Plasma Instabilities
&
Azimuthal fluctuations

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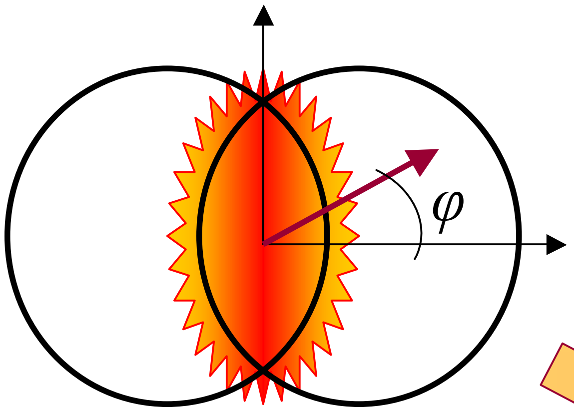
- elliptic flow & fast equilibration
- instabilities driven isotropization
- equilibration vs. isotropization
- azimuthal fluctuations & preequilibrium

Evidence of the early stage equilibration

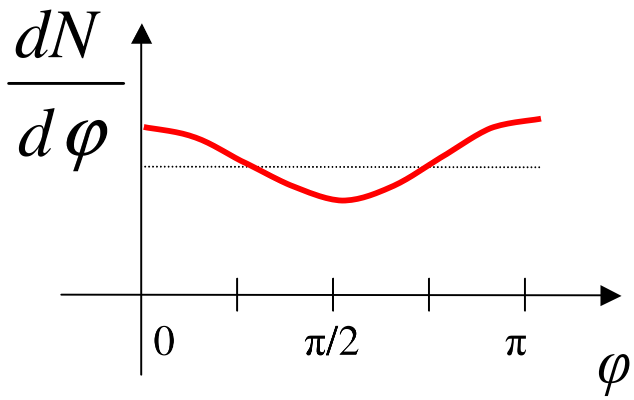
Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)\mathbf{v}=-\frac{\nabla p}{\rho}$$

Hydrodynamic requires local thermodynamical equilibrium!



Equilibration is fast

$$v_2 \sim \varepsilon = \left\langle \frac{x^2 - y^2}{x^2 + y^2} \right\rangle$$

Eccentricity decays due to the free streaming!

$\varepsilon \searrow \;\Rightarrow\; v_2 \searrow$ ➡ $t_{eq} \leq 0.6 \text{ fm}/c$

time of equilibration

U. Heinz, AIP Conf. Proc.**739,** 163 (2004)

Collisions are too slow

Time scale of hard parton-parton scattering

hard scattering ~ momentum transfer of order of T

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

either single hard scattering or multiple soft scatterings

$$t_{\text{eq}} \approx t_{\text{hard}} \geq 2.6 \text{ fm}/c$$

R. Baier, A.H. Mueller, D. Schiff & D.T. Son, Phys. Lett. **B539**, 46 (2002)

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

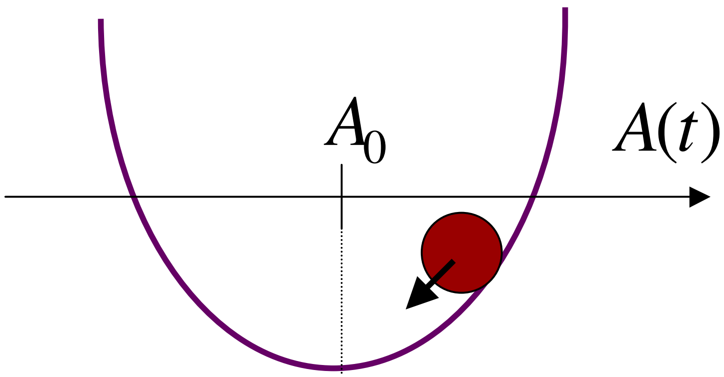
fluctuation

Instability

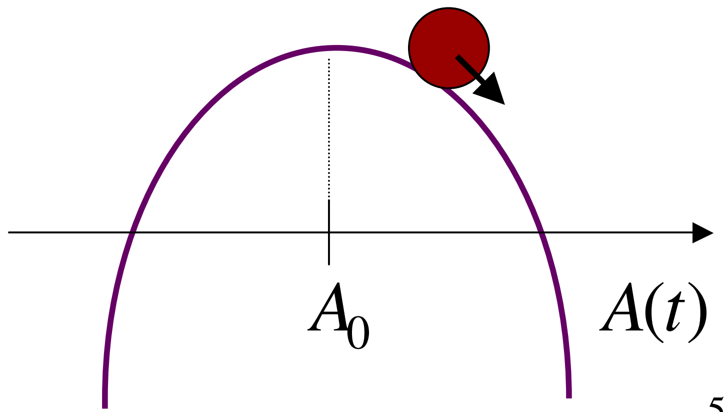
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \ \text{is not} \ \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

