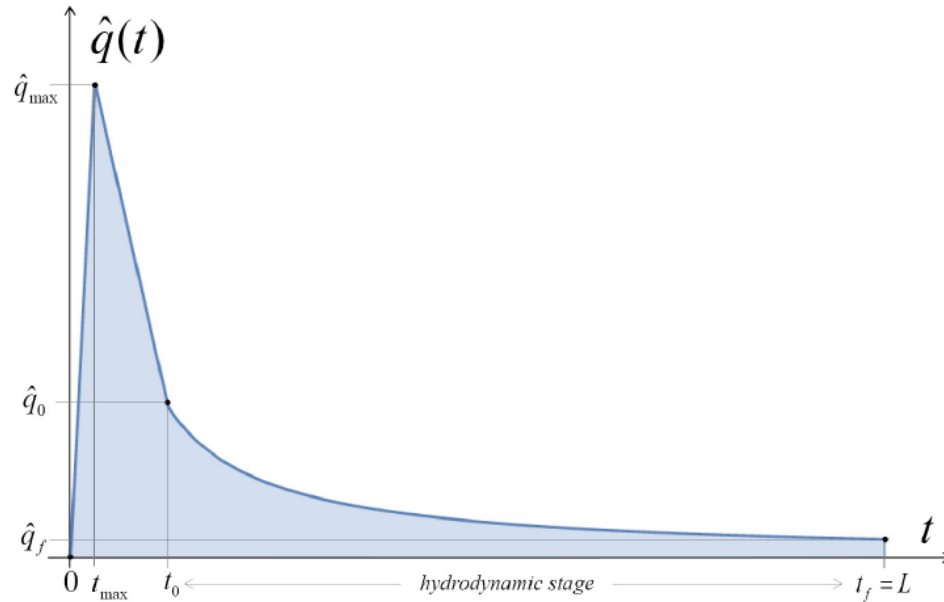
$$t_0 = 0.6 \text{ fm}$$

$$T_0 = 450 \text{ MeV}$$

$$\hat{q}_0 = 1.4 \text{ GeV}^2 / \text{fm}$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$$

$$L = 10 \text{ fm}$$



$$\Delta p_T^2 \Big|_{\text{non-eq}} = \int_0^{t_0} dt \hat{q}(t)$$

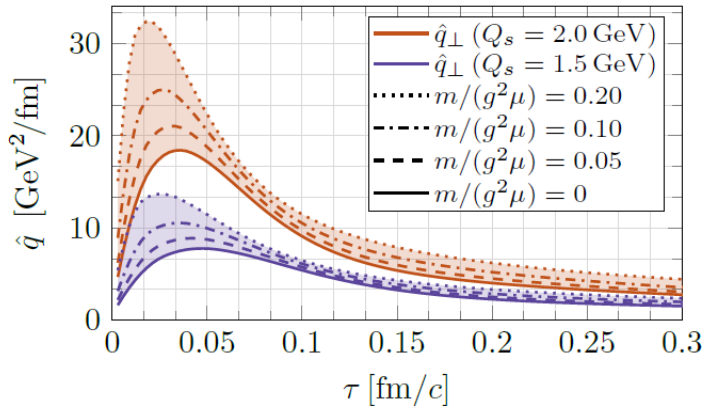
$$\Delta p_T^2 \Big|_{\text{eq}} = \int_{t_0}^L dt \hat{q}(t)$$

$$\frac{\Delta p_T^2 \Big|_{\text{non-eq}}}{\Delta p_T^2 \Big|_{\text{eq}}} = 0.93$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),

C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

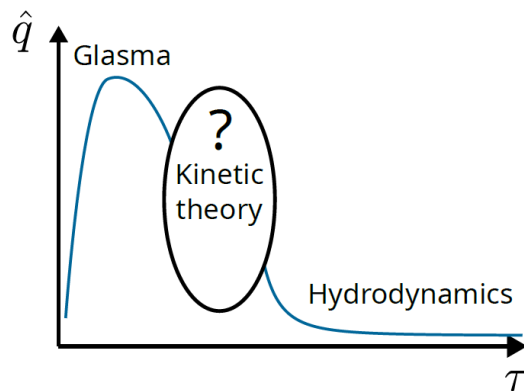
Glasma impact on jet quenching cont.



Full simulations of glasma

A. Ipp, D.I. Müller and D. Schuh, Phys. Lett. B **810**, 135810 (2020)

D. Avramescu, V. Băran, V. Greco, A. Ipp, D.I. Müller & M. Ruggieri, Phys. Rev. D **107**, 114021 (2023)



Kinetic theory interpolates between
glasma and equilibrium QGP

K. Boguslavski, A. Kurkela, T. Lappi, F. Lindenbauer & J. Peuron, Phys. Lett. B **850**, 138525 (2024)

Summary & Conclusions

- ▶ The Fokker-Planck equation of hard probes interacting with classical chromodynamic fields rather than with plasma constituents is derived.
- ▶ The known case of equilibrium plasma is reproduced.
- ▶ The field correlators are computed up to τ^7 or τ^8 .
- ▶ The momentum broadening and energy loss in the glasma are significantly bigger than in equilibrated QGP.
- ▶ In spite of its short lifetime the glasma provides a significant contribution to the jet quenching.