Energy loss in Unstable Quark-Gluon Plasma

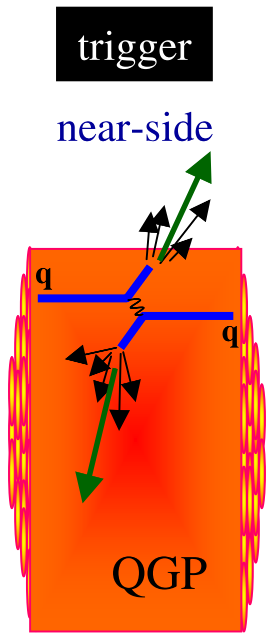
Stanisław Mrówczyński

Jan Kochanowski University, Kielce, Poland
& National Centre of Nuclear Reserach, Warsaw, Poland

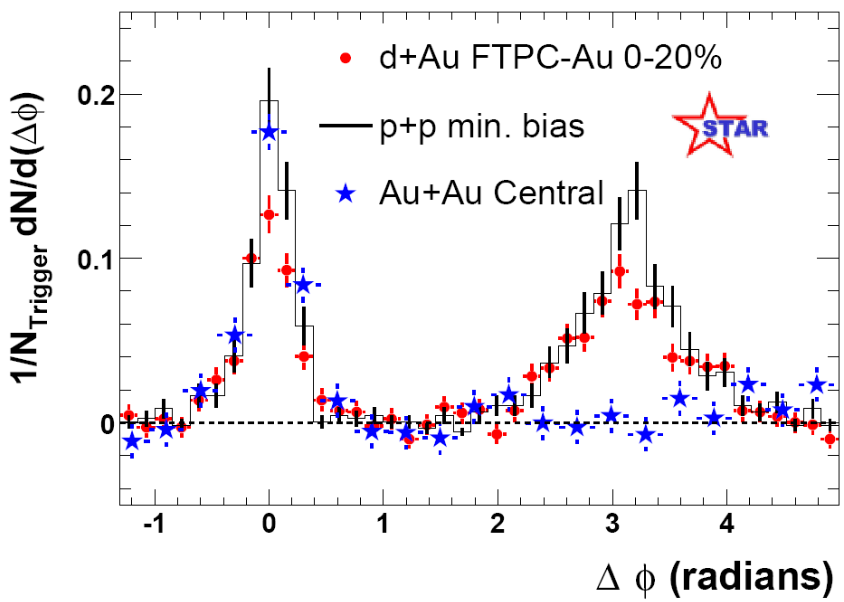
In collaboration with

Margaret Carrington & Katarzyna Deja

Motivation - jet quenching



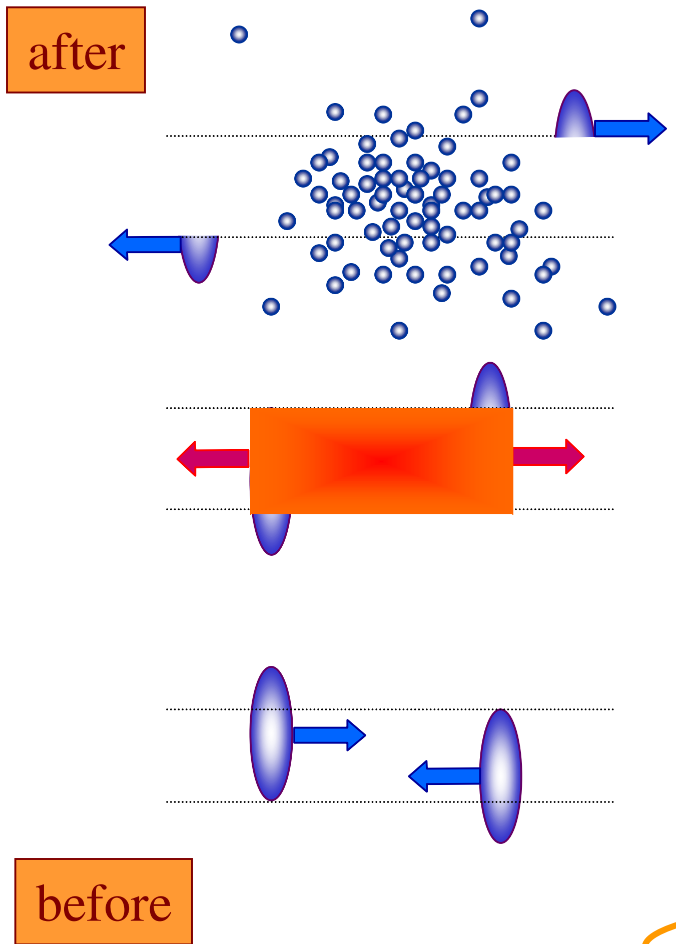
near-side away-side

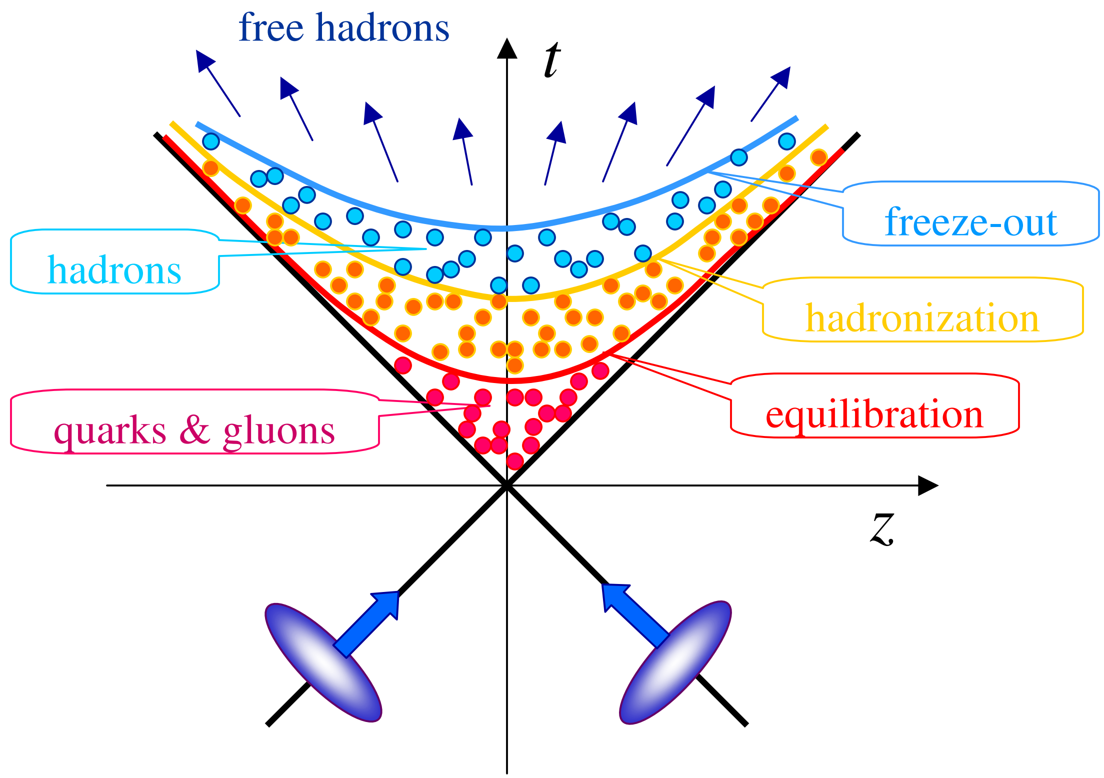


Away-side jet is suppressed
in central collisions

J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

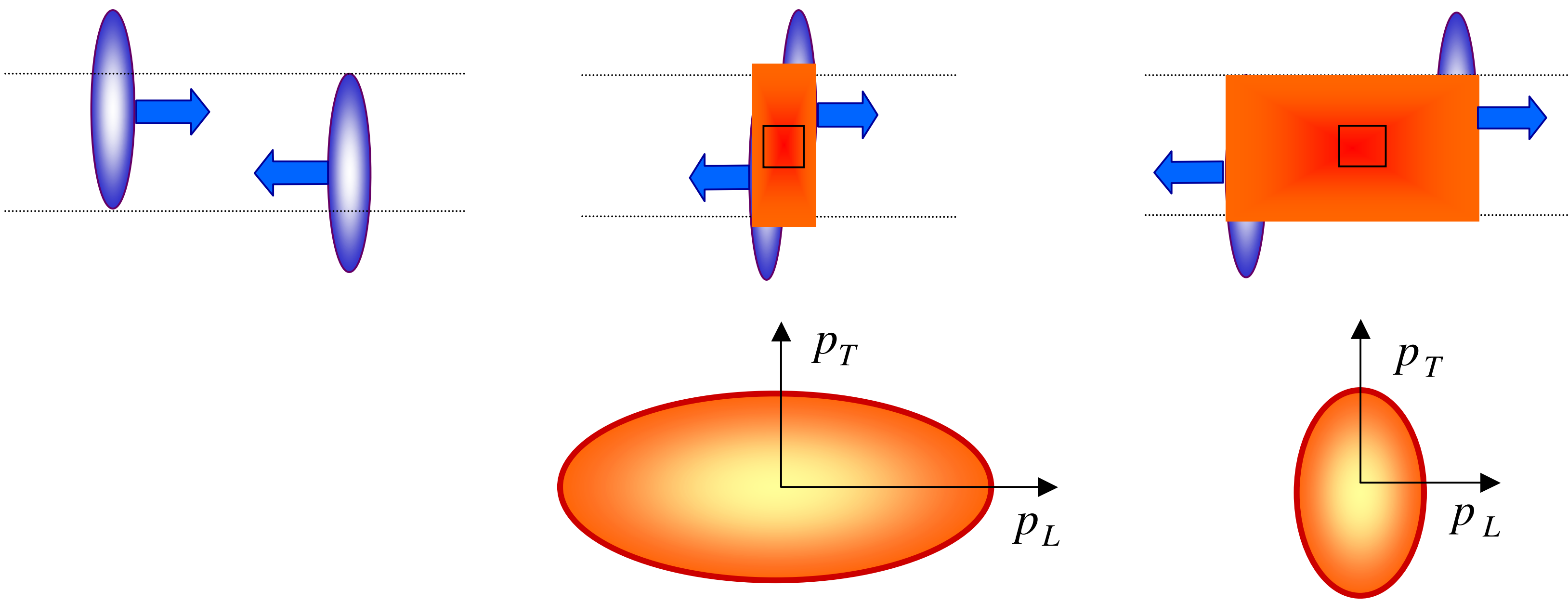
