Instabilities Driven Equilibration of the Quark-Gluon Plasma

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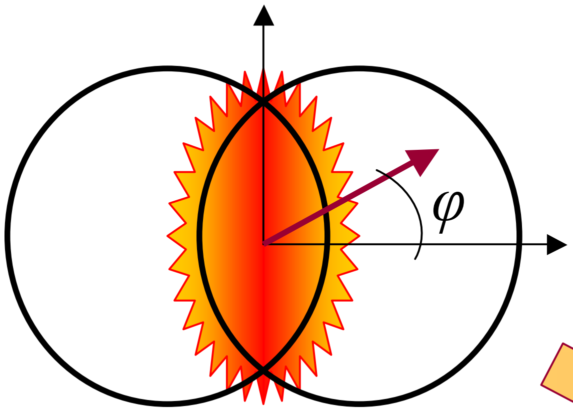
- Review focused on recent developments -

Evidence of the early stage equilibration

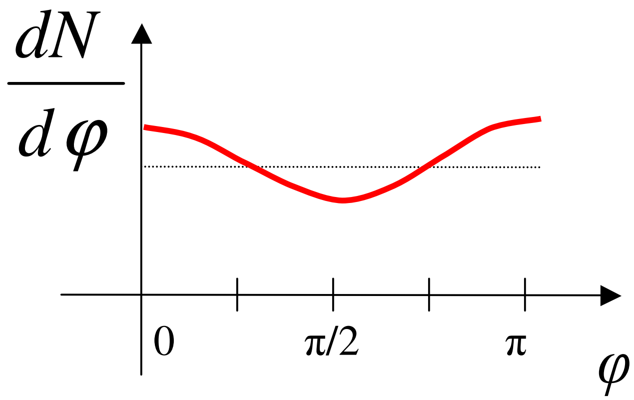
Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right)\mathbf{v} = -\frac{\nabla p}{\rho}$$

Hydrodynamic requires local thermodynamical equilibrium!



Equilibration is fast

$$v_2 \sim \varepsilon = \left\langle \frac{x^2 - y^2}{x^2 + y^2} \right\rangle$$

Eccentricity decays due to the free streaming!

$\varepsilon \searrow \;\Rightarrow\; v_2 \searrow$ ➡ $t_{eq} \leq 0.6 \;\; \mathrm{fm}/c$

time of equilibration

U. Heinz, AIP Conf. Proc.**739,** 163 (2004)

Collisions are too slow

Time scale of hard parton-parton scattering

hard scattering ~ momentum transfer of order of T

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

either single hard scattering or multiple soft scatterings

$$t_{\text{eq}} \approx t_{\text{hard}} \geq 2.6 \ \text{fm}/c$$

R. Baier, A.H. Mueller, D. Schiff & D.T. Son, Phys. Lett. **B539**, 46 (2002)

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

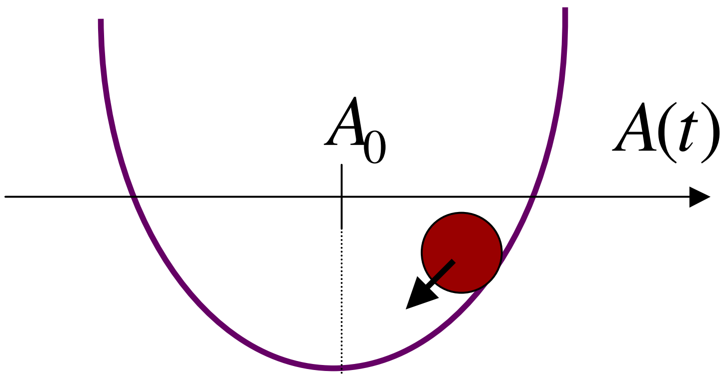
fluctuation

Instability

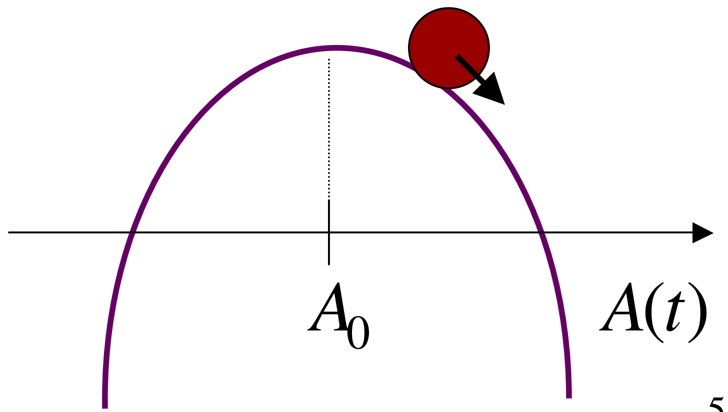
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Terminology

Plasma instabilities – interplay of particles and classical fields

Quantum Field Theory – no particles, no classical fields

$$p_{\text{hard}} \sim T$$

- particles – hard excitations, hard modes
- classical fields – highly populated soft excitations, soft modes

$$\sim 1/g^2 \qquad p_{\text{soft}} \sim gT$$

Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

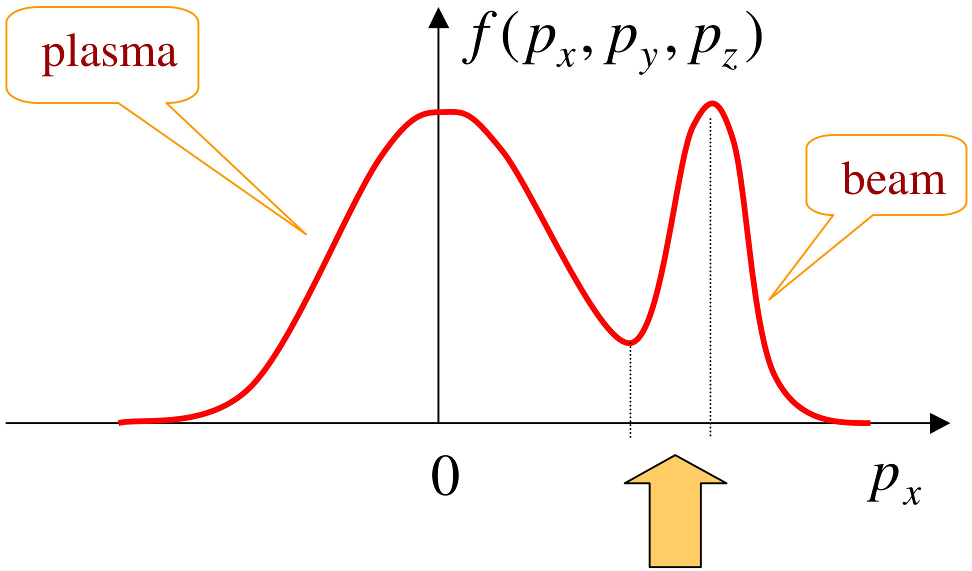
- **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}, \; \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$

- **transverse modes** – $\mathbf{k} \perp \mathbf{E}, \; \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