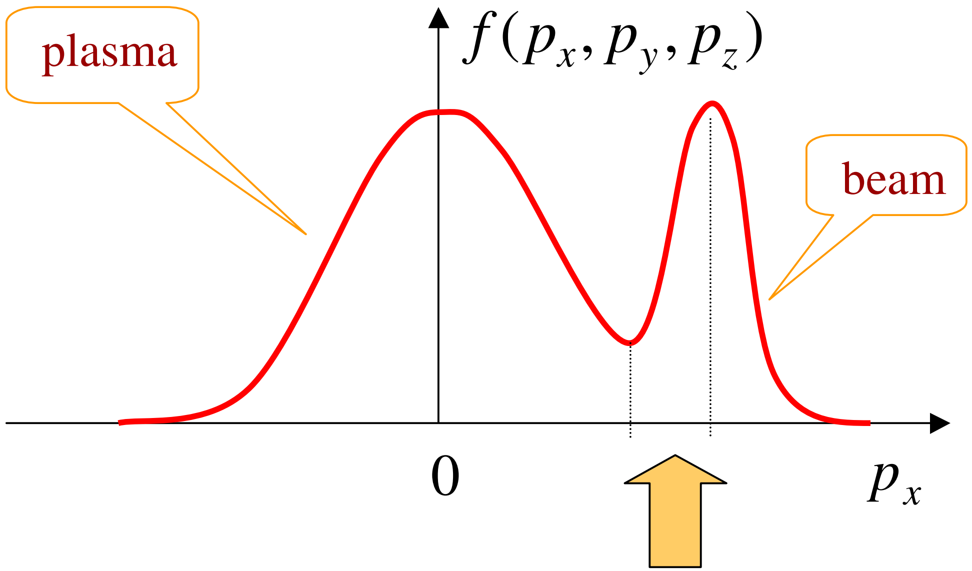
- **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}, \;\; \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$

- **transverse modes** – $\mathbf{k} \perp \mathbf{E}, \;\; \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