Scenario of relativistic heavy-ion collisions





At the early stage QGP is out of equilibrium

Anisotropic QGP



'prolate' 'oblate'

Anisotropic QGP is unstable due to magnetic plasma modes

Unstable QGP

How to compute *dE/dx* in unstable QGP?

This is initial value problem!

A test parton in QGP

Wong's equation of motion (HL approximation)

$$\begin{cases} \dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\ \dfrac{dp^{\mu}(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_{\nu}(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}\, p_{\mu}(\tau)\, A_b^{\mu}(x(\tau))\, Q_c(\tau) \end{cases}$$

<u>Simplifications</u>

Gauge condition: $p_{\mu}(\tau)\, A_b^{\mu}\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$

Parton travels with constant velocity: $u^{\mu} = (\gamma, \gamma\,\mathbf{v}) = \text{const}$

Parton's energy loss

$$\frac{dE(t)}{dt} = gQ_a\,\mathbf{E}_a(t,\mathbf{r}(t))\cdot\mathbf{v}$$

chromoelectric field: induced and spontaneously generated

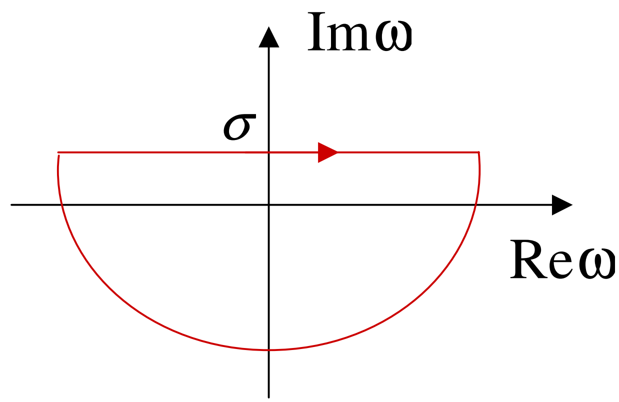
parton's current: $\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

$$\boxed{\frac{dE(t)}{dt} = \int d^3r\,\mathbf{E}_a(t,\mathbf{r})\cdot\mathbf{j}_a(t,\mathbf{r})}$$