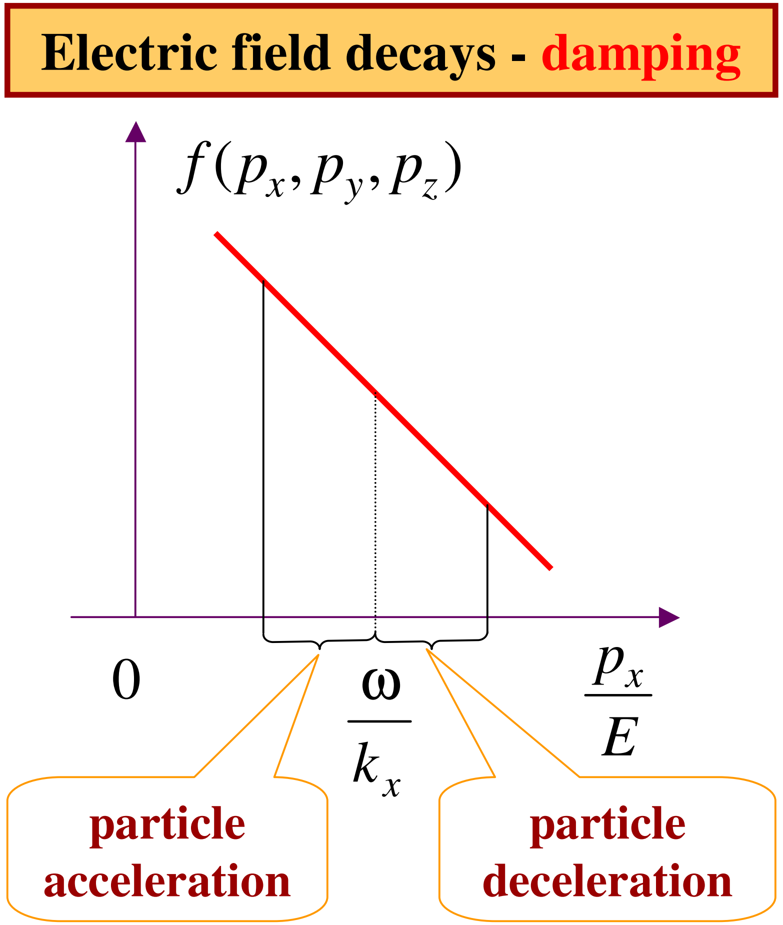
Logitudinal modes

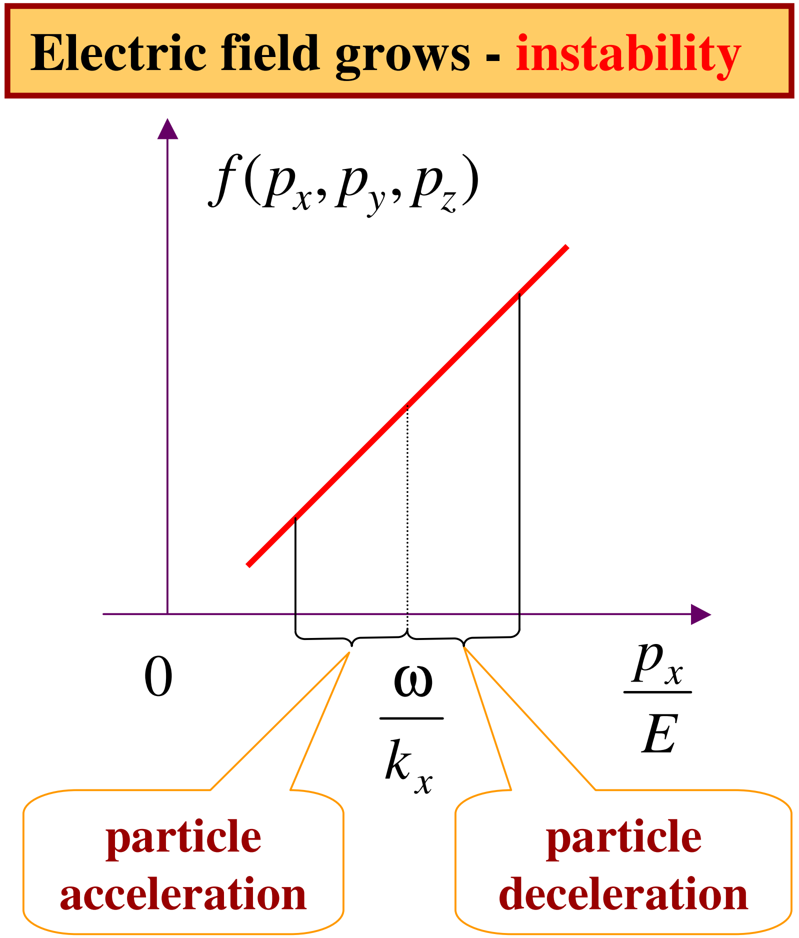
unstable configuration



Energy is transferred from particles to fields

Logitudinal modes



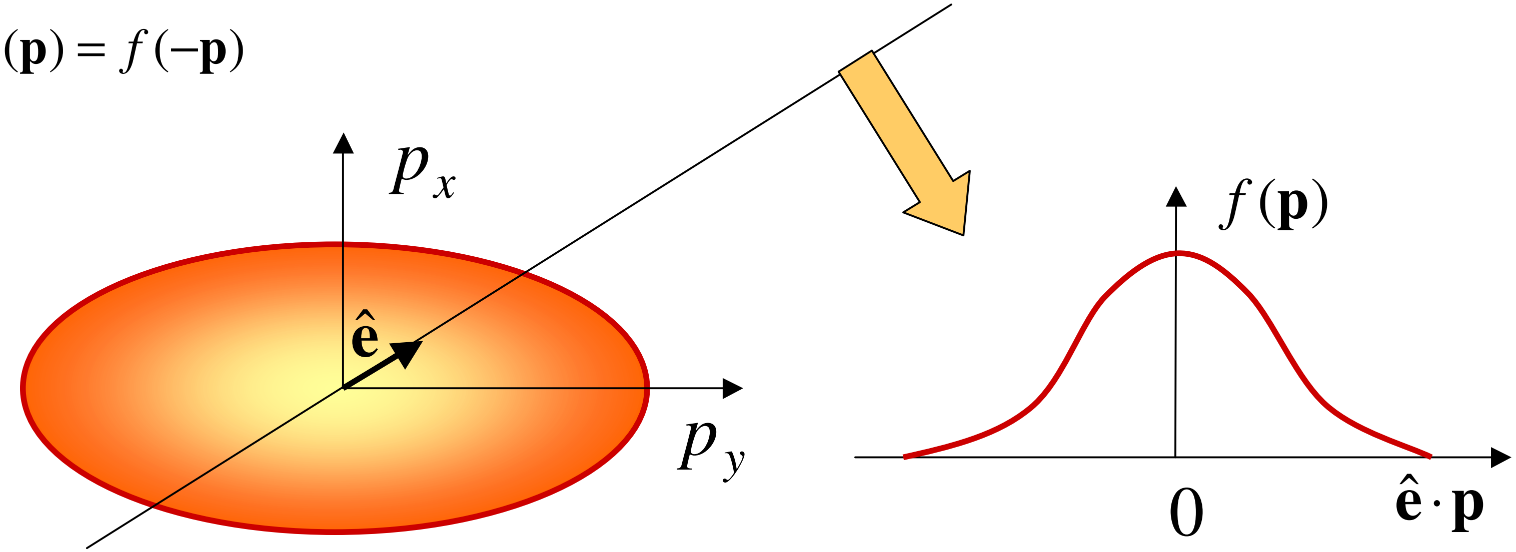


$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

Transverse modes

Unstable modes occur due to anisotropy of the momentum distribution

$f(\mathbf{p}) = f(-\mathbf{p})$

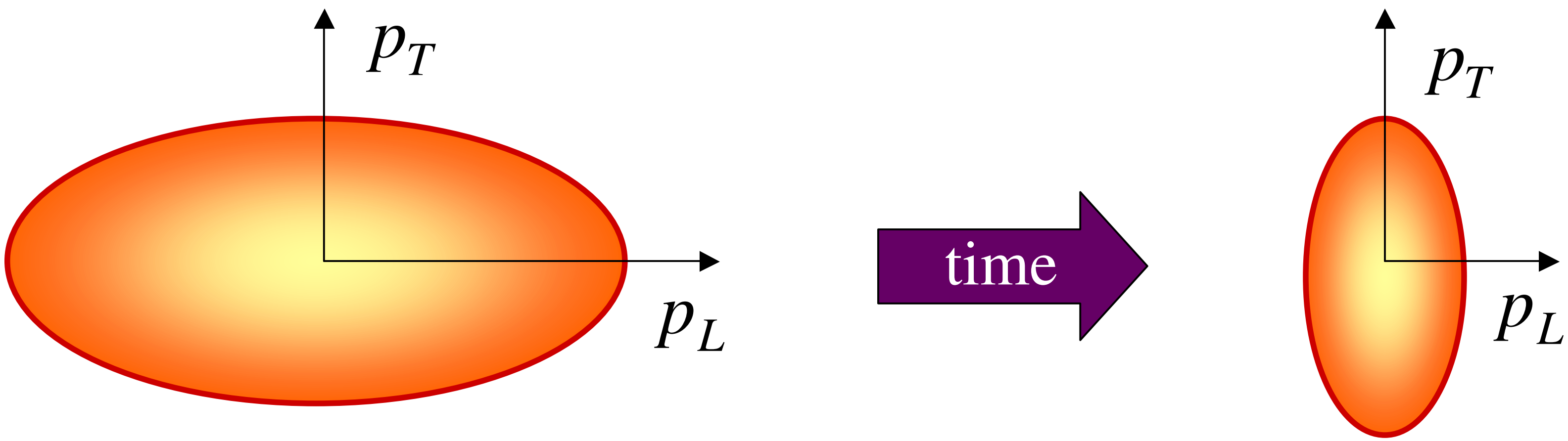


Momentum distribution distribution can monotonously decrease in every direction

Transverse modes are relevant for relativistic nuclear collisions!

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic



CM after 1-st collisions

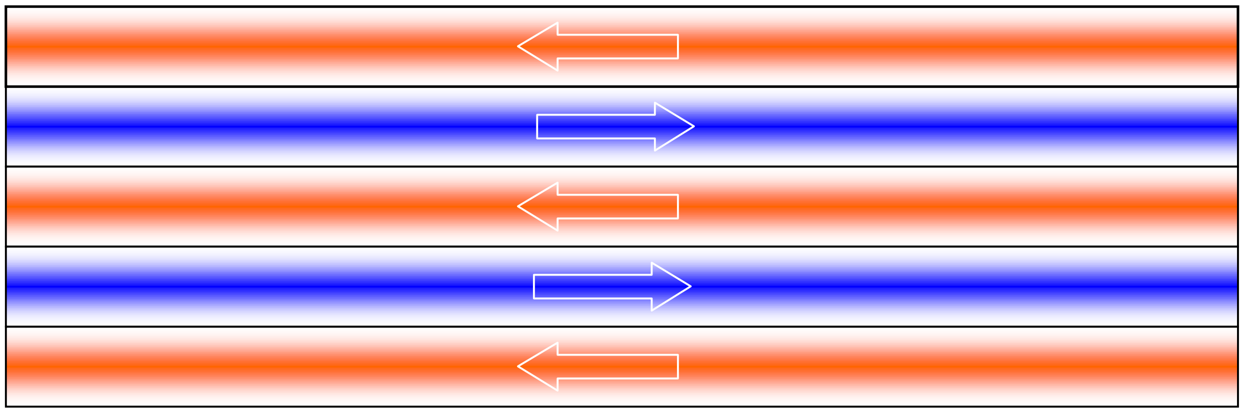
local rest frame

Seeds of instability

$\left\langle j_a^{\mu}(x) \right\rangle = 0$ **but current fluctuations are finite**

$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$

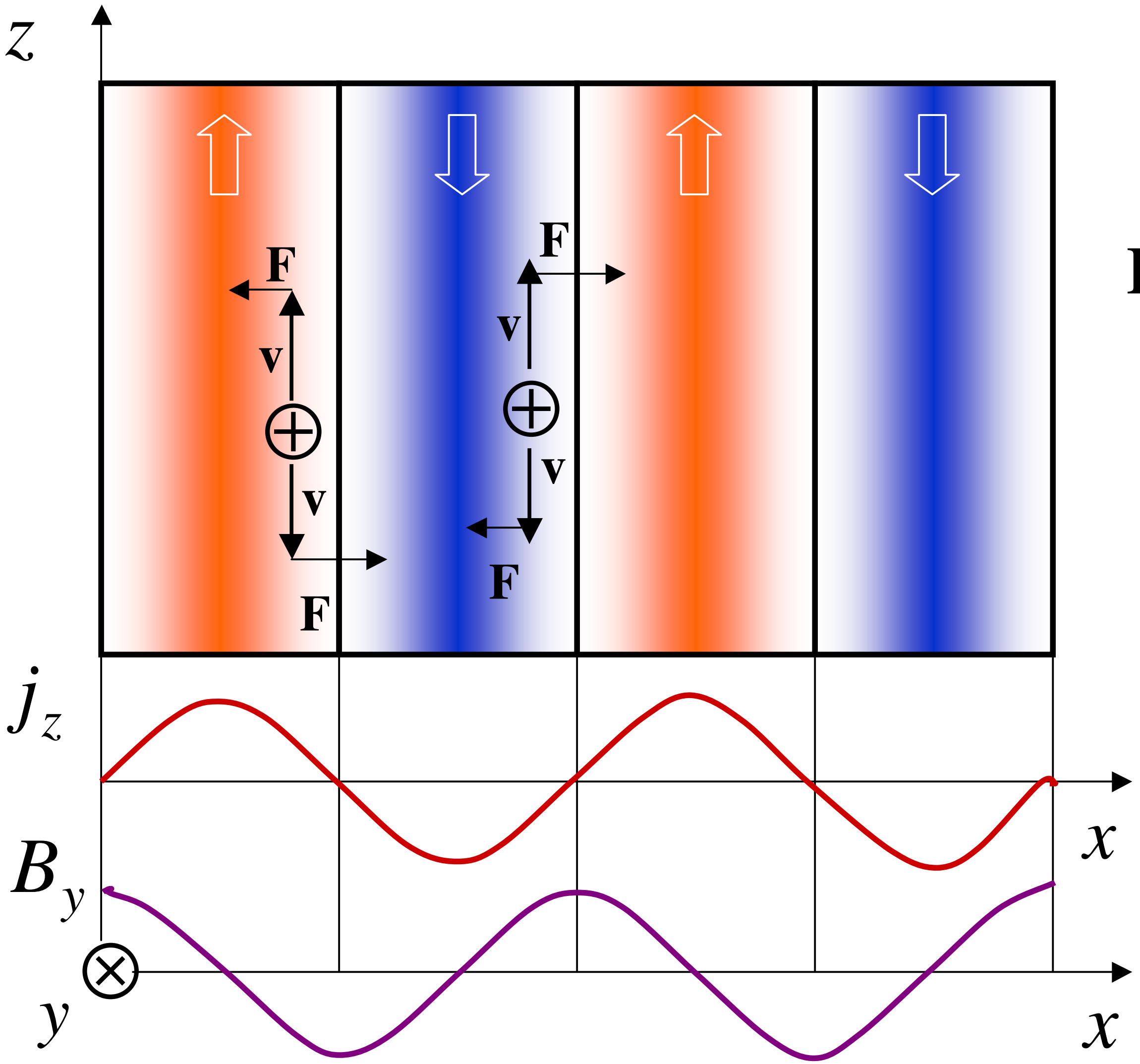


Direction of the momentum surplus

Mechanism of filamentation

Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j}$$



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Dispersion equation

Equation of motion of chromodynamic field A^μ in momentum space

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0$$

gluon self-energy

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with Im$\omega > 0$ $\Rightarrow$ $A^\mu(x) \sim e^{\mathrm{Im}\,\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. **How to get it?**

Transport theory – distribution functions

Distribution functions of quarks $Q(p,x)$ and antiquarks $\overline{Q}(p,x)$

are gauge dependent $N_c \times N_c$ matrices

The gauge transformation:

$$Q(p,x) \rightarrow U(x)\,Q(p,x)\,U^{-1}(x)$$

Distribution function of gluons $G(p,x)$ is $(N_c^2-1)\times(N_c^2-1)$ matrix

Transport theory – transport equations

fundamental:

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C \quad \text{quarks}$$

$$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = \overline{C} \quad \text{antiquarks}$$

adjoint:

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu},(x)\partial_p^\nu G\} = C_g \quad \text{gluons}$$

free streaming — mean-field force — collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, ...], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$ mean-field generation

collisionless limit: $C = \overline{C} = C_g = 0$

Transport theory - linearization

fluctuation

$$Q(p,x) = Q_0(p) + \delta Q(p,x)$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| >> |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p)| >> |\partial_p^\mu \delta Q(p,x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$
$$p_\mu D^\mu \delta \overline{Q}(p,x) + g p^\mu F_{\mu\nu}(x) \partial_p^\nu \overline{Q}_0(p) = 0$$
$$p_\mu \mathcal{D}^\mu \delta G(p,x) - g p^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu G_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p,x) = g \int d^4x' \Delta_p(x-x')\, p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p)$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$j^\mu[\delta Q, \delta\overline{Q}, \delta G]$$

$$j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$$

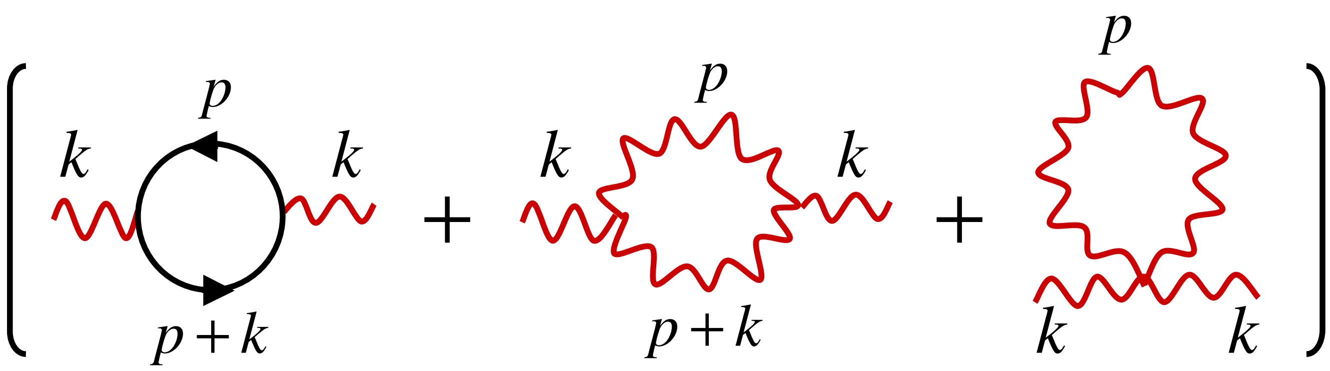
$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E} \Big[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+} \Big] \frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$\Pi^{\mu\nu}(k) =$



Hard loop approximation: $k^{\mu} << p^{\mu}$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}}{E} \left[g^{\nu\lambda} - \frac{p^{\nu} k^{\lambda}}{p^{\sigma} k_{\sigma} + i0^{+}} \right] \frac{\partial f(\mathbf{p})}{\partial p^{\lambda}}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_{\mu}\Pi^{\mu\nu}(k) = 0$$

St. M. & M. Thoma, Phys. Rev. C **62**, 036011 (2000)

Chromo-hydrodynamic approach

Transport equation of quark distribution function $Q(p,x)$

$$p_\mu D^\mu Q(p,x) - \frac{g}{2} p^\mu \{ F_{\mu\nu}, \partial^\nu_p Q(p,x) \} = 0$$

$$\int dP$$

Taking into account antiquarks and gluons is straightforward

$$dP \equiv \frac{d^4p}{(2\pi)^3} 2\Theta(p_0)\,\delta(p^2)$$

Covariant continuity

$$D_\mu n^\mu(x) = 0$$

$$n^\mu(x) \equiv \int dP\, p^\mu Q(p,x)$$

St. M. & C. Manuel, hep-ph/0606276

Chromo-hydrodynamic approach cont.

$$p_\mu D^\mu Q(p,x) - \frac{g}{2} p^\mu \{ F_{\mu\nu}, \partial_p^\nu Q(p,x) \} = 0$$

$$\int dP \, p^\mu \; \boxed{}$$

$$D_\mu T^{\mu\nu}(x) - \frac{g}{2} \{ F_{\mu\nu}, n^\mu(x) \} = 0$$

$$T^{\mu\nu}(x) \equiv \int dP \, p^\mu p^\nu Q(p,x)$$

$$p^2 = 0$$

$$T_\mu^\mu(x) = 0$$

Chromo-hydrodynamic equations

$$D_\mu n^\mu(x) = 0$$

$$D_\mu T^{\mu\nu}(x) - \frac{g}{2}\{F^{\mu\nu}, n_\mu(x)\} = 0$$

Postulated form of $n^\mu(x)$ and $T^{\mu\nu}(x)$:

$$n^\mu(x) = n(x)\, u^\mu(x)$$

$$T^{\mu\nu}(x) = \frac{1}{2}(\varepsilon(x) + p(x))\{u^\mu(x), u^\mu(x)\} - p(x)\, g^{\mu\nu}$$

$n(x)$, $\varepsilon(x)$, $p(x)$, $u^\mu(x)$ matrices! $\quad u^\mu(x)\, u_\mu(x) = 1$

To close the system of equations: $\nabla p = 0$ or $\varepsilon = 3p \Leftarrow T^\mu_\mu = 0$

Linear response approximation

Small perturbation of the space-time homogeneous & colorless state

$$n(x) = \tilde{n} + \delta n(x), \quad \varepsilon(x) = \tilde{\varepsilon} + \delta\varepsilon(x),$$ $$p(x) = \tilde{p} + \delta p(x), \quad u^{\mu}(x) = \tilde{u}^{\mu} + \delta u^{\mu}(x)$$
$\tilde{n}, \tilde{\varepsilon}, \tilde{p}, \tilde{u}^{\mu}$ unit matrices in color space
$$\tilde{n} >> \delta n, \quad \tilde{\varepsilon} >> \delta\varepsilon, \quad \tilde{p} >> \delta p, \quad \tilde{u}^{\mu} >> \delta u^{\mu}$$
$$F^{\mu\nu} \sim A^{\mu} \sim \delta n$$

Solutions of the linearized equations

- $D^{\mu} \rightarrow \partial^{\mu}$ full linearization $A^{\mu} \sim \delta n$
- Fourier transformations
- $\partial^{\nu}\delta p \approx 0$

continuity	$k_{\mu}\tilde{u}^{\mu}\,\delta n(k) + \tilde{n}\,k_{\mu}\delta u^{\mu}(k) = 0$
Euler	$i(\tilde{\varepsilon} + \tilde{p})\,\tilde{u}^{\mu}k_{\mu}\delta u^{\nu}(k) - g\tilde{n}\,\tilde{u}_{\mu}F^{\mu\nu}(k) = 0$

Solutions

$$\delta n(k) = ig\,\frac{\tilde{n}^2}{\tilde{\varepsilon} + \tilde{p}}\,\frac{\tilde{u}_{\nu}\,k_{\mu}}{(\tilde{u}\cdot k)^2}\,F^{\mu\nu}(k)$$

$$\delta u^{\nu}(k) = ig\,\frac{\tilde{n}}{\tilde{\varepsilon} + \tilde{p}}\,\frac{\tilde{u}_{\mu}}{\tilde{u}\cdot k}\,F^{\mu\nu}(k)$$

Color current & polarization tensor

$$j^{\mu}(x) = -\frac{g}{2}\left(n(x)u^{\mu}(x) - \frac{1}{N_c}\mathrm{Tr}[\,n(x)u^{\mu}(x)]\right)$$

$$j^{\mu}(x) = \tilde{j}^{\mu} + \delta j^{\mu}(x), \qquad \tilde{j}^{\mu} = 0$$

$$\delta j^{\mu}(x) = -\frac{g}{2}\left(\tilde{n}\,\delta u^{\mu}(x) + \tilde{u}^{\mu}\delta n(x)\right)$$

$\mathrm{Tr}[F^{\mu\nu}] = 0$

polarization tensor
$$\Pi^{\mu\nu}(x, y) = -\frac{\delta j^{\mu}(x)}{\delta A_{\nu}(y)}$$

Polarization tensor

$$\Pi^{\mu\nu}(k) = -\frac{g^2}{2}\frac{\tilde{n}^2}{\tilde{\varepsilon}+\tilde{p}}\frac{(\tilde{u}\cdot k)(\tilde{u}^\mu k^\nu + \tilde{u}^\nu k^\mu) - k^2\tilde{u}^\mu\tilde{u}^\nu - (\tilde{u}\cdot k)^2 g^{\mu\nu}}{(\tilde{u}\cdot k)^2}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)\,, \qquad k_\mu \Pi^{\mu\nu}(k) = 0$$

From one- to multi-stream system

There are several streams in the plasma

Transport equation of quark distribution function of stream α

$$p_\mu D^\mu Q_\alpha(p,x) - \frac{g}{2} p^\mu \{F_{\mu\nu}, \partial_p^\nu Q_\alpha(p,x)\} = 0$$

•
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All previous steps for each stream

$$D_\mu n_\alpha^\mu(x) = 0$$

$$D_\mu T_\alpha^{\mu\nu}(x) - \frac{g}{2}\{F_\mu{}^\nu, n_\alpha^\mu(x)\} = 0$$

$$D_\mu F^{\mu\nu}(x) = \sum_\alpha j_\alpha^\nu(x)$$

$$n_\alpha^\mu(x) = n_\alpha(x)\, u_\alpha^\mu(x)$$

Streams interact via mean field

Polarization tensor for multi-stream system

$$\Pi^{\mu\nu}(k) = -\frac{g^2}{2}\sum_{\alpha}\frac{\tilde{n}_{\alpha}^2}{\tilde{\varepsilon}_{\alpha}+\tilde{p}_{\alpha}}\frac{(\tilde{u}_{\alpha}\cdot k)(\tilde{u}_{\alpha}^{\mu}k^{\nu}+\tilde{u}_{\alpha}^{\nu}k^{\mu})-k^2\tilde{u}_{\alpha}^{\mu}\tilde{u}_{\alpha}^{\nu}-(\tilde{u}_{\alpha}\cdot k)^2 g^{\mu\nu}}{(\tilde{u}_{\alpha}\cdot k)^2}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)\,, \qquad k_{\mu}\Pi^{\mu\nu}(k) = 0$$

Connection with the kinetic theory

$$f(p) = \sum_{\alpha}\tilde{n}_{\alpha}\,\tilde{u}_{\alpha}^{0}\delta^{(3)}(\mathbf{p}-m_{\alpha}\tilde{\mathbf{u}}_{\alpha}) \qquad m_{\alpha} \equiv \frac{\tilde{\varepsilon}_{\alpha}+\tilde{p}_{\alpha}}{\tilde{n}_{\alpha}}$$

Effect of pressure gradients

The set of fluid equations is closed by the relation $p_\alpha(x) = \frac{1}{3}\varepsilon_\alpha(x)$

$$\Pi^{\mu\nu}(k) = -\frac{3g^2}{8}\sum_\alpha \frac{\tilde{n}_\alpha^2}{\tilde{\varepsilon}_\alpha}\Big[\frac{(\tilde{u}_\alpha\cdot k)(\tilde{u}_\alpha^\mu k^\nu + \tilde{u}_\alpha^\nu k^\mu) - k^2\tilde{u}_\alpha^\mu\tilde{u}_\alpha^\nu - (\tilde{u}_\alpha\cdot k)^2 g^{\mu\nu}}{(\tilde{u}_\alpha\cdot k)^2}$$

$$-\frac{(\tilde{u}_\alpha\cdot k)k^2(\tilde{u}_\alpha^\mu k^\nu + \tilde{u}_\alpha^\nu k^\mu) - (\tilde{u}_\alpha\cdot k)^2 k^\mu k^\nu - k^4\tilde{u}_\alpha^\mu\tilde{u}_\alpha^\nu}{k^2 + 2(\tilde{u}_\alpha\cdot k)^2}\Big]$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)\,, \qquad k_\mu\Pi^{\mu\nu}(k) = 0$$

