Energy Loss in Unstable Quark-Gluon Plasma

Stanisław Mrówczyński

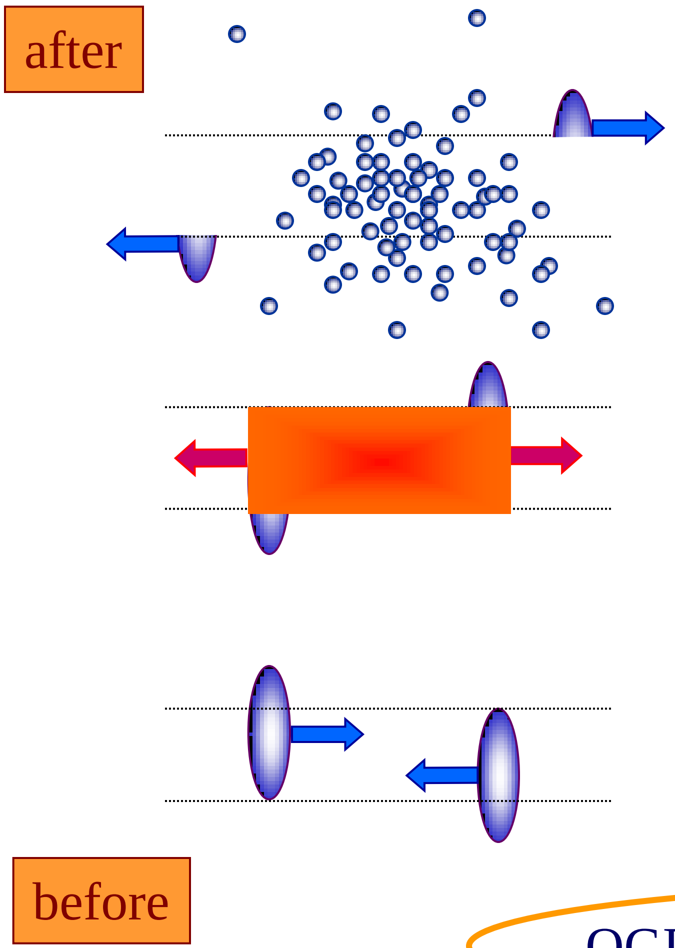
Jan Kochanowski University, Kielce, Poland
& National Centre of Nuclear Reserach, Warsaw, Poland

