My adventures with SUSY & Joe

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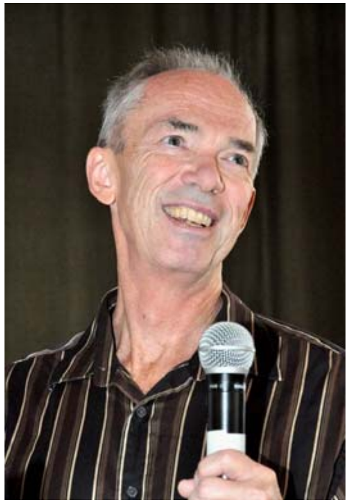
Happy birthday, Joe!



My first encounter with SUSY

PHYSICAL REVIEW D VOLUME 28, NUMBER 12 15 DECEMBER 1983

Finite-temperature and supercharged ideal supersymmetric matter

J. Kapusta, S. Pratt, and V. Višnjić
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 23 May 1983)

We investigate the properties of simple, ideal supersymmetric matter in 0+1 and 3+1 dimensions at finite temperature and finite supercharge density. The functional dependence of the partition function Z on the temperature T and chemical potential μ is qualitatively different from the usual situation of a net U(1) charge density, such as electric charge or fermion number. This is a consequence of the fact that the Hamiltonian H is the square of the supercharge Q, $H=Q^2$.

My first encounter with Joe

PHYSICAL REVIEW D VOLUME 31, NUMBER 4 15 FEBRUARY 1985

Erratum

Erratum: Finite-temperature and supercharged ideal supersymmetric matter [Phys. Rev. D 28, 3093 (1983)]

J. Kapusta, S. Pratt, and V. Višnjić

A factor of $\frac{1}{2}$ was missing from the first-order correction in $\beta^2\mu^2 E$ in Eqs. (2.39) and (2.40). They should read

$$\langle H \rangle = E\left[\frac{1}{e^{\beta E}-1} + \frac{}{e}\right.$$

We thank S. Mrówczyński for bringing these

My first paper with SUSY

Vol. B17 (1986) ACTA PHYSICA POLONICA No 1

RADIATION OF A SUPERCHARGED SYSTEM

BY ST. MRÓWCZYŃSKI*

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(*Received January 2, 1985*)

A simple supersymmetric system of noninteracting photons and photinos at finite values of temperature and supercharge is studied. Particular attention is drawn to properties of photons. Analogs of the Planck formula and the Stefan-Boltzmann law are considered.

PACS numbers: 05.30.Ch, 11.30.Pb

Supercharged system

Partition function

supercharge –fermionic operator

$$Z = \mathrm{Tr}\exp[-\beta(H-\mu Q)]$$

Mode decomposition of free system

$$\exp[-\beta\sum_p (H(p)-\mu Q(p))] \neq \prod_p \exp[-\beta(H(p)-\mu Q(p))]$$

$$[Q(p),Q(q)] \neq 0$$

$$\{Q(p),Q(q)\} = 0$$

Supercharged system cont.

Auxiliary fermionic field

$$\{c(p), c(q)\} = 2\delta(p-q)$$

Modified supercharge

$$\tilde{Q}(p) \equiv c(p)Q(p) \qquad [\tilde{Q}(p), \tilde{Q}(q)]_{p\neq q} = 0$$

Mode decomposition of free system

$$\exp[-\beta\sum_{p}(H(p)-\mu\tilde{Q}(p))] = \prod_{p}\exp[-\beta(H(p)-\mu\tilde{Q}(p))]$$

Radiation of supercharged system

Free SUSY QED of photons & photinos

$$\boxed{\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{i}{2}\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}D^2}$$

