Deuteron formation & hadron-deuteron correlations

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Producion of light nuclei in nucleus-nucleus collisions

Genuine production



Shattering of incoming nuclei



hard process

Final state interaction



S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963) A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a deuteron



$$\frac{1}{2}\frac{dN^{np}}{d^{3}\mathbf{p}_{n}d^{3}\mathbf{p}_{p}} \approx \frac{dN^{pp}}{d^{3}\mathbf{p}_{p}d^{3}\mathbf{p}_{p}} \approx \left(\frac{dN^{p}}{d^{3}\mathbf{p}_{p}}\right)^{2}$$

$$\frac{dN^{D}}{d^{3}\mathbf{P}_{D}} = A_{D} \left(\frac{dN^{p}}{d^{3}\mathbf{p}_{p}}\right)^{2}$$

Deuteron formation rate



H. Sato and K. Yazaki, Phys. Lett. B 98, 153 (1981)

Quantum-mechanical meaning of the formation rate formula



Transition matrix element

$$M = \left| \int d^{3}\mathbf{r} \psi^{*}(\mathbf{r}) \varphi(\mathbf{r}) \right|^{2} = \int d^{3}\mathbf{r} d^{3}\mathbf{r}' \varphi^{*}(\mathbf{r}') \psi(\mathbf{r}') \psi^{*}(\mathbf{r}) \varphi(\mathbf{r})$$

$$M = \int d^{3}\mathbf{r} d^{3}\mathbf{r}' \varphi^{*}(\mathbf{r}') \rho(\mathbf{r}',\mathbf{r}) \varphi(\mathbf{r})$$

If density matrix is diagonal

$$\rho(\mathbf{r}',\mathbf{r}) = S(\mathbf{r})\,\delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad M = \int d^3\mathbf{r}\,S(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2$$

Diagonal density matrix

$$\langle \psi | \hat{A} | \psi \rangle = \sum_{i,j} c_i^* c_j \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_{i,j} \rho_{ji} A_{ij}$$
$$| \psi \rangle = \sum_i c_i | \alpha_i \rangle \qquad \rho_{ji} \equiv c_i^* c_j \qquad A_{ij} \equiv \langle \alpha_i | \hat{A} | \alpha_j \rangle$$

density matrix

- averaging over time or events

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$$\overline{\langle \psi | \hat{A} | \psi \rangle} = \sum_{i,j} \overline{c_i^* c_j} \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_i |c_i|^2 A_{ii}$$
$$\overline{\rho_{ji}} = \overline{c_i^* c_j} = \delta^{ij} |c_i|^2 \quad \text{random phase approximation}$$

diagonal density matrix

Energy-momentum conservation



Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G 11, 1087 (1987)

Emission time

Instantaneous emission

$$A_{D} = \frac{3}{4} (2\pi)^{3} \int d^{3}\mathbf{r}_{1} d^{3}\mathbf{r}_{2} S^{N}(\mathbf{r}_{1}) S^{N}(\mathbf{r}_{2}) |\psi(\mathbf{r}_{1},\mathbf{r}_{2})|^{2}$$



Emission extended in time

 $S_r^{np}(\mathbf{r}) \equiv \int dt \; S_r^{np}\left(t, \mathbf{r} - \mathbf{v}t\right)$

$$A_{D} = \frac{3}{4} (2\pi)^{3} \int dt_{1} d^{3} \mathbf{r}_{1} dt_{2} d^{3} \mathbf{r}_{2} S^{N}(t_{1}, \mathbf{r}_{1}) S^{N}(t_{2}, \mathbf{r}_{2}) \left| \psi(\mathbf{r}_{1} + \mathbf{v}t_{1}, \mathbf{r}_{2} + \mathbf{v}t_{2}) \right|^{2}$$

$$S_r^{np}(t,\mathbf{r}) \equiv \int dT \, d^3 \mathbf{R} \, S^N \left(T - \frac{1}{2}t, \mathbf{R} - \frac{1}{2}\mathbf{r} \right) S^N \left(T + \frac{1}{2}t, \mathbf{R} + \frac{1}{2}\mathbf{r} \right)$$

 $A_D = \frac{3}{4} (2\pi)^3 \int d^3 \mathbf{r} \, S_r^{np}(\mathbf{r}) \left| \varphi(\mathbf{r}) \right|^2$