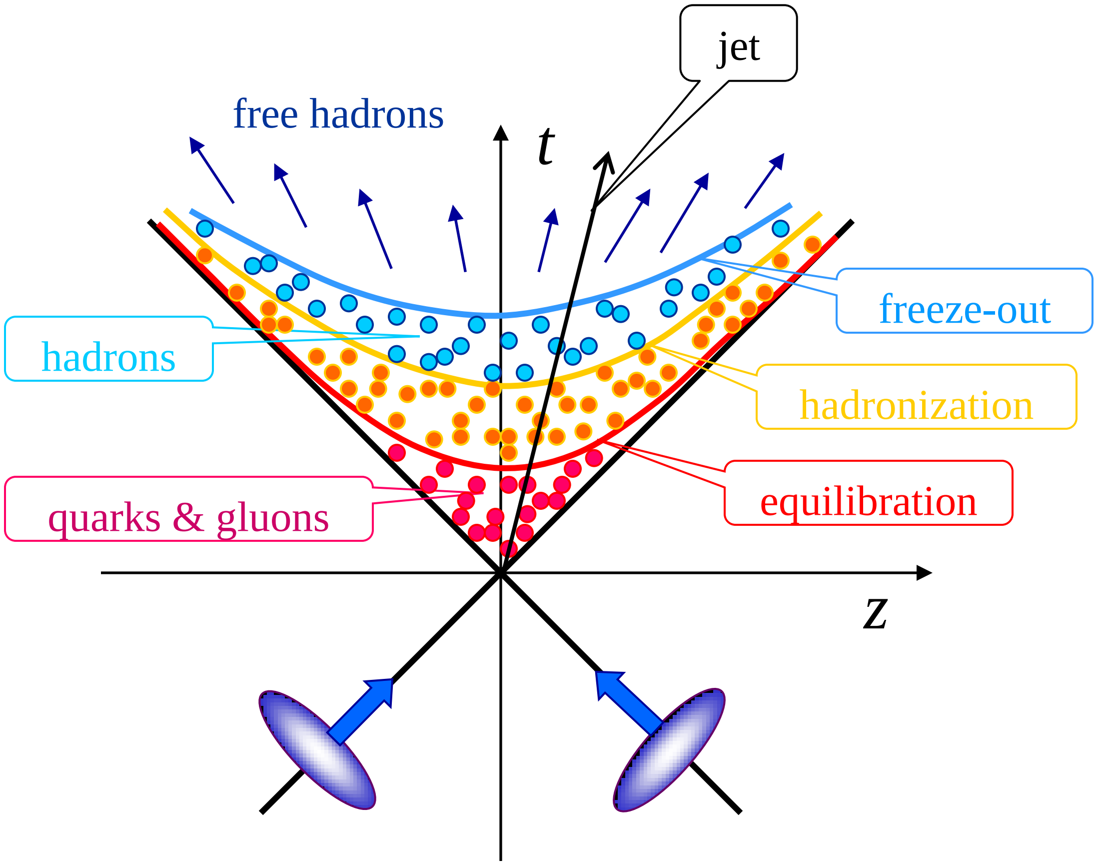
in collaboration with

Margaret Carrington & Katarzyna Deja

based on Physical Review C **92**, 044914 (2015) and arXiv:1607.02359

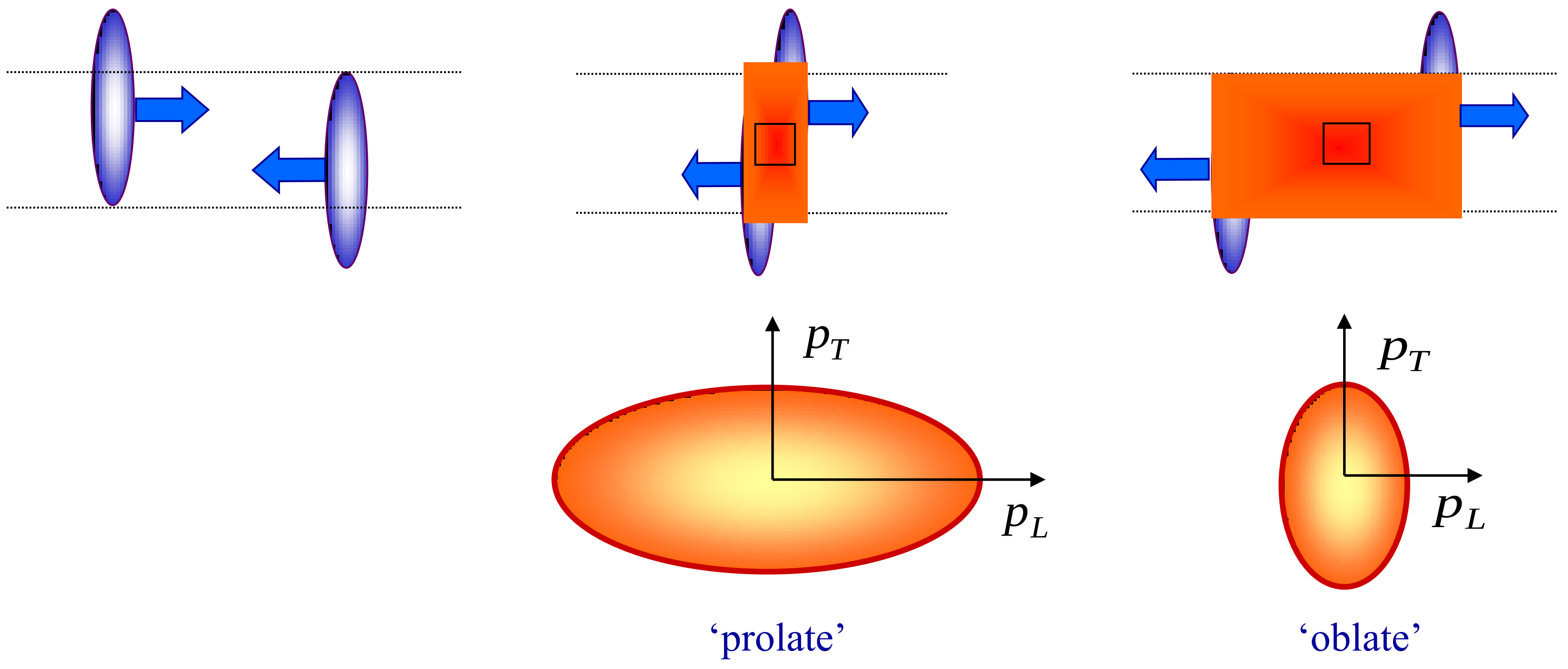
Scenario of relativistic heavy-ion collisions





QGP is out of equilibrium at the collision early stage

Anisotropic QGP



Anisotropic weakly copuled QGP is unstable
due to magnetic plasma modes

Questions

What is a magnitude of energy loss in a high-energy parton when it is traversing an unstable QGP?

- What is a magnitude of collisional energy loss?

- What is a magnitude of radiative energy loss?

$$\frac{dE^{\text{rad}}}{dx} = -\frac{g^2 N_c}{32\pi} L\,\hat{q}\,, \qquad \hat{q} = ? \qquad g << 1$$

R. Baier, *et al.* Nucl. Phys. B **484**, 265 (1997)

A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$
\begin{cases}
\dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\
\dfrac{dp^{\mu}(\tau)}{d\tau} = gQ_a(\tau)\,F_a^{\mu\nu}(x(\tau))\,u_{\nu}(\tau) \\
\dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}p_{\mu}(\tau)\,A_b^{\mu}(x(\tau))\,Q_c(\tau)
\end{cases}
$$

<u>Simplifications</u>

Gauge condition: $p_{\mu}(\tau)\,A_b^{\mu}\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$

Parton travels with constant velocity: $u^{\mu} = (\gamma, \gamma\mathbf{u}) = \text{const} \;\;\&\;\; \mathbf{u}^2 = 1$

Collisional energy loss

$$\frac{dE(t)}{dt} = gQ_a \, \mathbf{E}_a(t, \mathbf{r}(t)) \cdot \mathbf{u}$$

induced & spontaneously generated chromoelectric field

parton's current: $\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{u}t)$

$$\frac{dE(t)}{dt} = \int d^3r \, \mathbf{E}_a(t, \mathbf{r}) \cdot \mathbf{j}_a(t, \mathbf{r})$$

Transverse momentum broadening

$$\hat{q}(t) \equiv \frac{d}{dt}\left(\delta^{ij} - u^i u^j\right)\left\langle p^i(t)\, p^j(t)\right\rangle$$

$$\left\langle p^i(t)\, p^j(t)\right\rangle = \left\langle p^i(0)\, p^j(0)\right\rangle + g^2 \frac{C_R}{N_c^2 - 1}\int_0^t dt_1 \int_0^t dt_2 \left\langle F_a^i(t_1, \mathbf{r}_1)\, F_a^j(t_2, \mathbf{r}_2)\right\rangle$$

Lorentz force

test parton trajectory

$$\mathbf{F}_a(t,\mathbf{r}) \equiv \mathbf{E}_a(t,\mathbf{r}) + \mathbf{u}\times\mathbf{B}_a(t,\mathbf{r}), \qquad \mathbf{r}_i \equiv \mathbf{r}_0 + \mathbf{u}\, t_i\,, \quad i = 1, 2$$

color factor

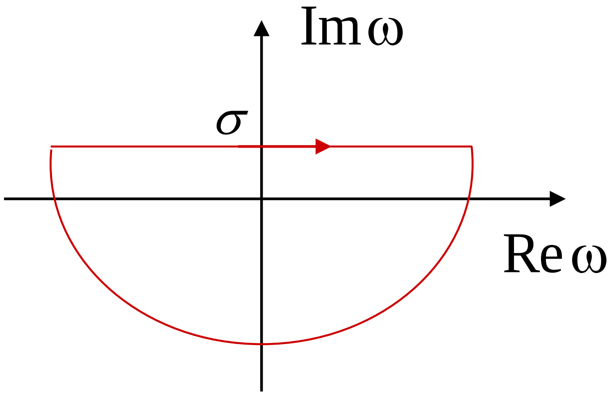
$$C_R \equiv \begin{cases} \dfrac{N_c^2 - 1}{2N_c} & \text{for quark} \\ N_c & \text{for gluon} \end{cases}$$

Initial value problem

One-sided Fourier transformation

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^{\infty} dt \int d^3r\, e^{i(\omega t-\mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t-\mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{u}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a\mathbf{u}}{\omega-\mathbf{k}\cdot\mathbf{u}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega-\mathbf{ku})t} \mathbf{E}_a(\omega,\mathbf{k})\cdot\mathbf{u}$$

Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$

$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k})$$

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1-\frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

dynamical information about medium

$$E^i(\omega,\mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})[\omega\mathbf{j}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Magnetic Field

Faraday law

$$\mathbf{B}(\omega,\mathbf{k}) = \frac{1}{\omega}\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) + \frac{i}{\omega}\mathbf{B}_0(\mathbf{k})$$