$$j_\mu \equiv -\frac{1}{2}F_{\nu\rho}\gamma^\nu\gamma^\rho\gamma_\mu\Psi$$

$$Q \equiv \int d^3x \; j_0(x)$$

$$Z = \mathrm{Tr}\exp[-\beta(H - \mu\tilde{Q})]$$

$$[Q] \sim \sqrt{m} \qquad [\mu] \sim \sqrt{m}$$

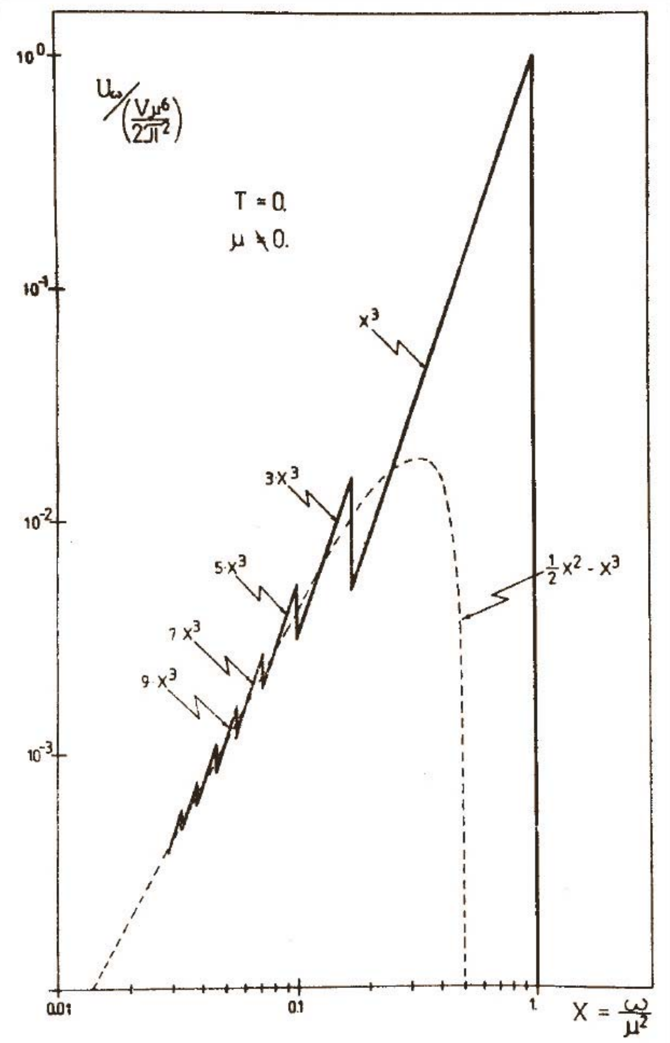

Fig. 1. The energetic photon spectrum at zero temperature

Alternative supercharged system

PHYSICAL REVIEW D **68**, 025007 (2003)

Hydrodynamic fluctuations, long-time tails, and supersymmetry

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(Received 15 March 2003; published 7 July 2003)

Hydrodynamic fluctuations at a nonzero temperature can cause slow relaxation toward equilibrium even in observables which are not locally conserved. A classic example is the stress-stress correlator in a normal fluid, which, at zero wave number, behaves at large times as $t^{-3/2}$. A novel feature of the effective theory of hydrodynamic fluctuations in supersymmetric theories is the presence of Grassmann-valued classical fields describing macroscopic supercharge density fluctuations. We show that hydrodynamic fluctuations in supersymmetric theories generate essentially the same long-time power-law tails in real-time correlation functions that are known in simple fluids. In particular, a $t^{-3/2}$ long-time tail must exist in the stress-stress correlator of $\mathcal{N}=4$ supersymmetric Yang-Mills theory at non-zero temperature, regardless of the value of the coupling. Consequently, this feature of finite-temperature dynamics can provide an interesting test of the AdS/CFT correspondence. However, the coefficient of this long-time tail is suppressed by a factor of $1/N_c^2$. On the gravitational side, this implies that these long-time tails are not present in the classical supergravity limit; they must instead be produced by one-loop gravitational fluctuations.

fermionic chemical potential

$$Z = \mathrm{Tr}\exp[-\beta(H - \tilde{\mu}Q)] \qquad \frac{\partial Z}{\partial \tilde{\mu}} = 0 \quad !$$

SUSY & instabilities

In QED plasma there are various unstable photon modes
but no unstable electron modes are known.

Are there unstable photino modes in SUSY QED plasma?

Does rudimentary SUSY induce
instabilities in fermionic sector?