Dispersion equation

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(k)$$ **chromodielectric tensor**

$$k^\mu \equiv (\omega, \mathbf{k})$$

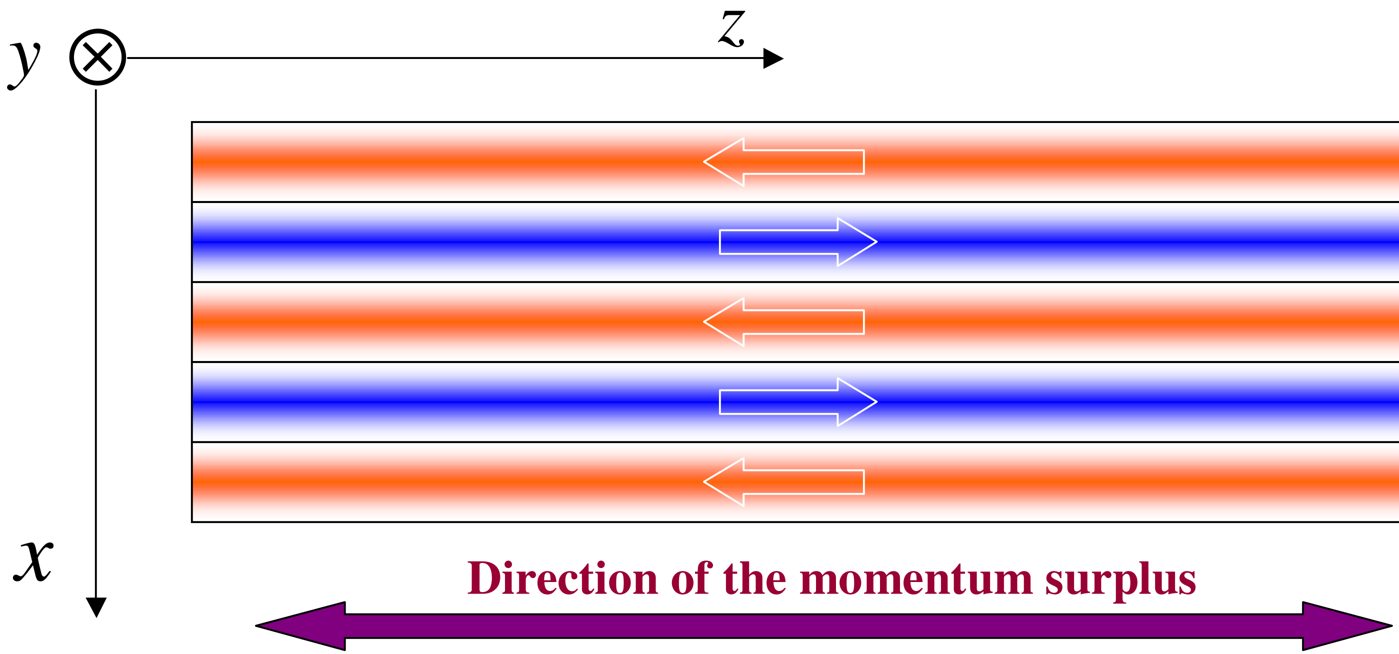
Dispersion equation

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

$$\mathbf{v} \equiv \mathbf{p}/E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0,0,j), \quad \mathbf{E} = (0,0,E), \quad \mathbf{k} = (k,0,0)$$

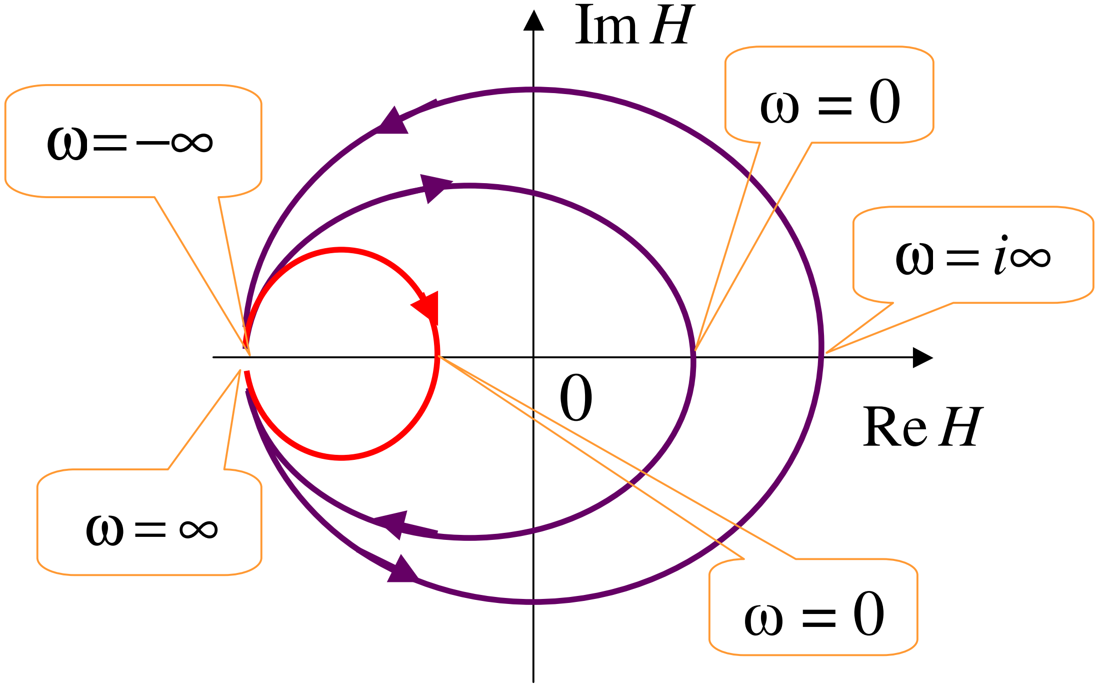
Dispersion equation

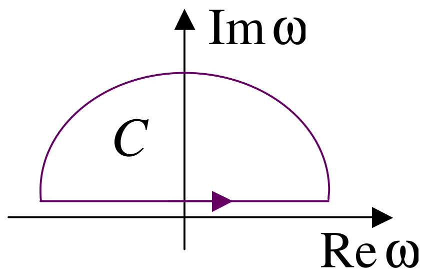
$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

Existence of unstable modes – Penrose criterion

$$H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)$$

$$\oint_C \frac{d\omega}{2\pi i} \frac{1}{H(\omega)} \frac{dH(\omega)}{d\omega} = \begin{cases} \oint_C \frac{d\omega}{2\pi i} \frac{d \ln H(\omega)}{d\omega} = \ln H(\omega)\Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{cases}$$





There are unstable modes if

$$H(\omega = 0) < 0$$

Anisotropy!

Unstable solutions

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho\sigma_\perp^4}{\sigma_\parallel} \frac{1}{(p_\perp^2 + \sigma_\perp^2)^3} e^{-\frac{p_\parallel^2}{2\sigma_\parallel^2}}$$

$$\rho = 6\ \mathrm{fm}^{-3}$$

$$\alpha_s = g^2/4\pi = 0.3$$

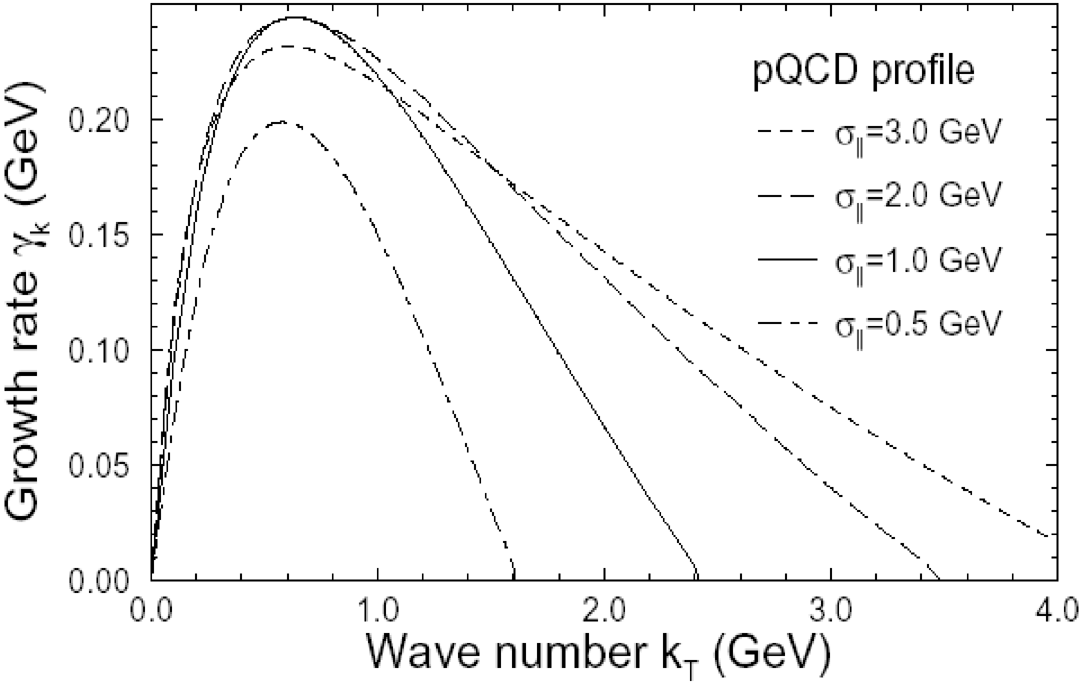
$$\sigma_\perp = 0.3\ \mathrm{GeV}$$

$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

solution

$$\omega(k) = \pm i\,\gamma_k$$

$$0 < \gamma_k \in \Re$$



J. Randrup & St. M., Phys. Rev. C **68**, 034909 (2003)

Hard-Loop dynamics

Soft fields in the passive background of hard particles

Braaten-Pisarski action generalized to anisotropic momentum distribution:

$$L_{\text{eff}} = \frac{g^2}{2} \int \frac{d^3 p}{(2\pi)^3} \Big[f(\mathbf{p}) F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p \cdot D)^2} \right)_{ab} F^{b\mu}_{\rho}(x) + i \frac{C_F}{3} \tilde{f}(\mathbf{p}) \psi(x) \frac{p \cdot \gamma}{p \cdot D} \psi(x) \Big]$$

$$k_\mu \Pi^{\mu\nu}(k) = 0, \qquad k_\mu \Lambda^\mu(p,q,k) = \Sigma(p) + \Sigma(q)$$

St. M., A. Rebhan & M. Strickland, Phys. Rev. D **74**, 025004 (2004)