Collisional energy loss

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}$$

$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[\frac{igQ_a \omega \mathbf{v}}{\omega - \overline{\omega}} + \mathbf{k} \times \mathbf{B}_0^a(\mathbf{k}) - \omega \mathbf{D}_0^a(\mathbf{k}) \right]^j$$

current of the test parton

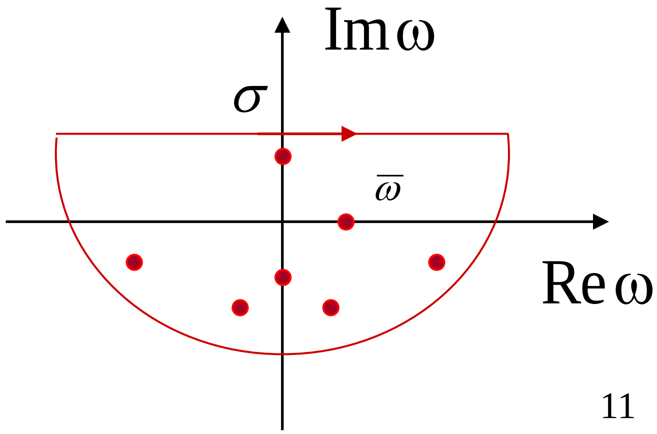
initial values of the fields

$\overline{\omega} \equiv \mathbf{k} \cdot \mathbf{u}$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\det[\Sigma(\omega,\mathbf{k})] = 0$$



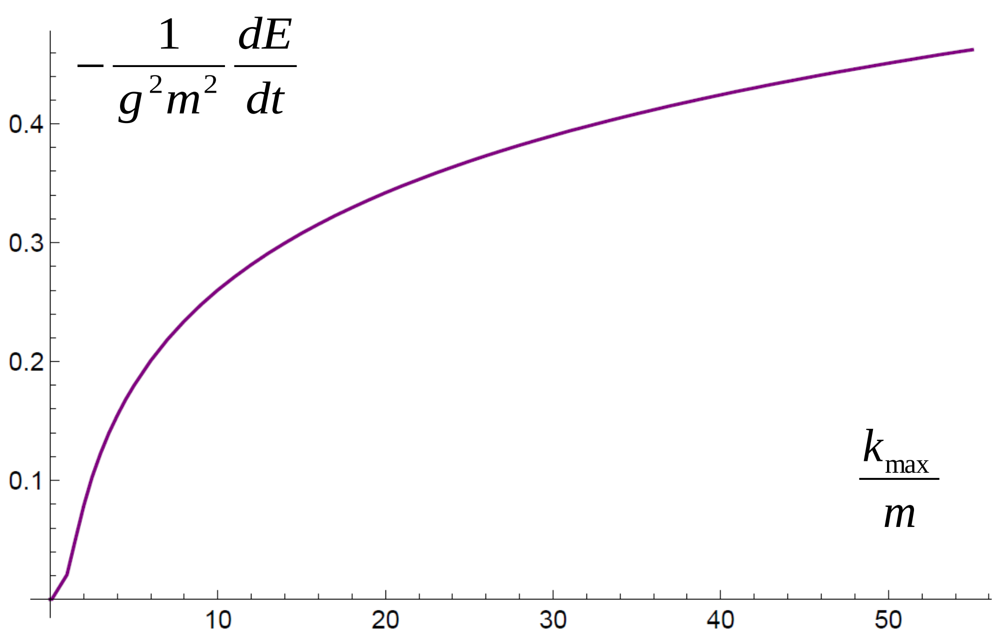
Collisional energy loss in equilibrium QGP

The initial conditions are *forgotten*

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right]$$

$$\overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$$

equivalent to the standard result by Braaten & Thoma



Debye mass

$$m^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

How to choose the field initial values?

1) The initial fields vanish: $\mathbf{D}_0(\mathbf{k}) = \mathbf{B}_0(\mathbf{k}) = 0$

2) The initial fields are independent of the parton's current.

1) is equivalent to 2)

The effect of the initial fields cancels out after an averaging over parton's colors.

$$\int dQ\, Q_a = 0, \qquad \int dQ\, Q_a\, Q_b = C_2 \delta^{ab}, \qquad C_2 \equiv \begin{cases} \frac{1}{2} & \text{for} \quad \text{quark} \\ N_c & \text{for} \quad \text{gluon} \end{cases}$$

How to choose the field initial values?

State of the test parton is, in general, correlated with state of the plasma.

Maximal correlation: the initial fields are induced by the parton's current.

$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t), \quad t\in(-\infty,\infty)$$

Maxwell equations ↓ Two-sided Fourier transformation

Initial values:

$$D_0^i(\mathbf{k}) = -igQ_a\overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

Collisional energy loss

$$\frac{dE(t)}{dt} = ig^2 v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t}$$

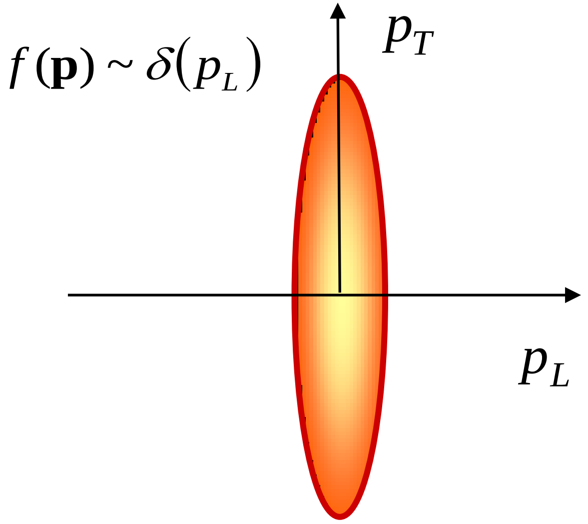
$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left\{ \frac{\omega\delta^{jl}}{\omega-\varpi} + \right.$$

$$\left. + \cos\varphi \Big[\underbrace{(k^j k^k - \mathbf{k}^2)(\Sigma^{-1})^{jk}(\varpi,\mathbf{k})}_{\mathbf{k}\times\mathbf{B}_0(\mathbf{k})} - \underbrace{\omega\varpi\varepsilon^{ij}(\varpi,\mathbf{k})(\Sigma^{-1})^{jk}(\varpi,\mathbf{k})}_{\omega\mathbf{D}_0(\mathbf{k})} \Big] \right\}$$