Bigger project motivated by AdS/CFT

**A systematic comparison
of supersymmetric plasma systems
to their non-supersymmetric counterparts
in a weak coupling domain**

in collaboration with **Alina Czajka**

A. Czajka & St. Mrówczyński, Physical Review **D83** (2011) 065021
A. Czajka & St. Mrówczyński, Physical Review **D84** (2011) 105020
} SUSY QED

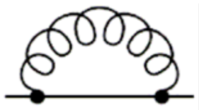
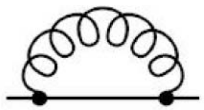
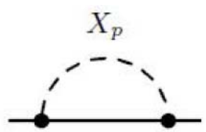
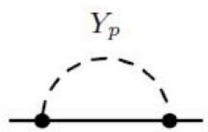
A. Czajka & St. Mrówczyński, arXiv: 1203.1856 [hep-th], Phys. Rev. **D** in print $\mathcal{N}$=4 super Yang-Mills

Fermionic collective modes

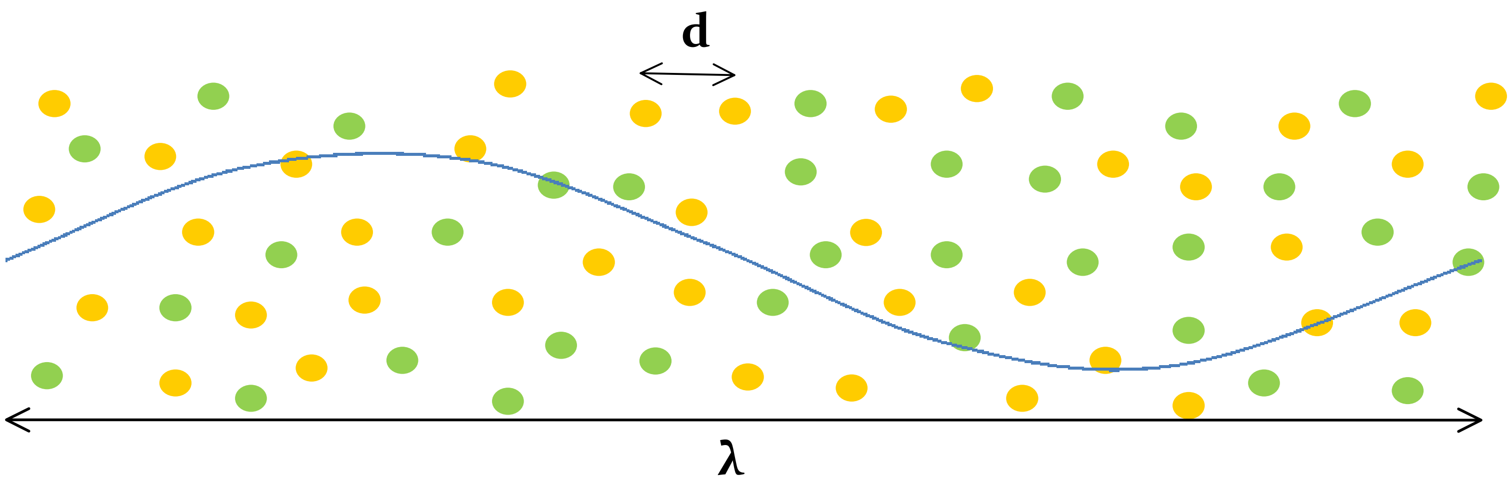
Dispersion equation

$$\boxed{\det[k_\mu \gamma^\mu - \Sigma(k)] = 0}$$

$\Sigma(k)$ - retarded self-energy

electron in QED			
electron in SUSY QED			
photino in SUSY QED			
quark in QCD			
fermion in $\mathcal{N}=4$ super Yang-Mills			

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma

$$\boxed{\lambda >> d}$$

Fermion self-energies in HLA

$$\boxed{\Sigma(k) = \# \, g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{p^\mu \gamma_\mu}{k \cdot p + i0^+}} \qquad k^\mu << p^\mu$$

The fermion self-energy has **the same structure** for QED, SUSY QED, QCD and $\mathcal{N}$=4 super Yang-Mills

There are identical spectra of collective excitations of fermions in all systems

No instabilities!

Supersymmetry does not change anything

Why self-energies in Hard Loop Approximation are universal?

Hard loop effective action

Self-energy constrains the form of effective action

$$\mathcal{L}_2(x) = \int d^4y\, \overline{\Psi}(x)\Sigma(x-y)\Psi(y)$$

$$\boxed{\Sigma(x,y) = \frac{\delta^2 S[\Psi,\overline{\Psi}]}{\delta\overline{\Psi}(x)\delta\Psi(y)}}$$

$$\boxed{\mathcal{L}_{\mathrm{HL}}(x) = \# g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \overline{\Psi}(x)\left(\frac{p\cdot\gamma}{p\cdot D}\right)\Psi(x)}$$

The structure of effective action is unique
and so is the structure of self-energy.