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t, z)$$

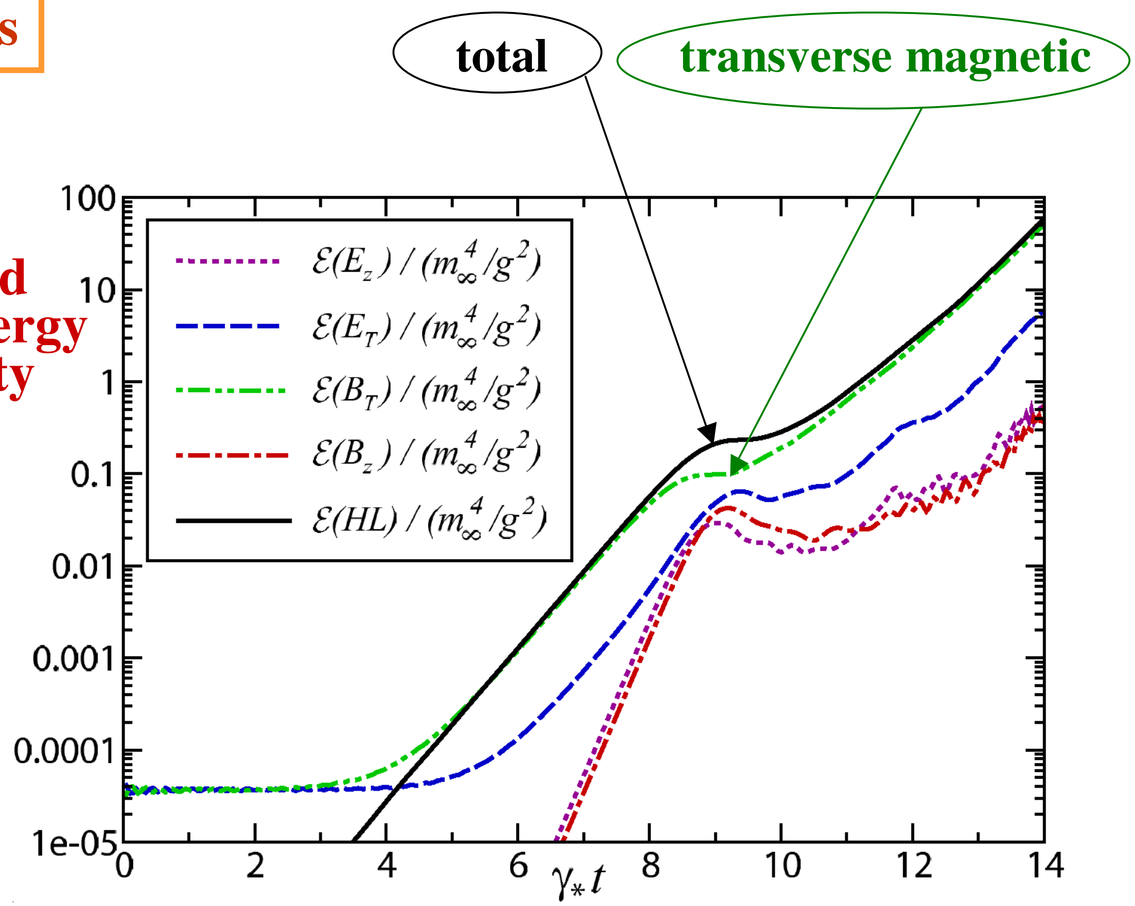
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\mathrm{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\, p^2 \frac{df_{\mathrm{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

Scaled field energy density



Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Growth of instabilities – 1+1 numerical simulations

Classical system of colored particles & fields

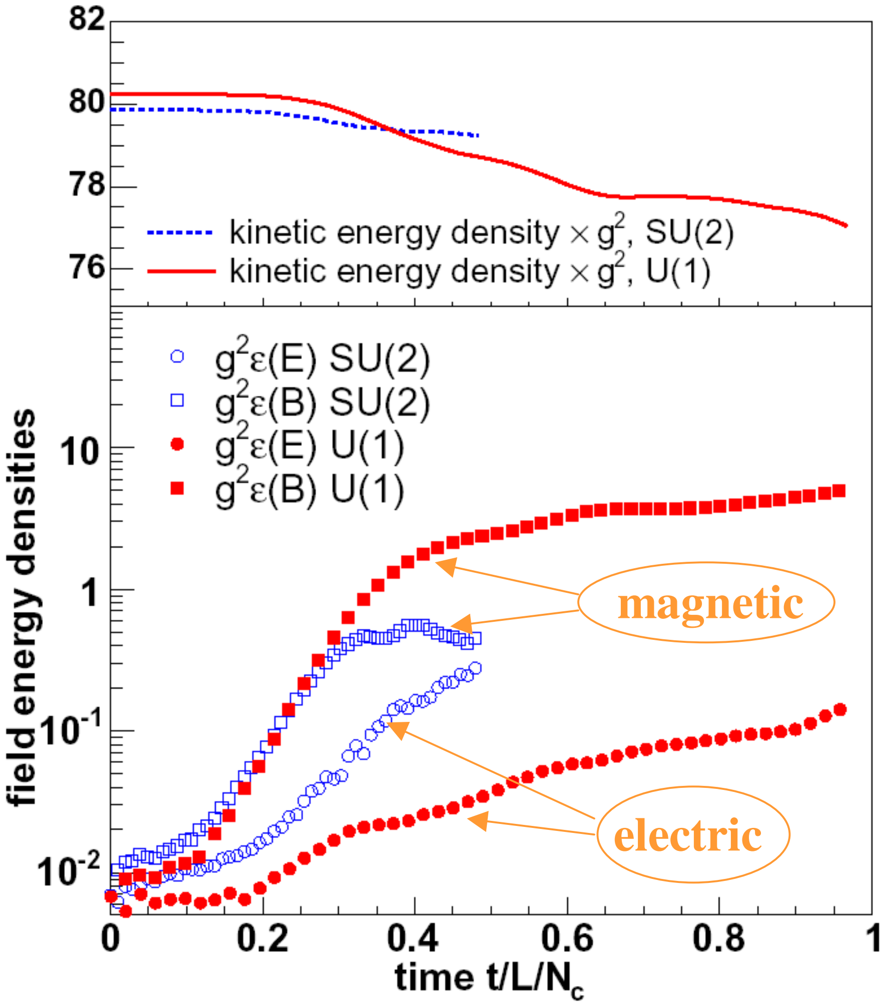
initial fields: **Gaussian noise as in Color Glass Condensate**

initial anisotropic particle distribution:

$$f_0(\mathbf{p},\mathbf{x}) \sim \delta(p_x)\,e^{-\frac{\sqrt{p_y^2+p_z^2}}{p_{\mathrm{hard}}}}$$

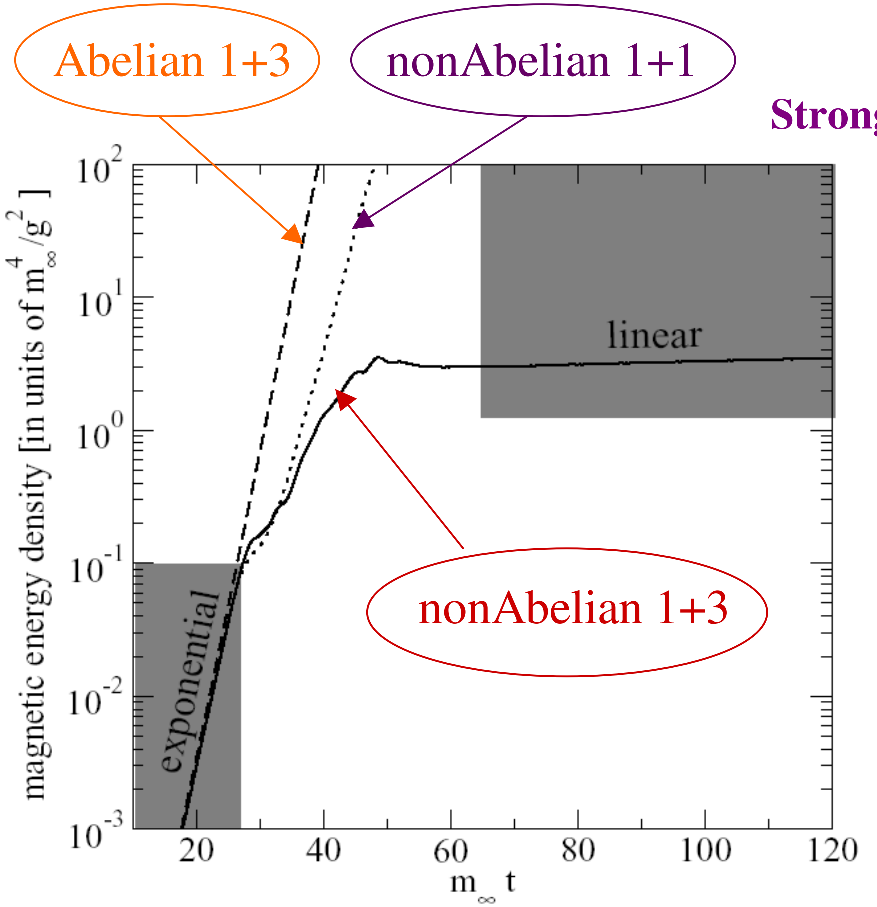
$$p_{\mathrm{hard}} = 10\ \mathrm{GeV}$$

$$L = 40\ \mathrm{fm} \qquad \rho = 10\ \mathrm{fm}^{-3}$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

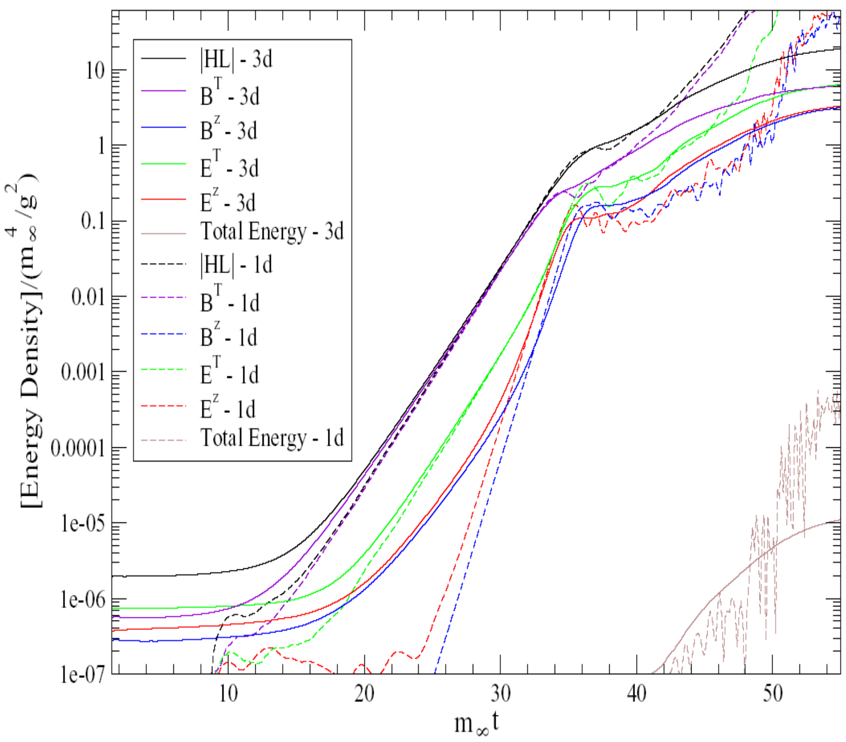
Growth of instabilities – 1+3 numerical simulations



P. Arnold, G.D. Moore & L.G. Yaffe,
Phys. Rev. **D72**, 054003 (2005)

SU(2) Hard Loop Dynamics

Strongly anisotropic particle's momentum distribution



A. Rebhan, P. Romatschke & M.Strickland,
JHEP **0509**, 041 (2005)

Abelanization

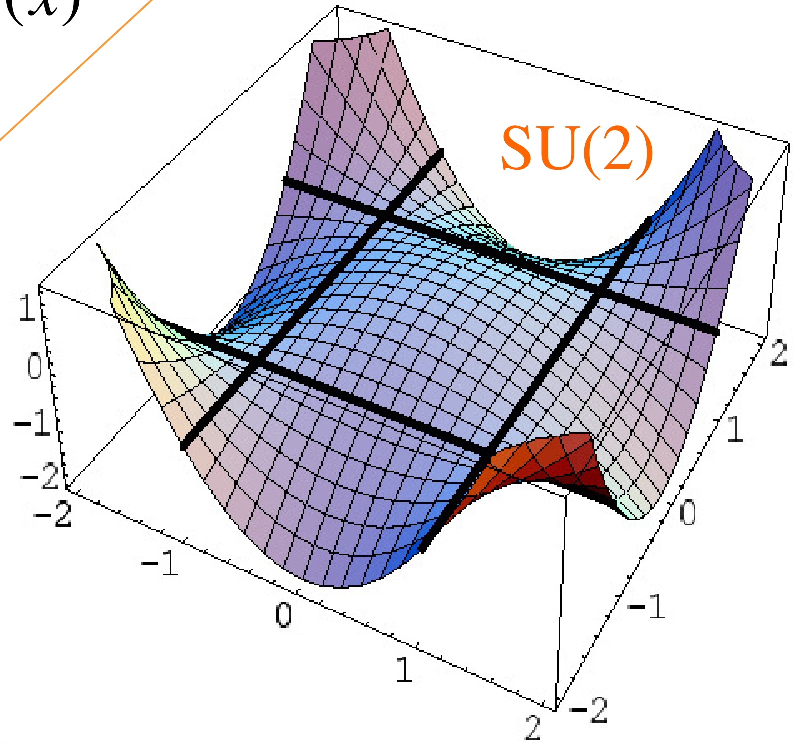
$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2 \mathbf{A}^a \cdot \mathbf{A}^a + \frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

the gauge $A_0^a = 0, \quad A_i^a(t, x, y, z) = A_i^a(x)$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} \mathbf{B}^a \mathbf{B}^a$$

$$= -\frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a + \frac{g}{2} f_{abc} \mathbf{A}^b \times \mathbf{A}^c$$