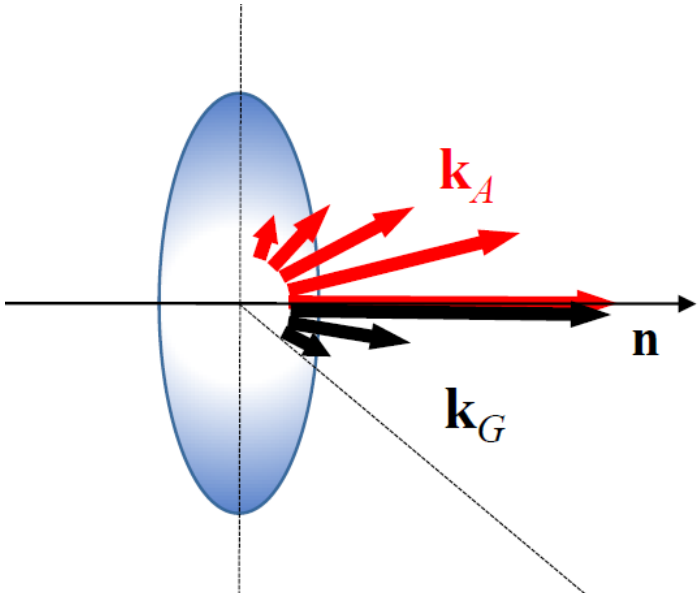
$-1 \le \cos\varphi \le 1$ arbitrary phase factor

- $\cos\varphi = 0$ ucorrelated initial fields
- $\cos\varphi = \begin{cases} +1 & \text{maximal correlation} \\ -1 & \text{maximal anticorrelation} \end{cases}$