J. Frenkel and J.C. Taylor, Nucl. Phys. **B374**, 156 (1992);
E. Braaten and R.D. Pisarski, Phys. Rev. **D45**, 1827 (1992).

Plasma transport characteristics

Elementary collisional processes

$$\Downarrow$$

transport properties of ultrarelativistic plasma

- ✓ Temperature T is the only dimensional parameter
- ✓ Coulomb-like scatterings $\left(|\mathcal{M}|^2 \sim t^{-2}\right)$ dominate the interaction

shear viscosity $$\eta \sim \frac{T^3}{g^4 \ln g^{-1}}$$

S. C. Huot, S. Jeon, and G. D. Moore, Phys. Rev. Lett. **98**, 172303 (2007)

Energy loss

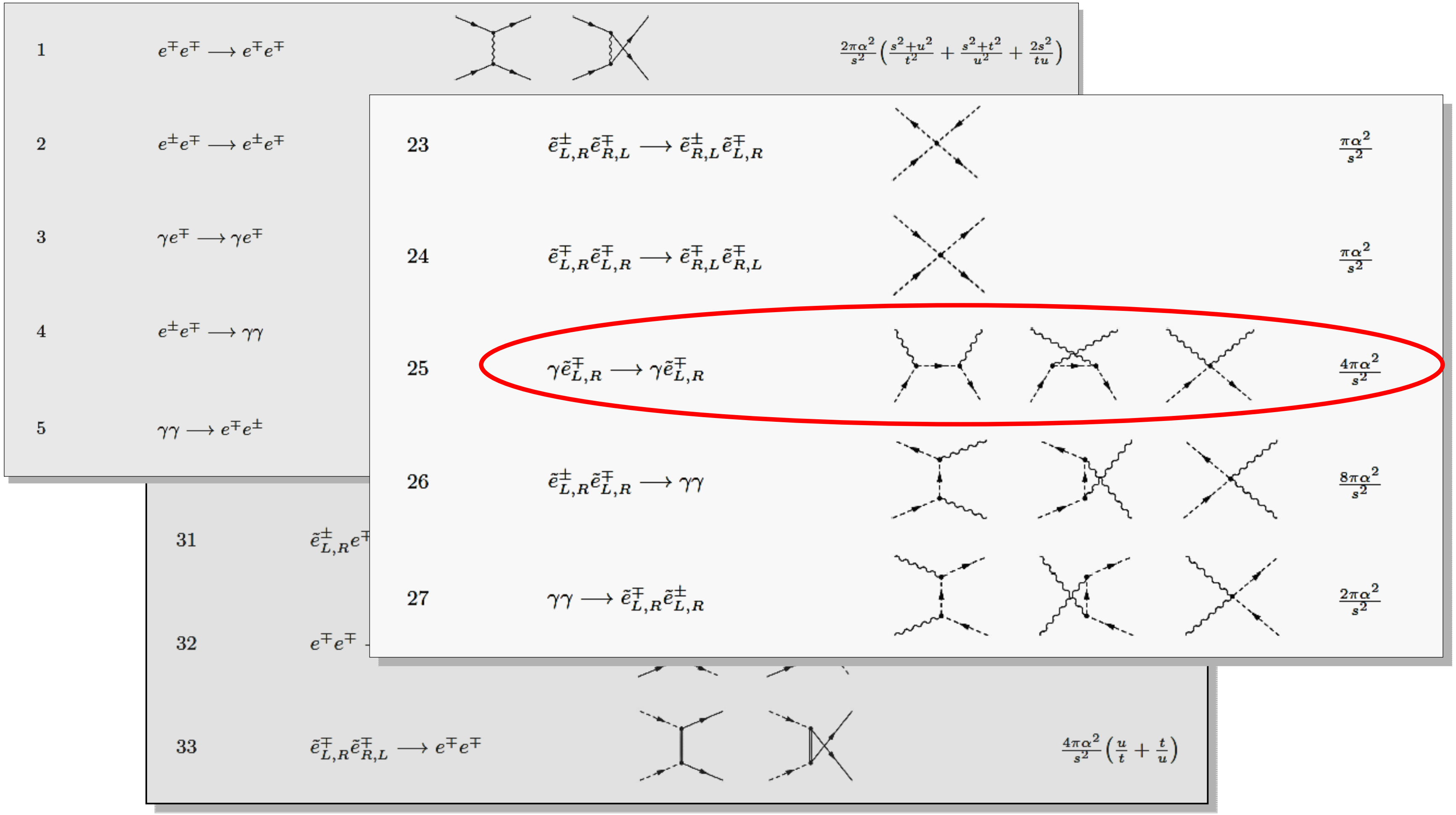
- **not constrained by dimensional arguments**