 $R_s \rightarrow \sqrt{R_s^2 + v^2 \tau^2}$

 $S^{N}(t,\mathbf{r}) = \left(\frac{1}{2\pi\tau^{2}}\right)^{1/2} \left(\frac{1}{2\pi R_{c}^{2}}\right)^{3/2} \exp\left(-\frac{t^{2}}{2\tau^{2}}\right) \exp\left(-\frac{\mathbf{r}^{2}}{2R_{c}^{2}}\right)$

Deuteron formation & n-p correlations



Sum rule due to completeness of quantum states

Lednicky-Lyuboshitz formula

St. Mrówczyński, Phys. Lett. B 277, 43 (1992)

R. Maj & St. Mrówczyński, Phys. Rev. C 101, 014901 (2020) R. Maj & St. Mrówczyński, Phys. Rev. C 71, 044905 (2005) St. Mrówczyński, Phys. Lett. B 345, 393 (1995)

Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



St. Mrówczyński & P. Słoń, Acta Phys. Pol. B 51, 1739 (2020)

1) Deuteron is treated as an elementary particle

Experimental definition



$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$

Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h \, d^3 r_D \, S^h(\mathbf{r}_h) \, S^D(\mathbf{r}_D) \left| \psi(\mathbf{r}_h, \mathbf{r}_D) \right|^2$$

distribution of emission points

h-D wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977) R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

1) Deuteron is treated as an elementary particle cont.

Separation of CM and relative motion

$$C(\mathbf{q}) = \int d^3 r S_r^{hD}(\mathbf{r}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$

"Relative" source function

h

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) A_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_h} \frac{dN_p}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$

Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) A_D = \int d^3 r_h d^3 r_n d^3 r_p S^h(\mathbf{r}_h) S^N(\mathbf{r}_n) S^N(\mathbf{r}_p) \left| \psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) \right|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = A_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \qquad \qquad \frac{1}{2} \mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$A_{D} = \frac{3}{8} (2\pi)^{3} \int d^{3}\mathbf{r}_{n} d^{3}\mathbf{r}_{p} S^{N}(\mathbf{r}_{n}) S^{N}(\mathbf{r}_{p}) \left| \psi_{D}(\mathbf{r}_{n},\mathbf{r}_{p}) \right|^{2} = \frac{3}{8} (2\pi)^{3} \int d^{3}r_{np} S^{np}_{r}(\mathbf{r}_{np}) \left| \phi_{D}(\mathbf{r}_{np}) \right|^{2}$$
spin-isopsin factor
$$\psi_{D}(\mathbf{r}_{n},\mathbf{r}_{p}) = e^{i\mathbf{P}\mathbf{R}} \phi_{D}(\mathbf{r}_{np})$$

$$R_{np} = \sqrt{2}R_{N}$$

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B **51**, 1739 (2020) St. Mrówczyński, Phys. Lett. B **864**, 139413 (2025) D

2) Deuteron is treated as a bound state of neutron and proton cont.

Separation of CM and relative motion

$$\mathbf{R} \equiv \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h}$$
$$\mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n$$
$$\mathbf{r}_{hD} \equiv \mathbf{r}_h - \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n}$$

$$\psi(\mathbf{r}_h,\mathbf{r}_n,\mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}}\psi_{hD}^{\mathbf{q}}(\mathbf{r}_{hD},\mathbf{r}_{np})$$

Gaussian source

$$S^{i}(\mathbf{r}) = \left(\frac{1}{2\pi R_{i}^{2}}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^{2}}{2R_{i}^{2}}\right), \quad i = h, N \qquad \qquad S_{3r}^{hD}(\mathbf{r}) = \left(\frac{1}{\pi (2R_{h}^{2} + R_{N}^{2})}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^{2}}{2R_{h}^{2} + R_{N}^{2}}\right)$$

$$\int d^3 R \ S^h(\mathbf{r}_h) S^N(\mathbf{r}_n) \ S^N(\mathbf{r}_p) = S_r^{np}(\mathbf{r}_{np}) \ S_{3r}^{hD}(\mathbf{r}_{hd}) \quad \longleftarrow \quad \frac{\text{Factorization}}{\text{of source functions}}$$

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 r_{hD} \, d^3 r_{np} \, S_r^{np}(\mathbf{r}_{np}) \, S_{3r}^{hD}(\mathbf{r}_{hd}) \left| \psi_{hD}^{\mathbf{q}}(\mathbf{r}_{hD}, \mathbf{r}_{np}) \right|^2$$

2) Deuteron is treated as a bound state of neutron and proton cont.