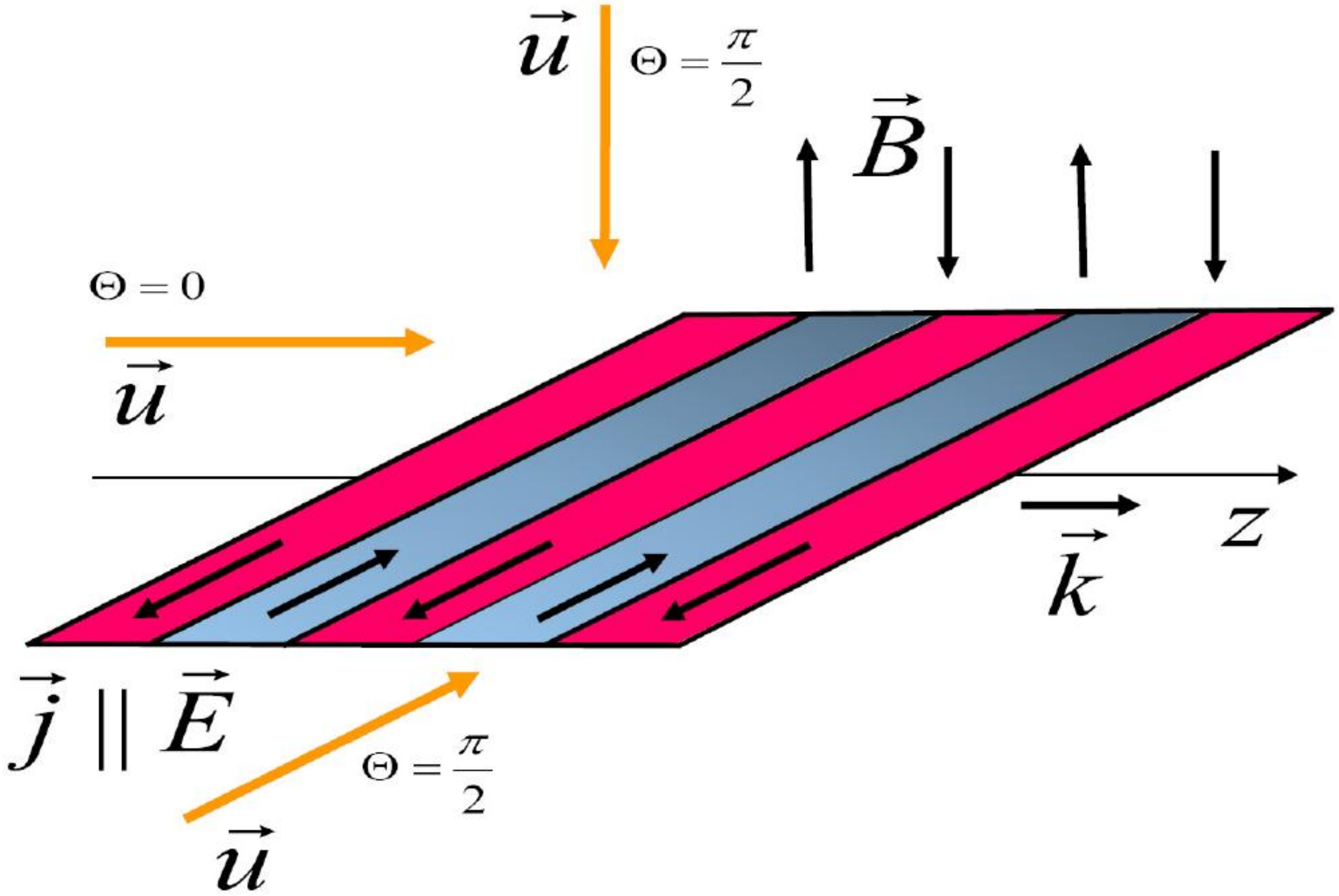
Extremely oblate QGP



2 unstable modes



Extremely oblate QGP



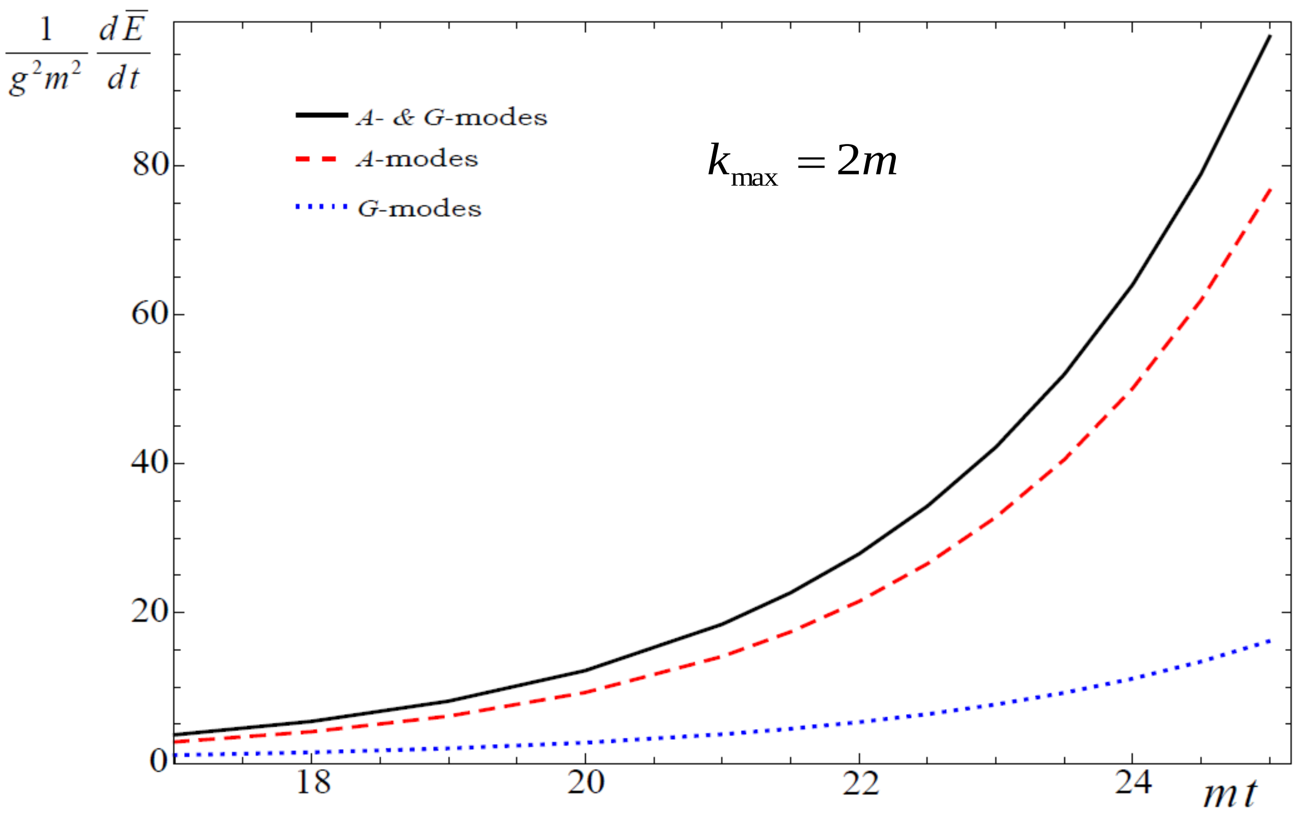
Collisional energy loss in extremely oblate QGP

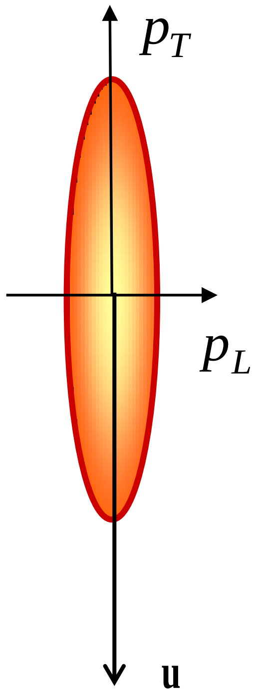
Nothing spectacular happens with *uncorrelated* initial fields!