Initial value problem

One-sided Fourier transformation

$$
\begin{cases}
f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t - \mathbf{k}\mathbf{r})} f(t,\mathbf{r}) \\
f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t - \mathbf{k}\mathbf{r})} f(\omega,\mathbf{k})
\end{cases}
$$



$$0 < \sigma \in R$$

$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\,\delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a \mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega - \mathbf{k}\mathbf{v})t}\, \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{v}$$

Induced Electric Field

Linearized Yang-Mills (Maxwell) equations

$$i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$
$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$
$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k})$$

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

chromodielectric tensor

$$\boxed{E^i(\omega,\mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})[\omega\mathbf{j}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Energy-Loss formula

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}$$
$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[\frac{igQ_a \omega \mathbf{v}}{\omega - \overline{\omega}} + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega \mathbf{D}_0(\mathbf{k}) \right]^j$$

$$\overline{\omega} \equiv \mathbf{k} \cdot \mathbf{v}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\boxed{\det[\Sigma(\omega,\mathbf{k})] = 0}$$

Initial values of the fields

Maxwell equations & $\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

Initial values:

$$D_0^i(\mathbf{k}) = -igQ_a\overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

Energy-Loss formula

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times \left[\underbrace{\frac{\omega\delta^{jl}}{\omega-\varpi}}_{\mathbf{j}(\omega,\mathbf{k})} \underbrace{- (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k})}_{\mathbf{B}_0(\mathbf{k})} \underbrace{+ \omega\varpi\varepsilon^{jk}(\varpi,\mathbf{k})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k})}_{\mathbf{D}_0(\mathbf{k})} \right]$$

Averaging over parton's colors: $\int dQ\, Q_a Q_b = C_2 \delta^{ab}, \quad C_2 \equiv \begin{cases} \frac{1}{2} & \text{for quark} \\ N_c & \text{for gluon} \end{cases}$

$$C_R = \begin{cases} C_2 \frac{N_c^2 - 1}{N_c} = \frac{N_c^2 - 1}{2N_c} & \text{for quark} \quad (\mathrm{R = F}) \\ C_2 = N_c & \text{for gluon} \quad (\mathrm{R = G}) \end{cases}$$

Stable isotropic plasma

- $$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

- $$(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}(\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times\left[\frac{\omega\delta^{jl}}{\omega-\overline{\omega}} - (k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})\right]$$

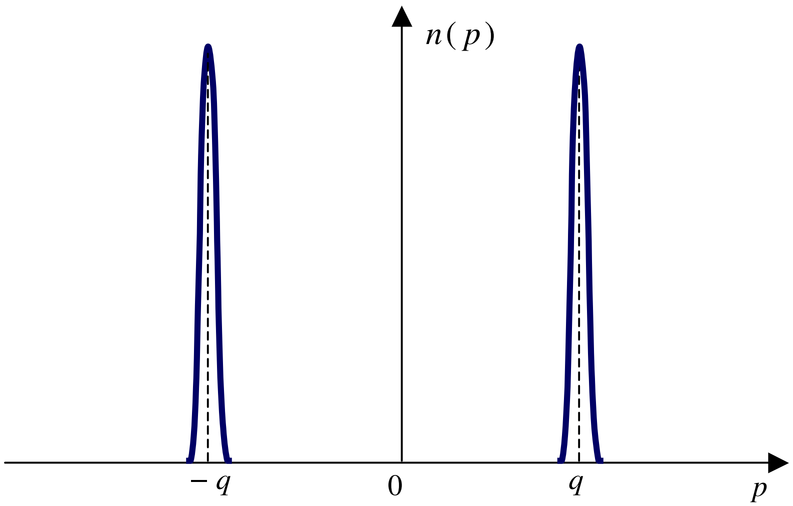
The only stationary contribution: $\omega = \overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$

$$\boxed{\frac{dE(t)}{dt} = ig^2 C_R\int\frac{d^3k}{(2\pi)^3}\frac{\overline{\omega}}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2\varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2}\right]}$$

equivalent to the standard result by Braaten & Thoma

Unstable two-stream system

$$n(\mathbf{p}) = (2\pi)^3 \rho \left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$$