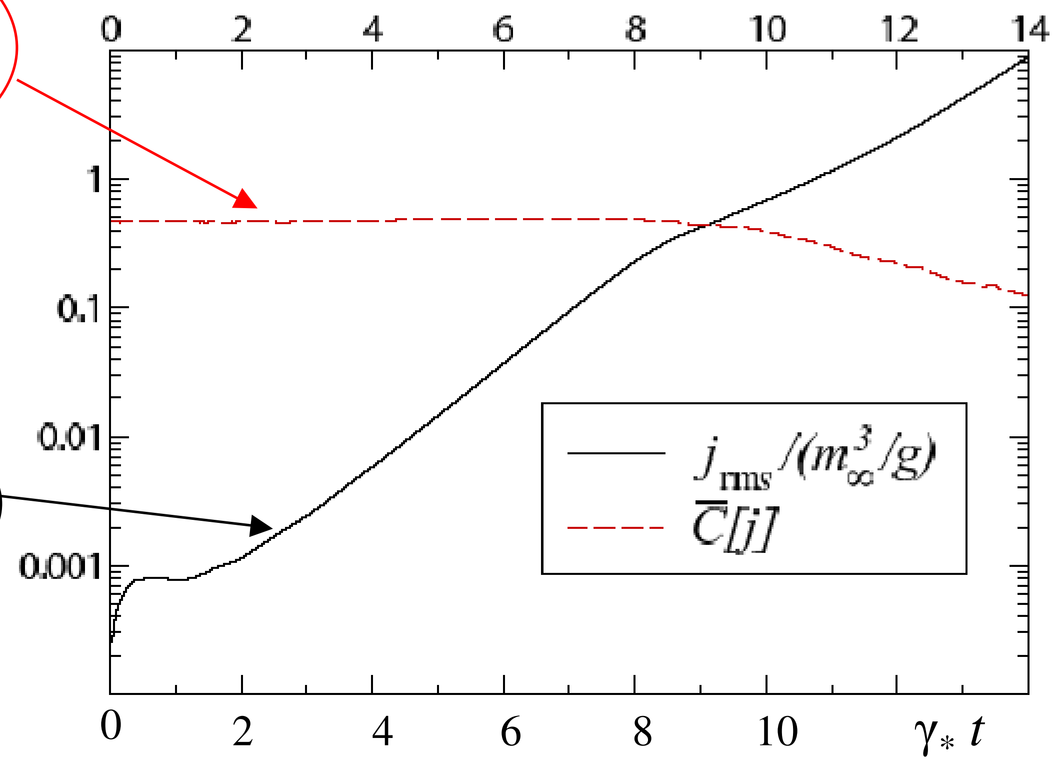
P. Arnold & J. Lenaghan, Phys. Rev. D **70**, 114007 (2004)

Abelanization – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

$$\bar{C} \equiv \int_0^L \frac{dz}{L} \frac{\sqrt{\mathrm{Tr}((i[j_x, j_y])^2)}}{\mathrm{Tr}[\mathbf{j}^2]}$$

$$j_{\mathrm{rms}} \equiv \sqrt{\int_0^L \frac{dz}{L} 2\,\mathrm{Tr}[\mathbf{j}^2]}$$



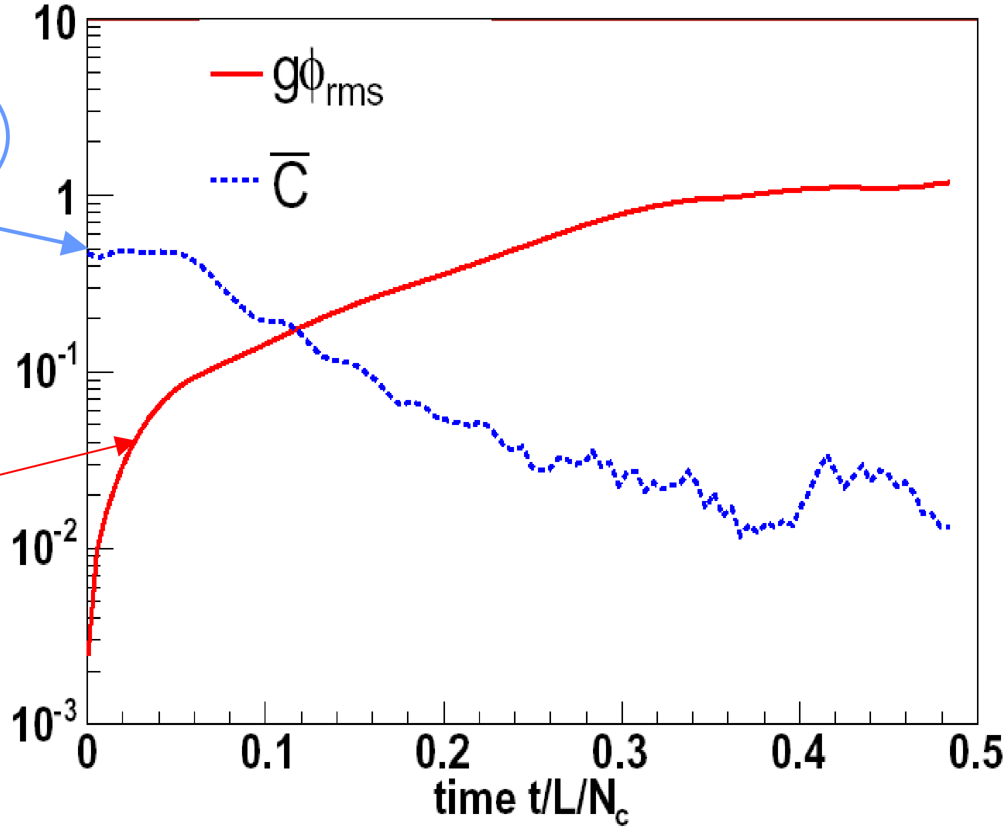
A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Abelanization – 1+1 numerical simulations

Classical system of colored particles & fields

$$\overline{C} \equiv \int_0^L \frac{dx}{L} \frac{\sqrt{\mathrm{Tr}((i[A_y, A_z])^2)}}{\mathrm{Tr}[\mathbf{A}^2]}$$

$$\phi_{\mathrm{rms}} \equiv \sqrt{\int_0^L \frac{dx}{2L} \mathrm{Tr}[\mathbf{A}^2]}$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

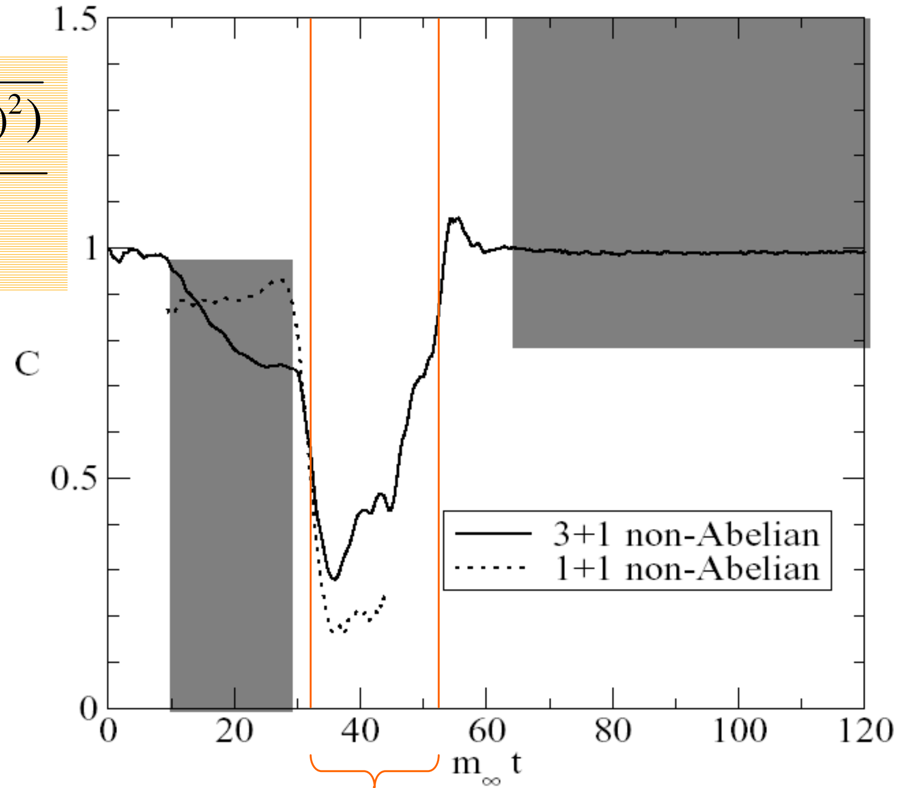
Abelanization – 1+3 numerical simulations

SU(2) Hard Loop Dynamics

$$C \equiv \frac{3}{\sqrt{2}} \frac{\int \frac{d^3x}{V} \sqrt{\mathrm{Tr}((i[j_x, j_y])^2 + (i[j_y, j_z])^2 + (i[j_z, j_x])^2)}}{\int \frac{d^3x}{V} \mathrm{Tr}(\mathbf{j}^2)}$$

$$A_i^a \sim e^{\gamma t}$$

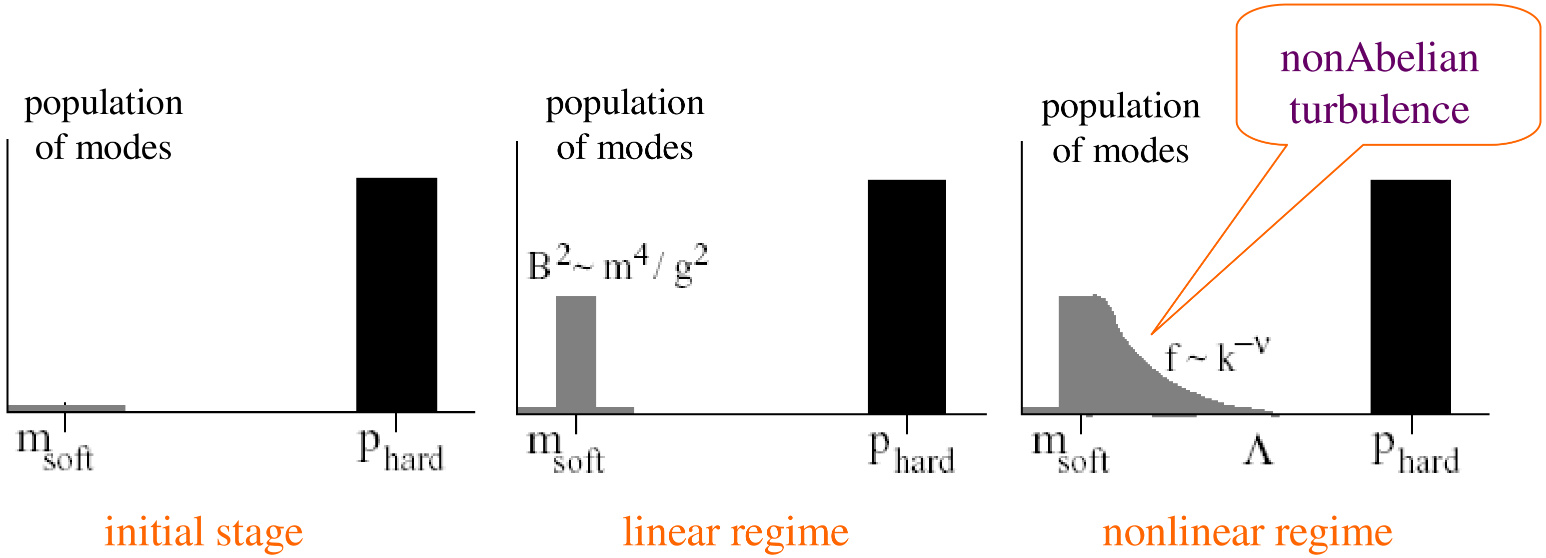
$$A_i^a \sim \frac{k_{\text{field}}}{g} << \frac{p_{\text{hard}}}{g}$$



P. Arnold, G.D. Moore & L.G. Yaffe, Phys. Rev. **D72**, 054003 (2005)

NonAbelian Turbulence

1+3 simulations of Hard Loops Dynamics



P. Arnold & G.D. Moore, Phys. Rev. **D73**, 025006 (2006);
P. Arnold & G.D. Moore, Phys. Rev. **D73**, 025013 (2006);
A. Dumitru,Y. Nara & M. Strickland, hep-ph/0604149.