$$\frac{dE}{dt} \approx \left.\frac{dE}{dt}\right|_{\mathrm{eq}}$$

Collisional energy loss in extremely oblate QGP

Correlated initial fields



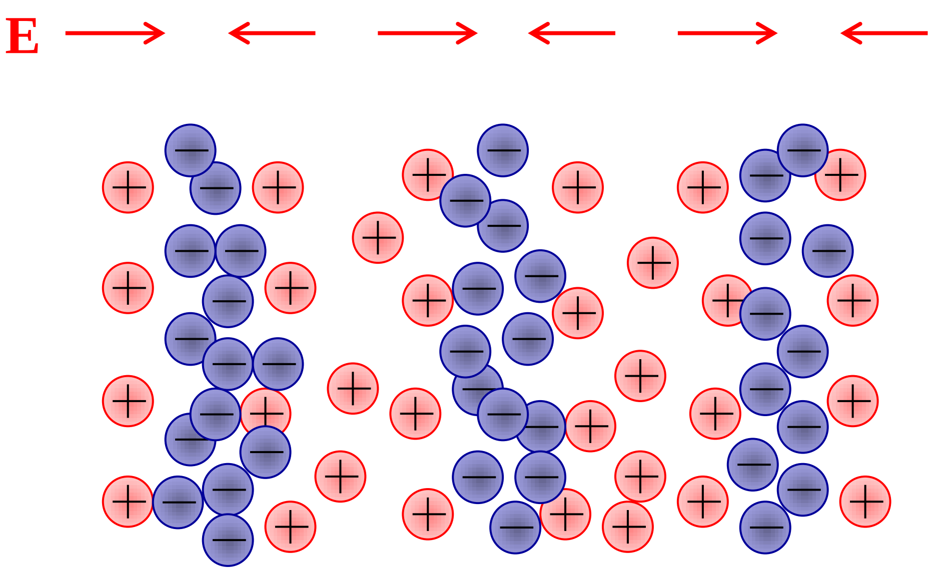


$$-\frac{1}{g^2 m^2}\frac{dE}{dt}\bigg|_{\text{eq}} = 0.08$$

energy gain for cosφ < 0

energy loss for cosφ > 0

Plasma accelerator



T. Tajima & J. M. Dawson,
Phys. Rev. Lett. **43**, 267 (1979)

$E_e = 1\,\mathrm{GeV}\ @\ 3.3\,\mathrm{cm}$

W. P. Leemans *et al.*,
Nature Phys. **2**, 696 (2006).

$$E^z(t,x) = E_0 \cos(\omega_0 t - kz)$$

$$v = v_\varphi = \frac{\omega_0}{k}$$

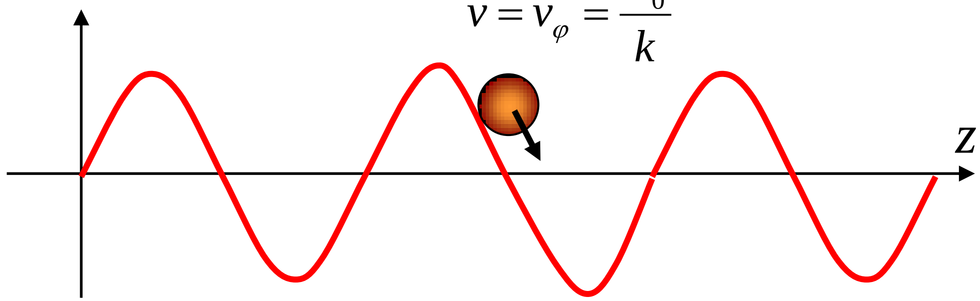
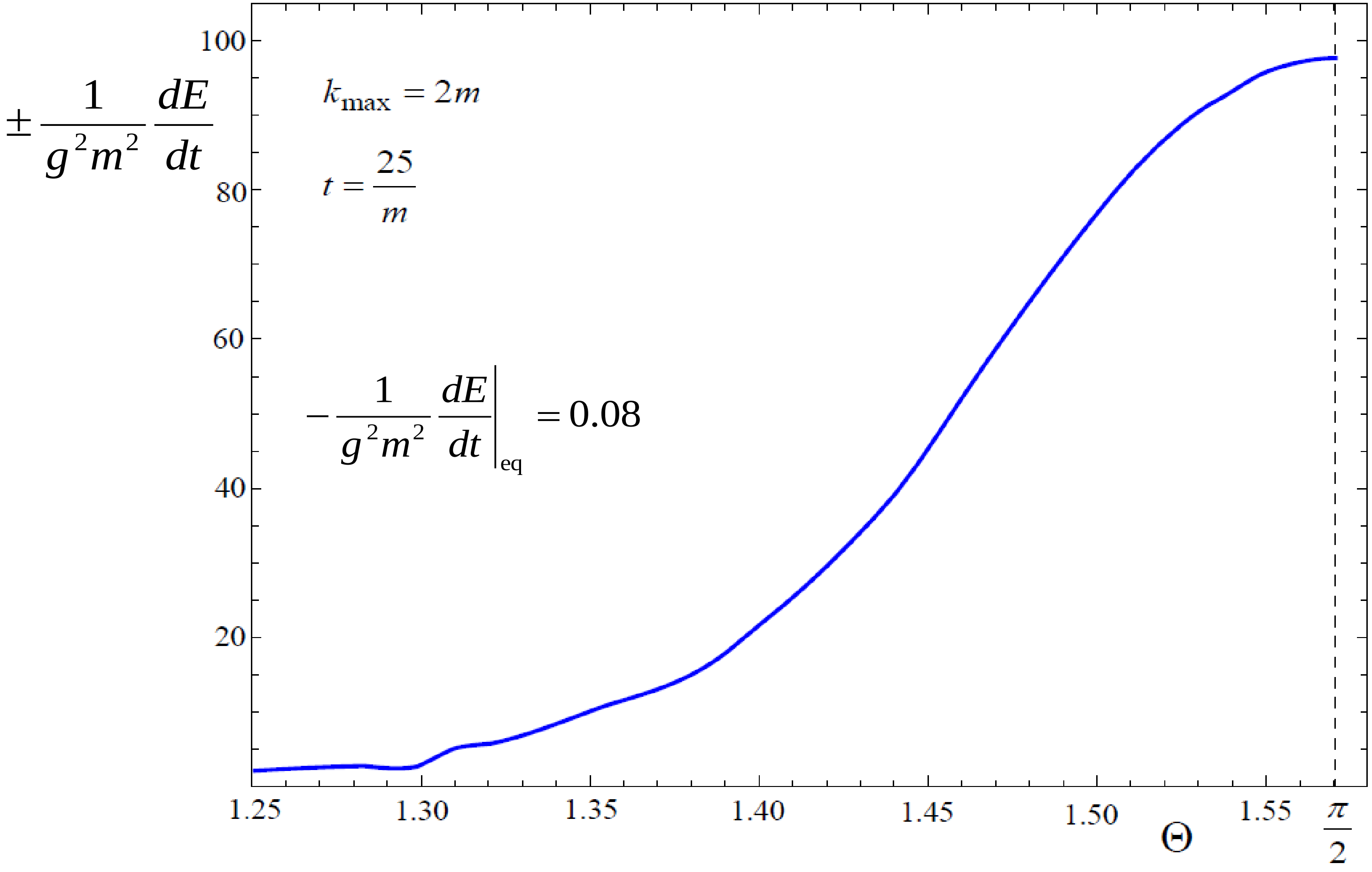
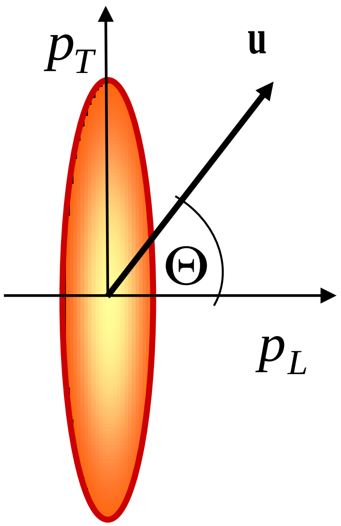


Table-top high-energy accelerator!