There is unstable longitudinal chromoelectric mode

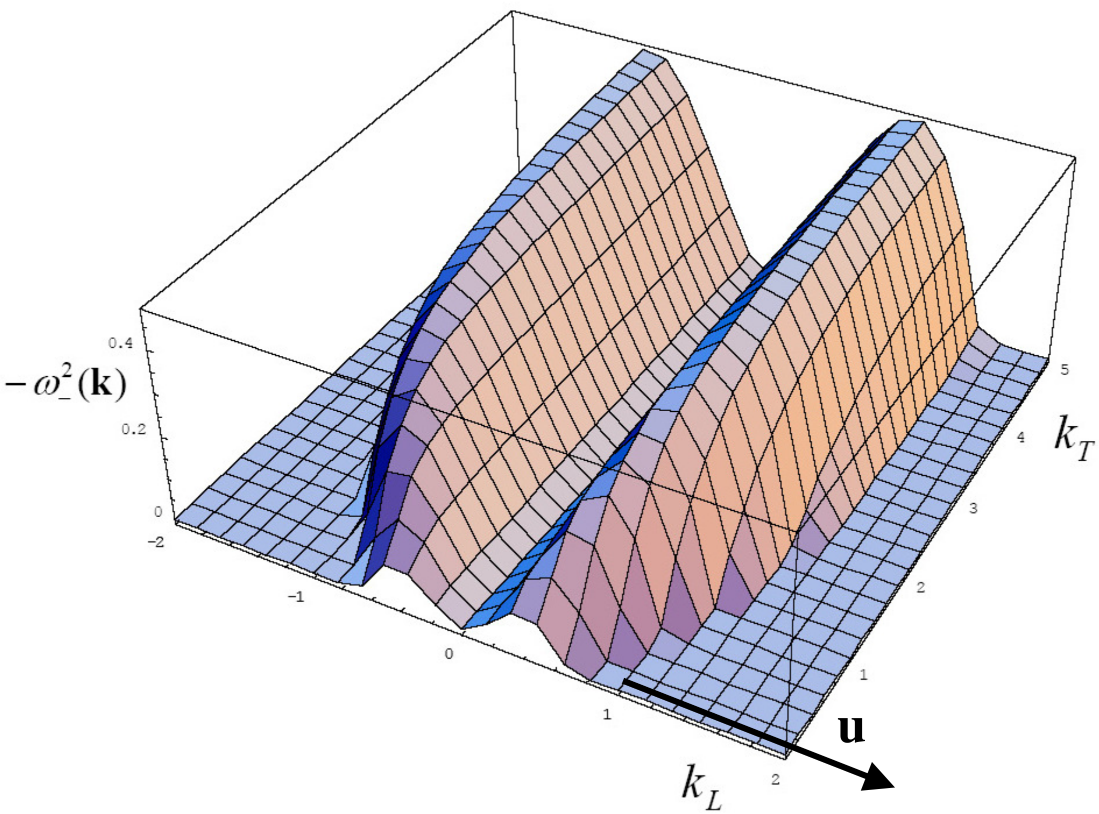
$$\left.\begin{aligned} &\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} \\ &(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2 \varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} \end{aligned}\right\}$$

longitudinal chromoelectric field only!

$$\boxed{\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\overline{\omega})t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\overline{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\overline{\omega}} + \frac{\overline{\omega}}{\omega}\right]}$$

Collective modes in two-stream system

$$\varepsilon_L(\omega,\mathbf{k}) = \frac{(\omega-\omega_+(\mathbf{k}))(\omega+\omega_+(\mathbf{k}))(\omega-\omega_-(\mathbf{k}))(\omega+\omega_-(\mathbf{k}))}{(\omega^2-(\mathbf{k}\cdot\mathbf{u})^2)}$$



$\mathbf{u}$ - stream velocity

$\omega_+(\mathbf{k})$ - stable longitudinal mode

$\omega_-(\mathbf{k})$ - unstable longitudinal mode

Energy loss in two-stream system

$$\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\bar{\omega})t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\bar{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\bar{\omega}} + \frac{\bar{\omega}}{\omega} \right]$$

There are 6 contributions corresponding to $\omega = \pm\omega_+(\mathbf{k}), \pm\omega_-(\mathbf{k}), \bar{\omega} = \mathbf{k}\cdot\mathbf{v}, 0$

Only one dimensional parameter: $\mu^2 \equiv \dfrac{g^2 n}{2E_q}$

Remaining parameters: $g = 1, \quad |\mathbf{v}| = 1, \quad |\mathbf{u}| = 0.9, \quad C_R = 3$