Factorized 3-body approach

Assumption of factorization

 $\psi_{hD}^{\mathbf{q}}(\mathbf{r}_{hD},\mathbf{r}_{np}) = \phi_{hD}^{\mathbf{q}}(\mathbf{r}_{hD})\,\varphi_{D}(\mathbf{r}_{np})$



$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 r_{np} d^3 r_{hD} S_r^{np}(\mathbf{r}_{np}) S_{3r}^{hD}(\mathbf{r}_{hd}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}_{hd}) \right|^2 \left| \varphi_D(\mathbf{r}_{np}) \right|^2$$

$$C(\mathbf{q}) = \int d^3 r S_{3r}^{hD}(\mathbf{r}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$

Correlation function as in 2-body approach but ...

$$S_{3r}^{hD}(\mathbf{r}) = \left(\frac{1}{2\pi R_{hD}^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_{hD}^2}\right)$$

$$R_{hD} = \sqrt{R_h^2 + \frac{1}{2}R_N^2}$$

For a non-Gaussian source, A_D remains in the correlation function!

Direct vs. final state interaction

Direct production – 2-body approach

$$C(\mathbf{q}) = \int d^{3}r S_{r}^{hD}(\mathbf{r}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}) \right|^{2}$$

$$S_{r}^{hD}(\mathbf{r}) = \left(\frac{1}{2\pi (R_{h}^{2} + R_{D}^{2})} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^{2}}{2(R_{h}^{2} + R_{D}^{2})} \right)$$

$$S_{3r}^{hD}(\mathbf{r}) = \left(\frac{1}{\pi (R_{N}^{2} + 2R_{h}^{2})} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^{2}}{R_{N}^{2} + 2R_{h}^{2}} \right)$$
Final state interaction
$$Factorized$$
3-body approach
$$C(\mathbf{q}) = \int d^{3}r S_{3r}^{hD}(\mathbf{r}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}) \right|^{2}$$

St. Mrówczyński, Phys. Lett. B 864, 139413 (2025)

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Direct vs. final state interaction in *p*-*D*

Direct production – 2-body approach

$$C(\mathbf{q}) = \int d^3 r S_r^{pD}(\mathbf{r}) \left| \phi_{pD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$

$$R_h = R_D = R_N$$

$$S_r^{hD}(\mathbf{r}) = \left(\frac{1}{4\pi R_N^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_N^2}\right)$$

$$S_{3r}^{hD}(\mathbf{r}) = \left(\frac{1}{3\pi R_N^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{\mathbf{3}R_N^2}\right)$$

Final state interaction & factorization

$$C(\mathbf{q}) = \int d^3 r S_{3r}^{pD}(\mathbf{r}) \left| \phi_{pD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B 51, 1739 (2020)





K-D & p-D correlations in 2-body approach



ALICE, Phys. Rev. X 14, 031051 (2024)

p-*D* correlations

$$\psi_{pD}^{\mathbf{q}}(\mathbf{r}_{pD},\mathbf{r}_{np})\neq\phi_{\mathbf{q}}(\mathbf{r}_{pD})\varphi_{D}(\mathbf{r}_{np})$$



Full three-body calculation is needed

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 r_{hD} \, d^3 r_{np} \, S_r^{np}(\mathbf{r}_{np}) \, S_{3r}^{hD}(\mathbf{r}_{hd}) \left| \psi_{hD}^{\mathbf{q}}(\mathbf{r}_{hD}, \mathbf{r}_{np}) \right|^2$$

p-D correlations in 3-body apprach



 $R_N = 1.43 \pm 0.16 \,\mathrm{fm}$

ALICE, Phys. Rev. X **14**, 031051 (2024) M. Viviani et al, Phys. Rev. C **108**, 064002 (2023)