Collisional energy loss in extremely oblate QGP

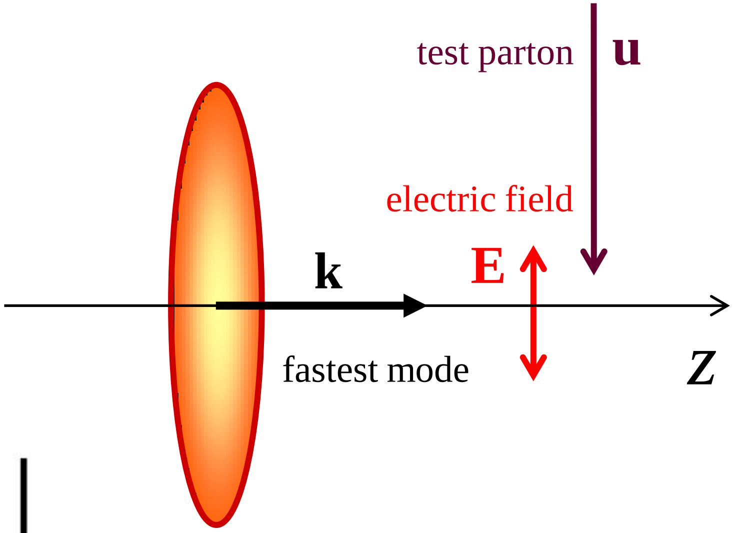
Angular dependence

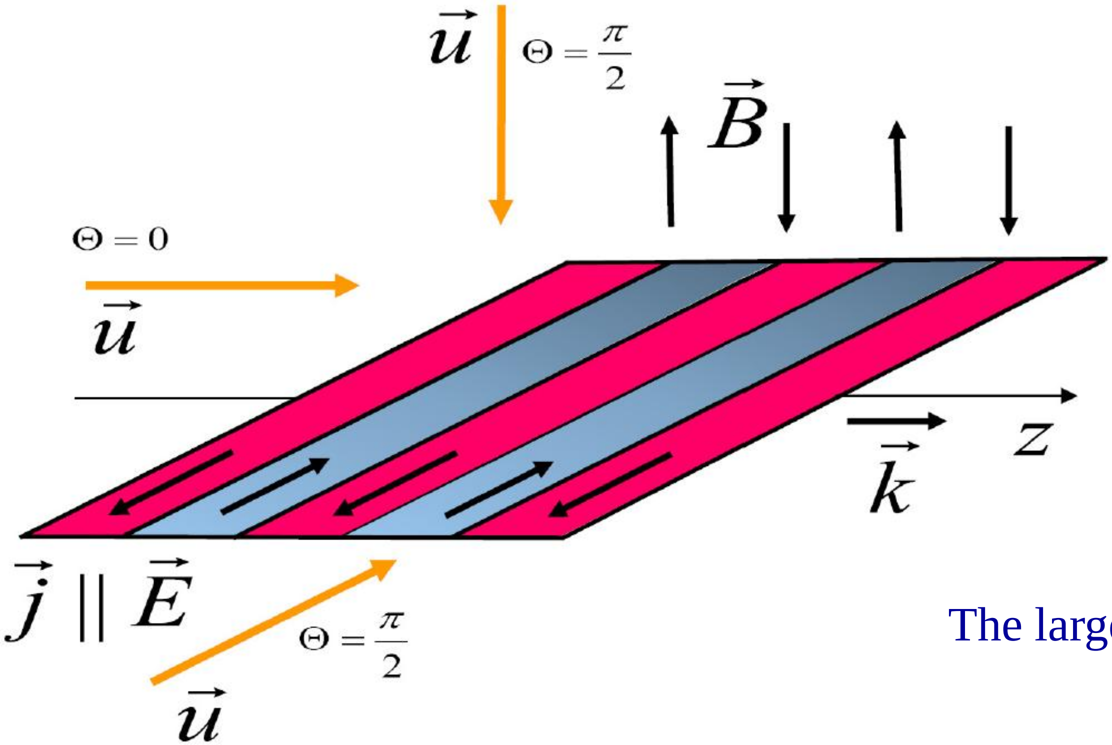




Angular dependence

$$\frac{dE}{dt} \sim \mathbf{E}\cdot\mathbf{j} \sim \mathbf{E}\cdot\mathbf{u}$$





The largest *dE/dt* for **u** transverse to *z* !

Momentum broadening

$$\hat{q}(t) = g^2 \frac{C_R}{N_c^2 - 1} \frac{d}{dt} \int_0^t dt_1 \int_0^t dt_2 \left[\left\langle E_a^i(t_1, \mathbf{r}_1) E_a^i(t_2, \mathbf{r}_2) \right\rangle + \left\langle E_a^i(t_1, \mathbf{r}_1) B_a^i(t_2, \mathbf{r}_2) \right\rangle + \ldots\ldots \right]$$

$$\mathbf{r}_i \equiv \mathbf{r}_0 + \mathbf{u}\, t_i\,, \quad i = 1, 2$$

Solution of the Maxwell equations

$$E_a^i(\omega, \mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega, \mathbf{k})[\omega \mathbf{j}_a(\omega, \mathbf{k}) + \mathbf{k} \times \mathbf{B}_a^0(\mathbf{k}) - \omega \mathbf{D}_a^0(\mathbf{k})]^j$$

Solution of the Vlasov equation

$$\mathbf{j}_a(\omega, \mathbf{k}) = g \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}} \delta N_a^0(\mathbf{k}, \mathbf{p})$$

Solution of the 3rd Maxwell equation

$$\mathbf{B}(\omega, \mathbf{k}) = \frac{1}{\omega} \mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) + \frac{i}{\omega} \mathbf{B}_0(\mathbf{k})$$

Initial condition

All correlators of initial values such as

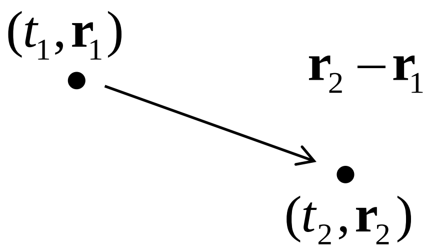
$$\left\langle \delta N_a^0(\mathbf{r}_1,\mathbf{p}_1)\,\delta N_a^0(\mathbf{r}_2,\mathbf{p}_2)\right\rangle, \left\langle \delta N_a^0(\mathbf{r}_1,\mathbf{p}_1)\,E_b^0(\mathbf{r}_2)\right\rangle,$$

$$\left\langle \delta N_a^0(\mathbf{r}_1)\,E_b^0(\mathbf{r}_2)\right\rangle, \left\langle \delta N_a^0(\mathbf{r}_1)\,E_b^0(\mathbf{r}_2)\right\rangle, \left\langle E_a^0(\mathbf{r}_1)\,E_b^0(\mathbf{r}_2)\right\rangle,$$

$$\left\langle E_a^0(\mathbf{r}_1)\,B_b^0(\mathbf{r}_2)\right\rangle, \left\langle B_a^0(\mathbf{r}_1)\,E_b^0(\mathbf{r}_2)\right\rangle, \ldots\ldots\ldots\ldots$$

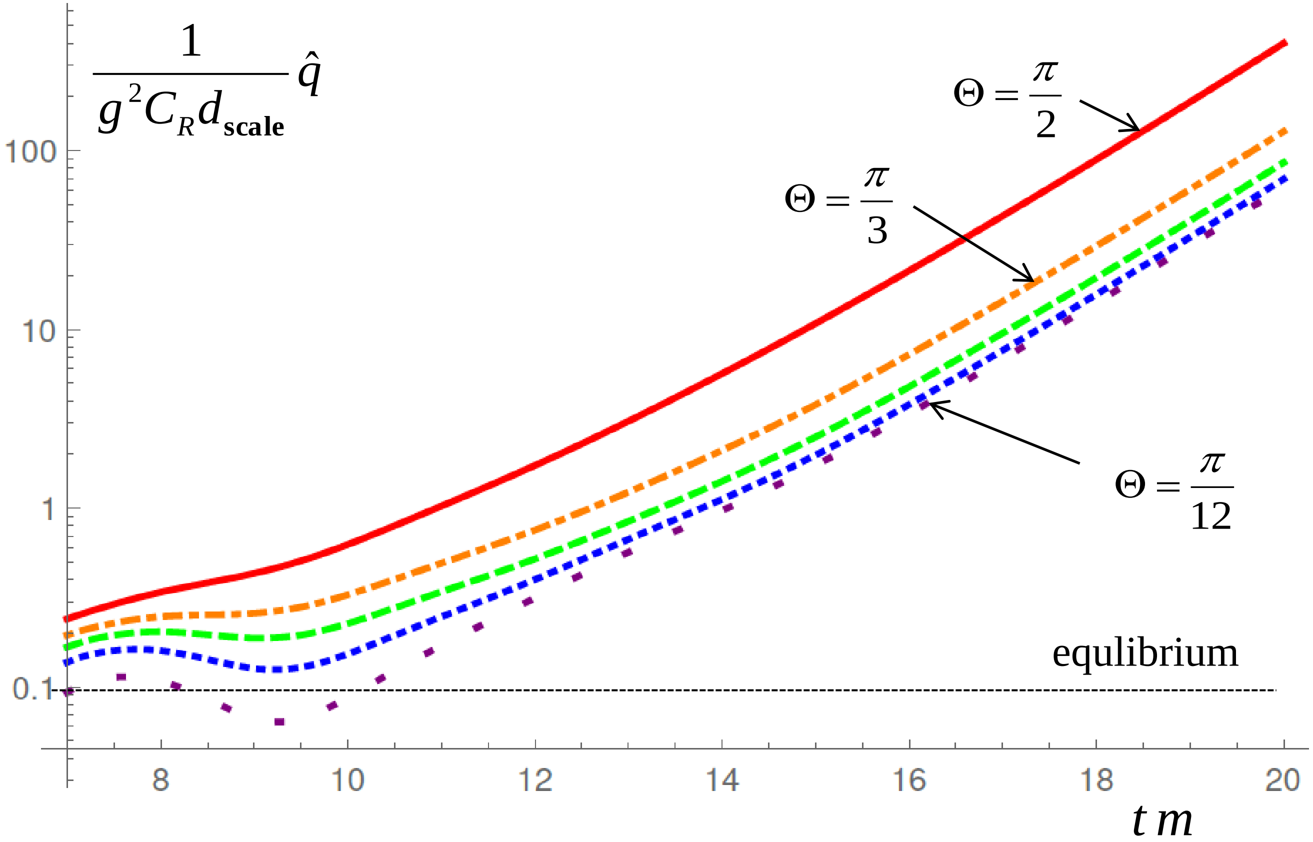
are computed with the correlation function of free particles