The integral over **k** performed numerically for

$$-k_{\max} < k_L < k_{\max}, \quad 0 < k_T < k_{\max}$$

Vacuum contribution to energy loss

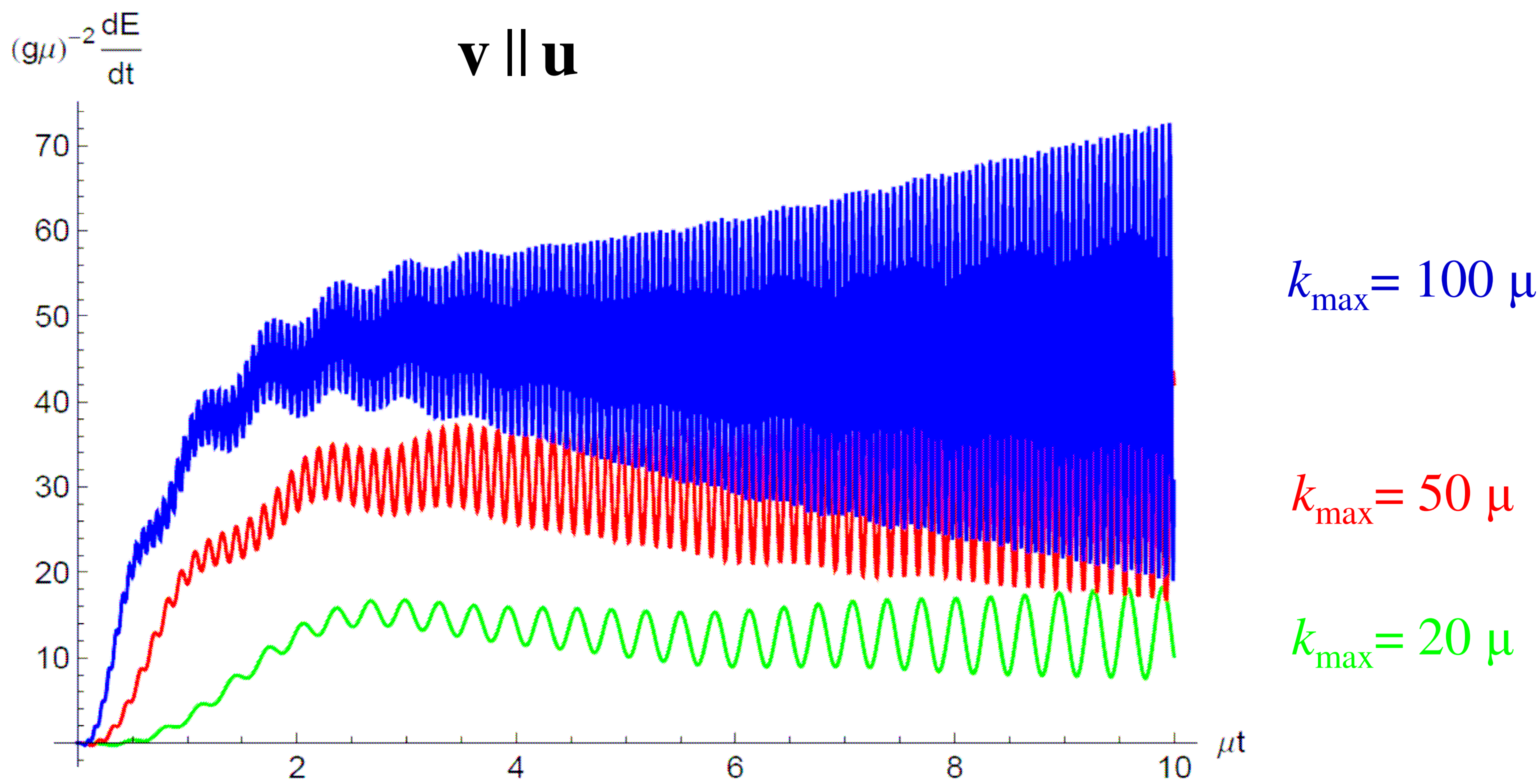
$$\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\overline{\omega})t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\overline{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\overline{\omega}} + \frac{\overline{\omega}}{\omega} \right]$$

Vacuum contribution: $\varepsilon_L(\omega,\mathbf{k}) \to 1$

$$\boxed{\left.\frac{dE(t)}{dt}\right|_{\text{vacuum}} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\overline{\omega})t}}{\omega^2} \frac{\overline{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\overline{\omega}} + \frac{\overline{\omega}}{\omega} \right] = \frac{g^2 C_R}{8\pi} \frac{1}{t^2}}$$