Hard Expanding Loops

fluctuation

$$Q(p,x) = Q_0(p,x) + \delta Q(p,x)$$

colorless expanding background $Q_0^{ij}(p,x) = \delta^{ij} n(p,x)$

$$|Q_0(p,x)| >> |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p,x)| >> |\partial_p^\mu \delta Q(p,x)|$$

$$p_\mu D^\mu Q_0(p,x) = 0$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p,x) = 0$$

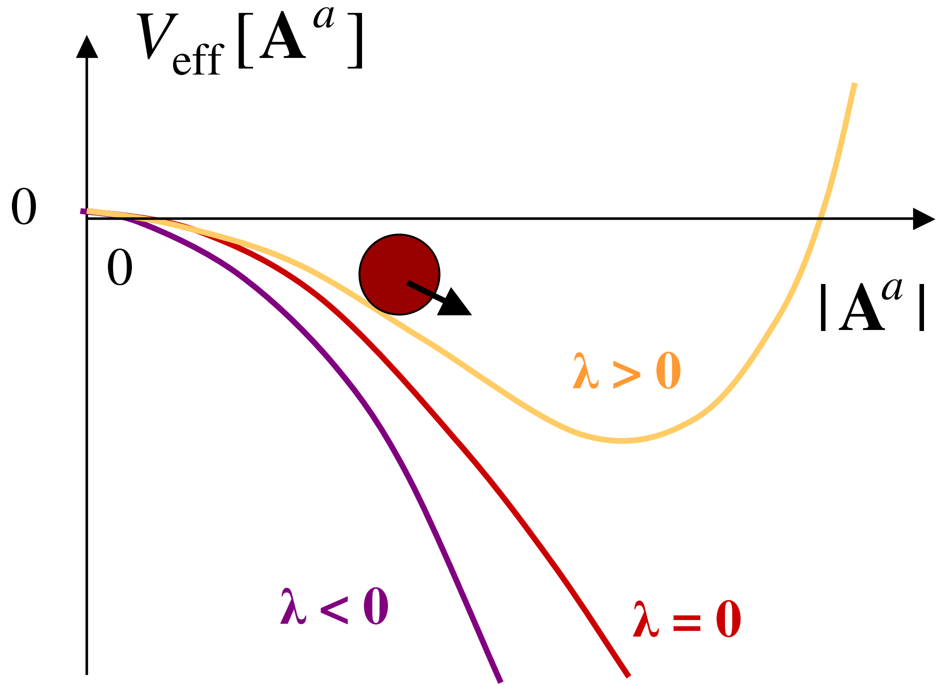
Expansion delays the onset of instability growth $A_i^a \sim e^{\lambda\sqrt{t}}$

A. Rebhan & P. Romatschke, hep-ph/0605064

Beyond Hard Loop level

$$V_{\mathrm{eff}}[\mathbf{A}^a] = -\mu^2\,\mathrm{Tr}[\mathbf{A}^2] + \lambda\,\mathrm{Tr}[\mathbf{A}^4] + \cdots$$

hard-loop term



$\lambda = ?$

C. Manuel & St. M., Phys. Rev. **D72**, 034005 (2005)

Beyond Hard Loop level cont.

Vlasov equation

$$\left(p_\mu D^\mu - g p^\mu F_{\mu\nu}(x)\partial_p^\nu\right) Q(p,x) = 0$$

Exact solution for a system homogeneous along α direction

$$\partial^\alpha Q(p,x) = 0 = \partial^\alpha A^\mu(x)$$

$$Q(p,x) = f\left(p^\alpha - g A^\alpha(x)\right) = \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} \left(A^\alpha(x)\right)^n \frac{\partial^n f(p^\alpha)}{\partial p_\alpha^n}$$

$$[D^\mu A^\alpha(x), A^\alpha(x)] = 0$$

Beyond Hard Loop level cont.

$$j^{\mu}[Q(x,p)] = j^{\mu}[f(p), A^{\alpha}(x)]$$

⇓

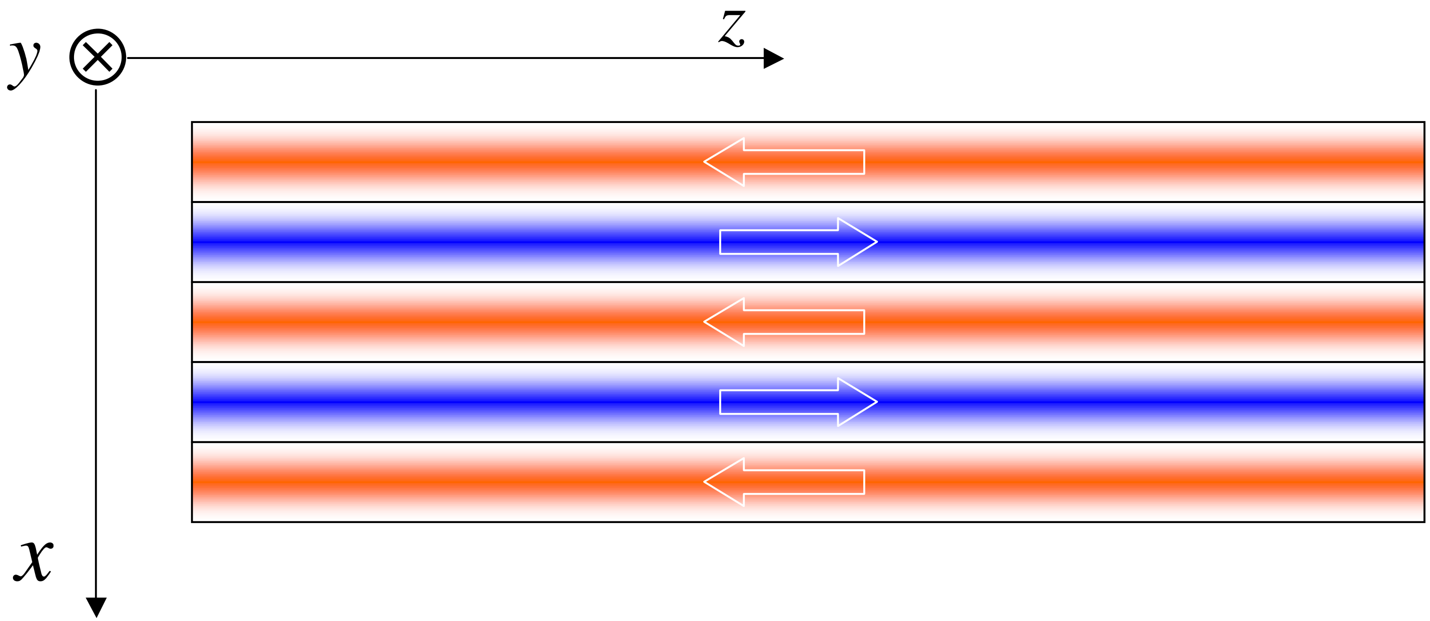
$$j^{\mu}(x) = -\frac{\delta S_{\text{eff}}}{A_{\mu}(x)}$$

$$S_{\text{eff}} \equiv -\int d^4x \, V_{\text{eff}}$$

⇓

$$V_{\text{eff}}[A^{\alpha}] = \sum_{n=0}^{\infty} \frac{(-g)^{n+1}}{(n+1)!} \text{Tr}[(A^{\alpha})^{n+1}] \int \frac{d^3p}{(2\pi)^3} \frac{p^{\alpha}}{E_p} \frac{\partial^n f(p^{\alpha})}{\partial p_{\alpha}^n}$$

Unstable configuration of interest



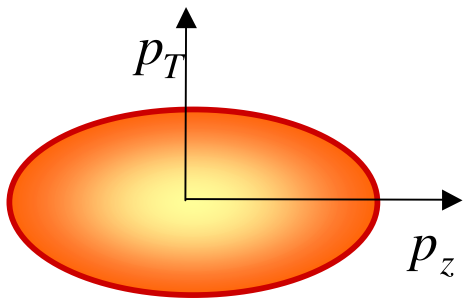
$$\mathbf{j}(x) = (0,0,\ j(x)), \quad \mathbf{B}(x) = (0, B(x), 0), \quad \mathbf{A}(x) = (0,0,A(x))$$

$$Q(p,x) = f\left(p_0, p_z - gA_z\right), \qquad A_0 = 0$$

Effective potential beyond Hard Loop

$$f\left(p_0, p_z\right) \sim \exp(-\beta p_0^2 + \alpha p_z^2) = \exp\left(-\beta(p_x^2 + p_y^2) - (\beta - \alpha) p_z^2\right)$$

$$p_0 = \sqrt{p_x^2 + p_y^2 + p_z^2}$$



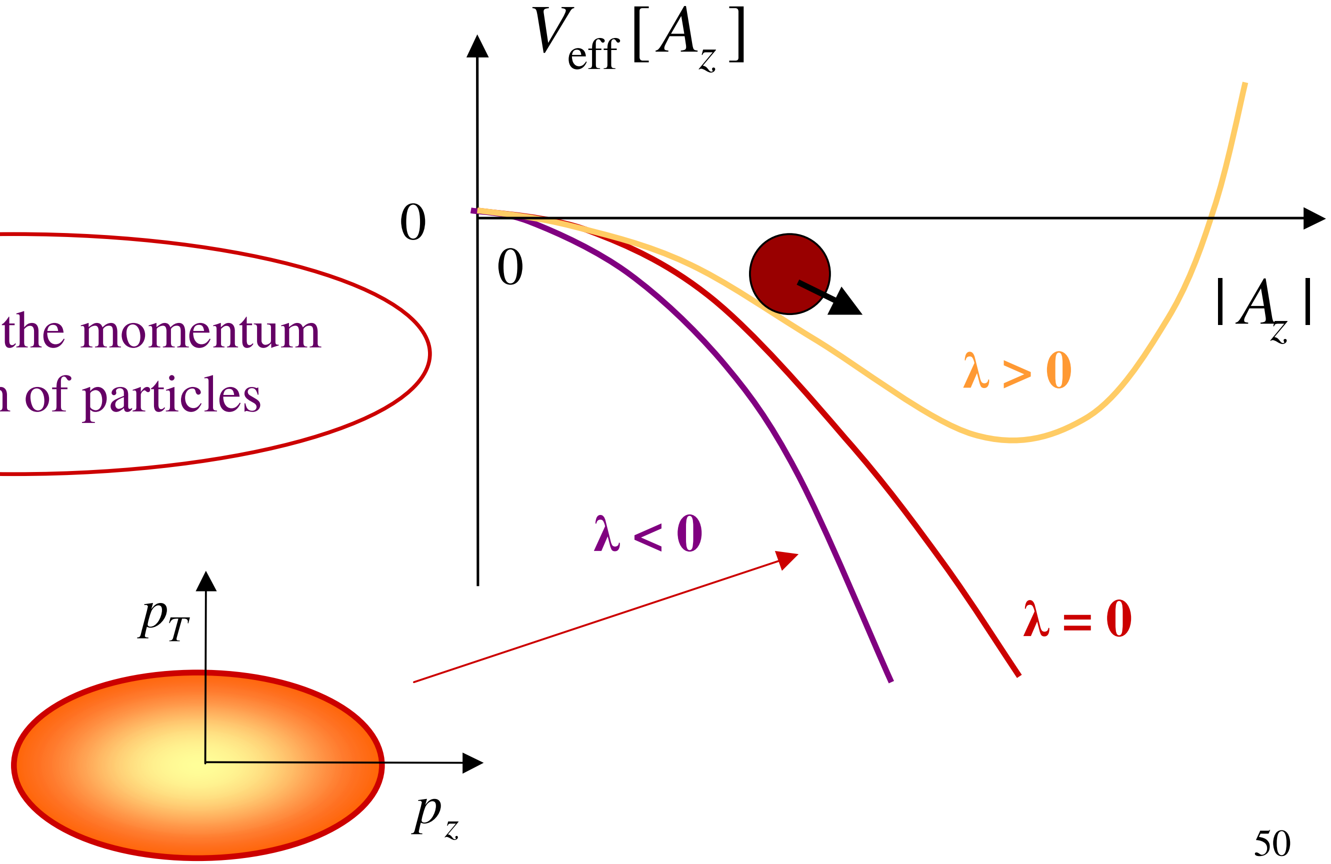
$$V_{\text{eff}}[A_z] = -g^2 \alpha \left\langle \frac{p_z^2}{E_p} \right\rangle \mathrm{Tr}[A_z^2]$$
$$- g^4 \left\{ \tfrac{1}{3} \alpha^3 \left\langle \frac{p_z^4}{E_p} \right\rangle + \tfrac{1}{2} \alpha^2 \left\langle \frac{p_z^2}{E_p} \right\rangle \right\} \mathrm{Tr}[A_z^4] - \ldots$$