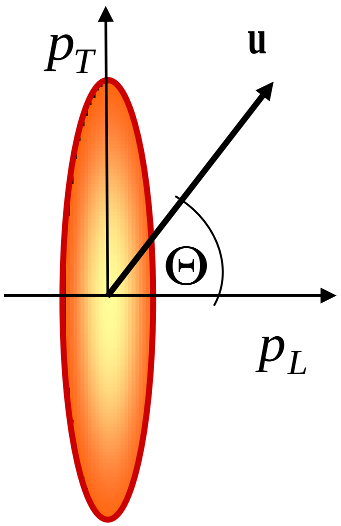
$$\left\langle \delta N_a(t_1,\mathbf{r}_1,\mathbf{p}_1)\,\delta N_b(t_2,\mathbf{r}_2,\mathbf{p}_2)\right\rangle = \delta^{ab}\delta^{(3)}(\mathbf{p}_1-\mathbf{p}_2)\,\delta^{(3)}(\mathbf{r}_1-\mathbf{r}_2-\mathbf{v}_1(t_1-t_2))\,f(\mathbf{p})$$



$$\mathbf{r}_2-\mathbf{r}_1=\mathbf{v}_1(t_2-t_1)$$

Momentum broadening in extremely oblate QGP



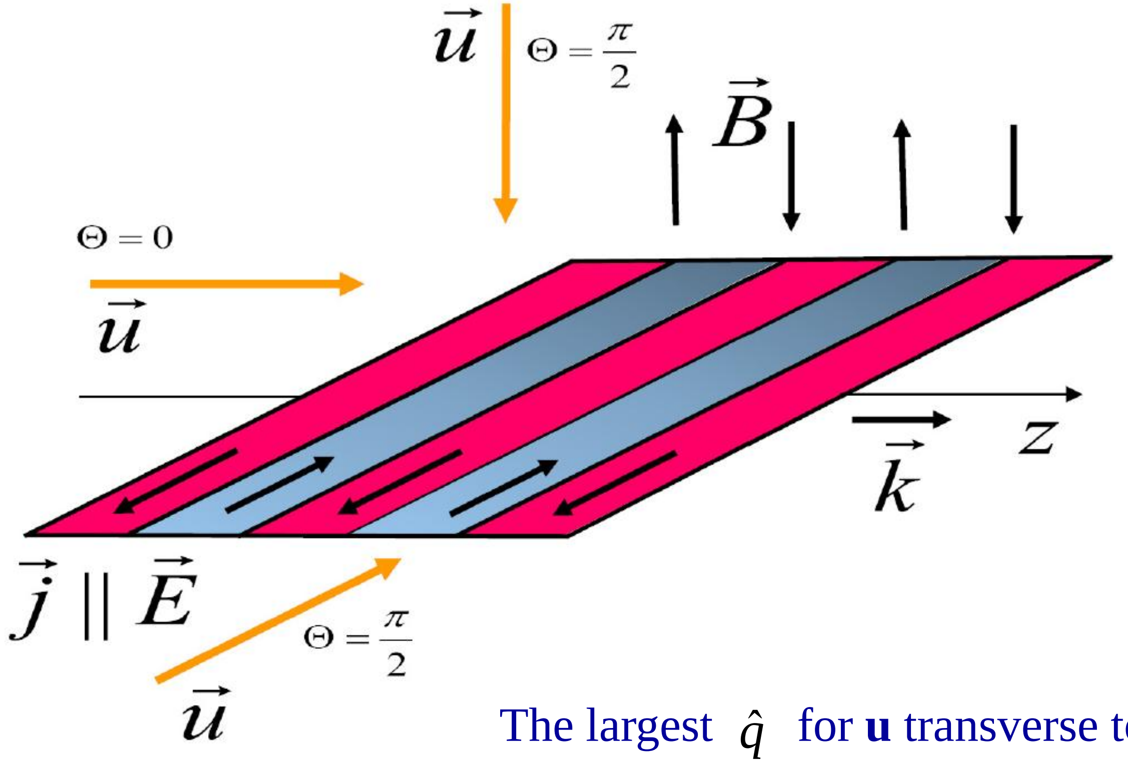


$\hat{q}^{\text{eq}} = 0.11 g^2 C_R d_{\text{scale}}$ $d_{\text{scale}} \equiv \frac{1}{2} \langle p_T \rangle m^2$ $k_{\max} = 2m$

Angular dependence

$$\hat{q} \sim \left\langle \mathbf{F}^2 - (\mathbf{F}\cdot\mathbf{u})^2 \right\rangle$$

$$\mathbf{F} = \mathbf{E} + \mathbf{u}\times\mathbf{B}$$



The largest $\hat{q}$ for **u** transverse to *z* !

Conclusions

- dE/dt crucially depends on initial conditions;
- $dE/dt > 0$ & $dE/dx < 0$;
- dE/dt strongly varies with time and direction;
- $|dE/dt|$ can be much bigger than in equilibrium QGP;
- $\hat{q}$ varies with time and direction;
- $\hat{q}$ can be much bigger than in equilibrium QGP;