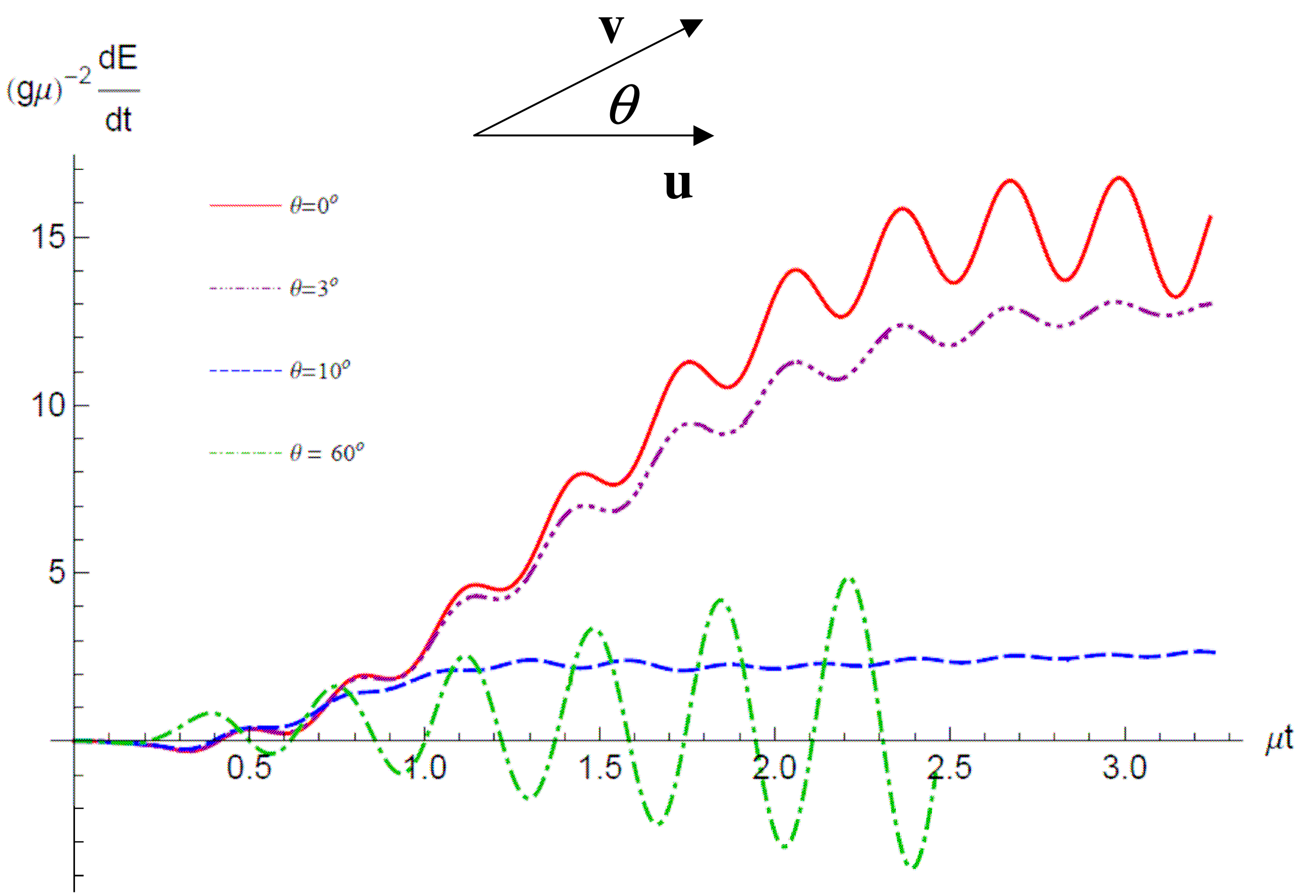
Vacuum contribution has to be subtracted

Energy loss in two-stream system



Strong ultraviolet dependence!

Energy loss in two-stream system



k_{max}= 50 μ

Conclusions

- Energy loss found as a solution of initial value problem
- Two-stream plasma system discussed as an example
- Strong time and directional dependence of *dE/dx* demonstrated

More details in:
M.E. Carrington, K. Deja and St. Mrówczyński,
arXiv:1110.4846 [hep-ph]; 1201.1486 [nucl-th].