Deuteron-deuteron correlation function

Direct production – 2-body approach

$$C(\mathbf{q}) = \int d^3 r S_r^{DD}(\mathbf{r}) \left| \phi_{DD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$

$$R_D = R_N$$

$$S_r^{DD}(\mathbf{r}) = \left(\frac{1}{4\pi R_N^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_N^2}\right)$$

$$S_{4r}^{DD}(\mathbf{r}) = \left(\frac{1}{2\pi R_N^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_N^2}\right)$$

Final state interaction & factorization

$$C(\mathbf{q}) = \int d^3 r S_{4r}^{DD}(\mathbf{r}) \left| \phi_{hD}^{\mathbf{q}}(\mathbf{r}) \right|^2$$



$$\sqrt{2} \approx 1.41$$



Proton-³He correlation function

Direct production – 2-body approach $C(\mathbf{q}) = \int d^3 r S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$ ³He $R_{\rm He} = R_N$ $S_r(\mathbf{r}) = \left(\frac{1}{4\pi R_v^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_v^2}\right)$ $\sqrt{\frac{3}{2}} \approx 1.22$ $S_{4r}(\mathbf{r}) = \left(\frac{3}{8\pi R_{y}^2}\right)^{3/2} \exp\left(-\frac{3\mathbf{r}^2}{8R_{y}^2}\right)$ Final state interaction Factorized 4-body approach & factorization $C(\mathbf{q}) = \int d^3 r S_{4r}(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$ ³He

S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Source radii in 2-body, 3-body & 4-body factorized approaches

$$C(\mathbf{q}) = \int d^3 r S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

$$S_r(\mathbf{r}) = \left(\frac{1}{2\pi R_{ab}^2}\right)^{s/2} \exp\left(-\frac{\mathbf{r}^2}{2R_{ab}^2}\right)$$

$$R_{pp} = \sqrt{2}R_{N}, \qquad R_{pD} = \sqrt{\frac{3}{2}}R_{N}, \qquad R_{p^{3}He} = \sqrt{\frac{4}{3}}R_{N} \qquad R_{DD} = R_{N}$$
$$R_{pp} > R_{pD} > R_{pD} > R_{p^{3}He} > R_{DD}$$

St. Mrówczyński, Phys. Lett. B 864, 139413 (2025)

Conclusions



The correlation functions of light nuclei depend on their production mechanism.



Light nuclei should not be treated as point-like objects.



The h-D, D-D, h- 3 He correlation functions in 3- and 4-body factorized approaches are as in the 2-buty approach but the source radii, which can be measured, differ.



The correlation functions of light nuclei can reveal an existence of various nuclear resonances.

Back-up slides

Production of light nuclei at RHIC & LHC



²H, ²H, ³H, ³H, ³He, ³He, ⁴He, ⁴He, ⁴He, ³AH, ³AH, ⁴AH, ⁴AH,

baryonless matter

Two approaches to production of light nuclei

- > Thermal model direct production from thermalized hadron matter
 - Coalescence model final state interactions of nucleons

Thermal model prediction



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321 (2018)

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is $\varepsilon_B = 2.2$ MeV.
- A characteristic time of deuteron formation t is longer than 2 fm/c.
- A hadron gas at T = 156 MeV is essentially a classical system.

Snowflakes in hell ? or Snowflakes from hell ?



p-D correlation function in 3-body factorized approach



St. Mrówczyński & P. Słoń, Acta Physica Polonica B 51, 1739 (2020)

p-D correlation function

Two-body approaches beyond the Lednicky-Lyuboshitz formula





- The asymptotic wave function is insufficient.
- The *p* & *d* waves are important.

There is an ambiguity of deuteron source radius.

W. Rzęsa, M. Stefaniak & S.Pratt, arXiv:2410.13983

J. M.Torres-Rincon, A.Ramos & J.Rufi, arXiv:2410.23853

Deuteron-deuteron correlation in 4-body factorized appraoch



St. Mrówczyński & P. Słoń, Phys. Rev. C 104, 024909 (2021)

Resonance 4Li





J. Pochodzala et al. Phys. Rev. C 35, 1695 (1987)



M. Stefaniak for HADES Collaboration, arXiv:2402.09280

Correlation function *p***-**³**He**



S. Bazak & St. Mrówczyński, Eur. Phys. J. A 56, 193 (2020)