$$\frac{dE}{dx} \sim T^2, ET, E^2, \ldots$$

E – energy of test particle

- **depend on a specific scattering process under consideration**

Elementary processes in SUSY QED



1	$e^{\mp}e^{\mp} \longrightarrow e^{\mp}e^{\mp}$	$\frac{2\pi\alpha^2}{s^2}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}+\frac{2s^2}{tu}\right)$
2	$e^{\pm}e^{\mp} \longrightarrow e^{\pm}e^{\mp}$	
3	$\gamma e^{\mp} \longrightarrow \gamma e^{\mp}$	
4	$e^{\pm}e^{\mp} \longrightarrow \gamma\gamma$	
5	$\gamma\gamma \longrightarrow e^{\mp}e^{\pm}$	

23	$\tilde{e}^{\pm}_{L,R}\tilde{e}^{\mp}_{R,L} \longrightarrow \tilde{e}^{\pm}_{R,L}\tilde{e}^{\mp}_{L,R}$	$\frac{\pi\alpha^2}{s^2}$
24	$\tilde{e}^{\mp}_{L,R}\tilde{e}^{\mp}_{L,R} \longrightarrow \tilde{e}^{\mp}_{R,L}\tilde{e}^{\mp}_{R,L}$	$\frac{\pi\alpha^2}{s^2}$
25	$\gamma\tilde{e}^{\mp}_{L,R} \longrightarrow \gamma\tilde{e}^{\mp}_{L,R}$	$\frac{4\pi\alpha^2}{s^2}$
26	$\tilde{e}^{\pm}_{L,R}\tilde{e}^{\mp}_{L,R} \longrightarrow \gamma\gamma$	$\frac{8\pi\alpha^2}{s^2}$
27	$\gamma\gamma \longrightarrow \tilde{e}^{\mp}_{L,R}\tilde{e}^{\pm}_{L,R}$	$\frac{2\pi\alpha^2}{s^2}$

31	$\tilde{e}^{\pm}_{L,R}e^{\mp}$	
32	$e^{\mp}e^{\mp}$	
33	$\tilde{e}^{\mp}_{L,R}\tilde{e}^{\mp}_{R,L} \longrightarrow e^{\mp}e^{\mp}$	$\frac{4\pi\alpha^2}{s^2}\left(\frac{u}{t}+\frac{t}{u}\right)$

Energy loss in SUSY QED

A selectron is traversing an equilibrium photon gas

$$\left|\mathcal{M}\right|^2 = 4e^4$$

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2 \left[1 - \frac{12\zeta(3)}{\pi^2}\frac{T}{E}\right] \underset{E>>T}{\approx} -\frac{e^4}{2^5 3\pi} T^2$$

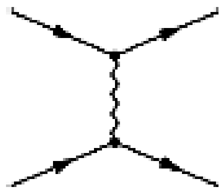
Comparison with Coulomb-like interaction

Energy loss for contact interaction

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2$$

Energy loss for Coulomb-like interaction

$$\frac{dE}{dx} = -\frac{e^4}{48\pi^3} T^2 \left(\ln \frac{E}{eT} + 2.031 \right)$$



E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991)

Energy loss

		Contact $\lvert\mathcal{M}\rvert^2 \sim e^4$	Coulomb $\lvert\mathcal{M}\rvert^2 \sim e^4 \dfrac{s^2}{t^2}$
energy change in single collision	ΔE	$\sim E$	$\sim e^2 T$
cross section	σ	$\sim \dfrac{e^4}{ET}$	$\sim \dfrac{e^2}{T^2}$
density	ρ	$\sim T^3$	$\sim T^3$
inverse mean path	$\lambda^{-1} = \sigma\rho$	$\sim \dfrac{e^4 T^2}{E}$	$\sim e^2 T$
energy loss	$\dfrac{dE}{dx} \sim \dfrac{\Delta E}{\lambda}$	$\sim e^4 T^2$	$\sim e^4 T^2$

Different interactions lead to the same energy loss!

Conclusions

Description of supercharged systems needs to be settled

Supersymmetry does not make fermionic modes unstable

In the weak coupling regime supersymmetric plasma systems are very similar to their non-supersymmetric counterparts