All terms are negative!

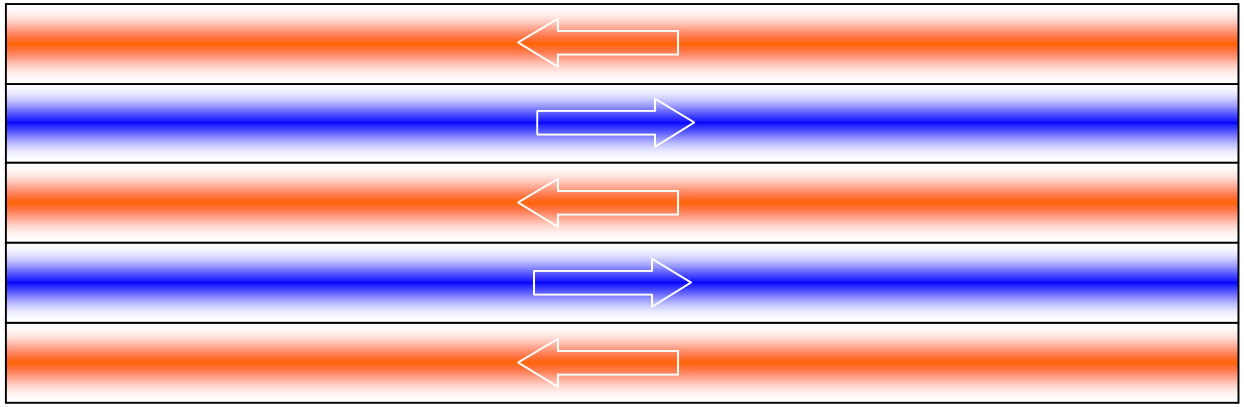
Effective potential beyond Hard Loop

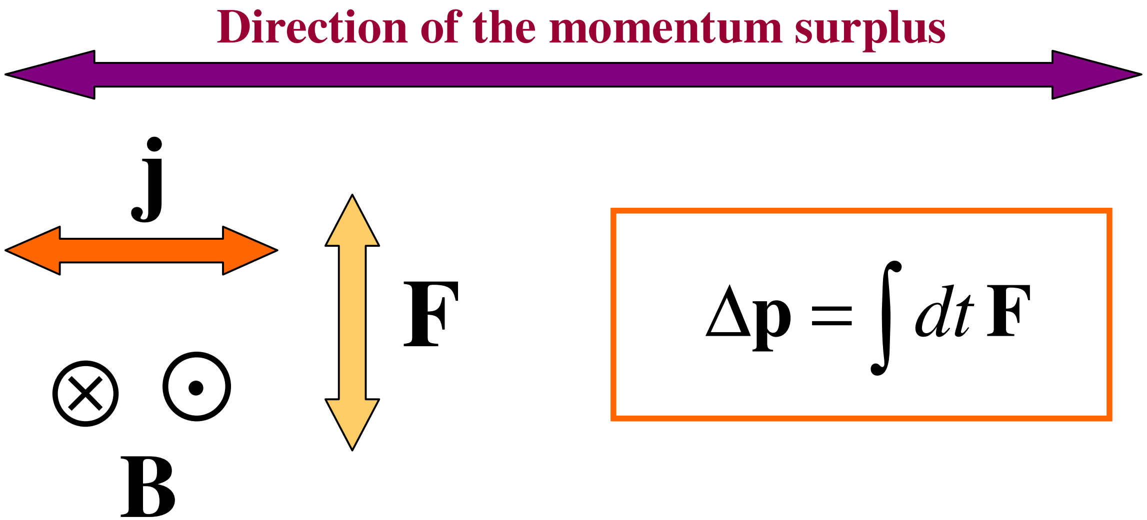
$$V_{\mathrm{eff}}[A_z] = -\mu^2\,\mathrm{Tr}[A_z^2] + \lambda\,\mathrm{Tr}[A_z^4] + \cdots$$

λ depends on the momentum distribution of particles



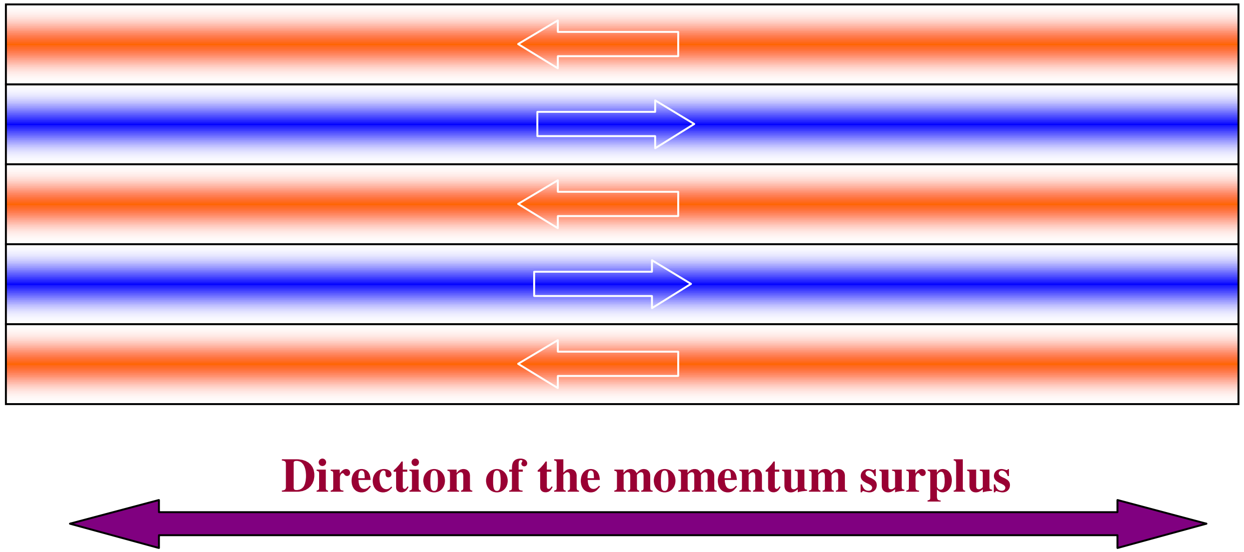
Isotropization - particles

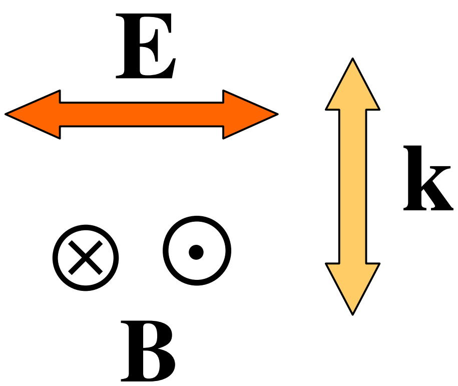




$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields





$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

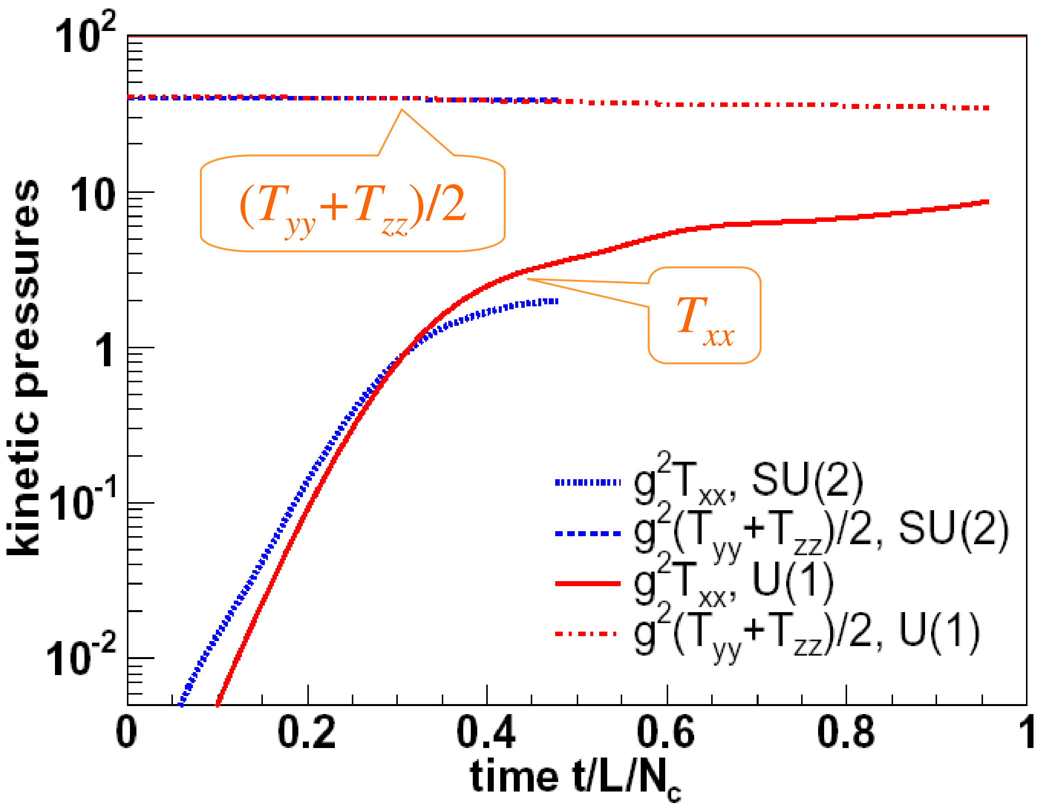
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

$$T_{xx} = (T_{yy} + T_{zz})/2$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Instabilities in CGC & Glasma

Expansion into vacuum of self-interacting classical nonAbelian fields

Instabilities well seen!

hard modes – particles
soft modes – classical fields

Transverse magnetic modes

P. Romatschke & R. Venugopalan, Phys. Rev. Lett. **96**, 062302 (2006);
T. Lappi & L. McLerran, Nucl. Phys. **A772**, 200 (2006)

Conclusion

The scenario of instabilities driven equilibration is dynamically very rich and it provides a plausible solution of the fast equilibration problem