Logitudinal modes

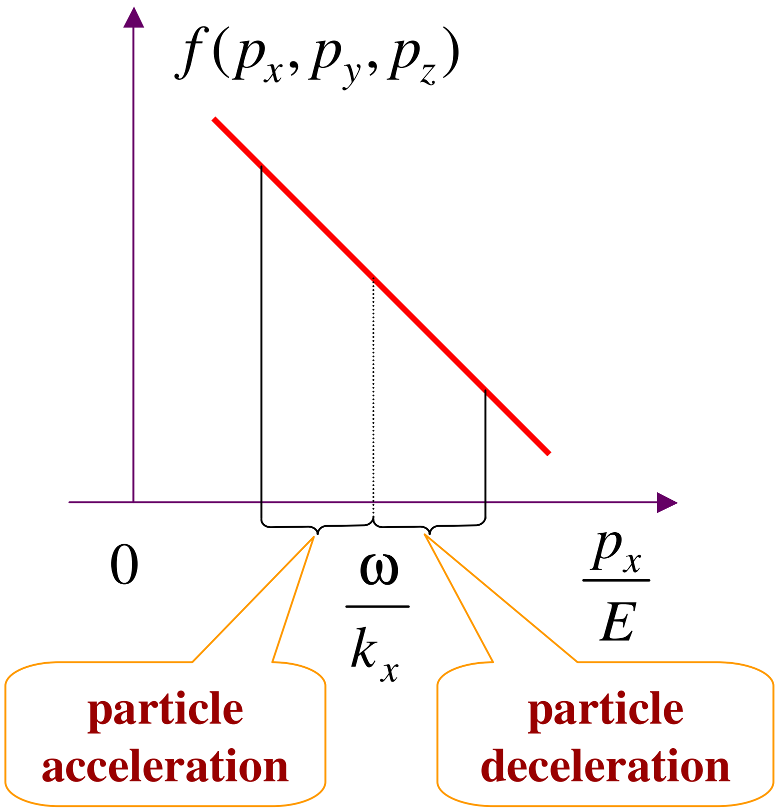
unstable configuration



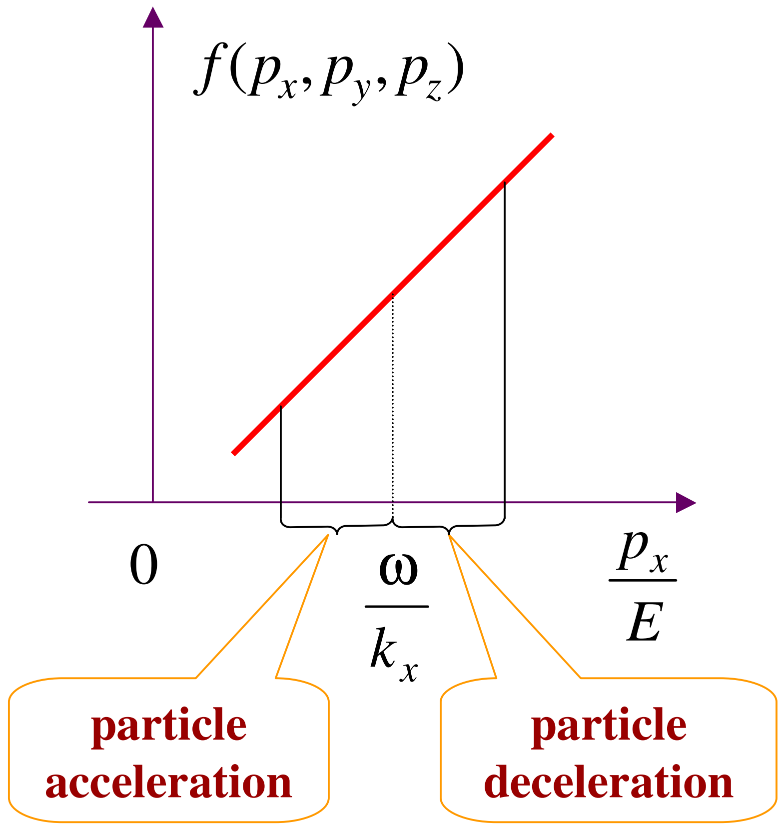
Energy is transferred from particles to fields

Logitudinal modes

Electric field decays - damping



Electric field grows - instability

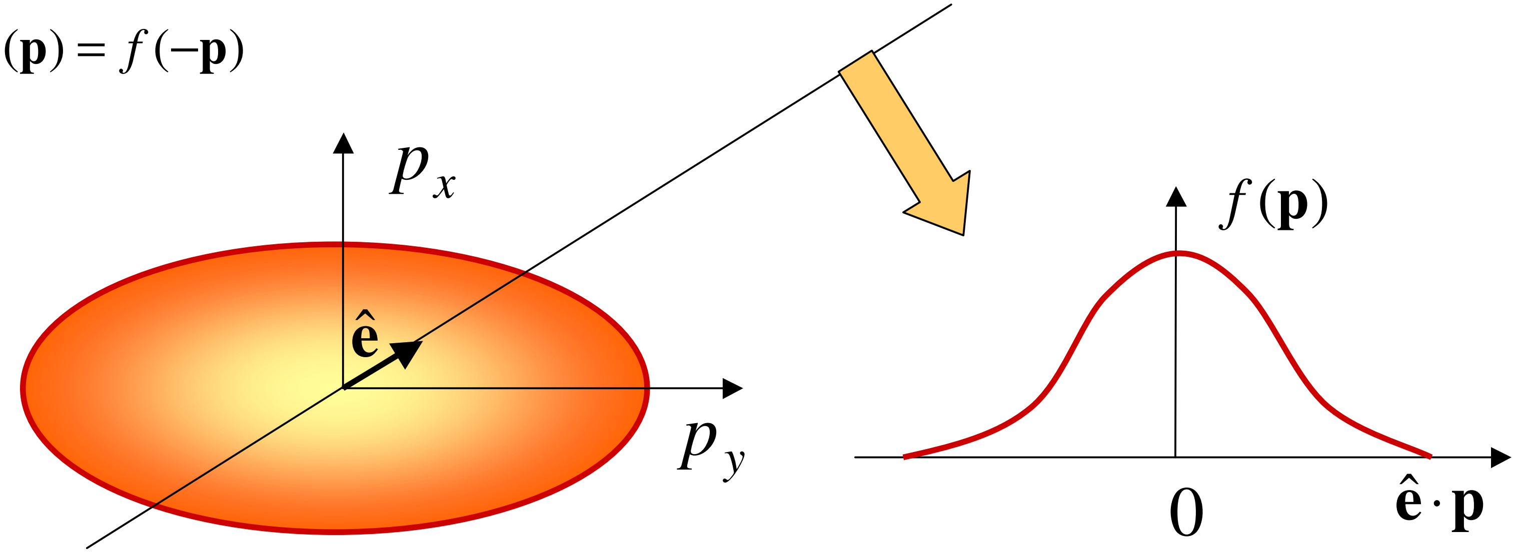


$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

Transverse modes

Unstable modes occur due to anisotropy of the momentum distribution

$$f(\mathbf{p}) = f(-\mathbf{p})$$

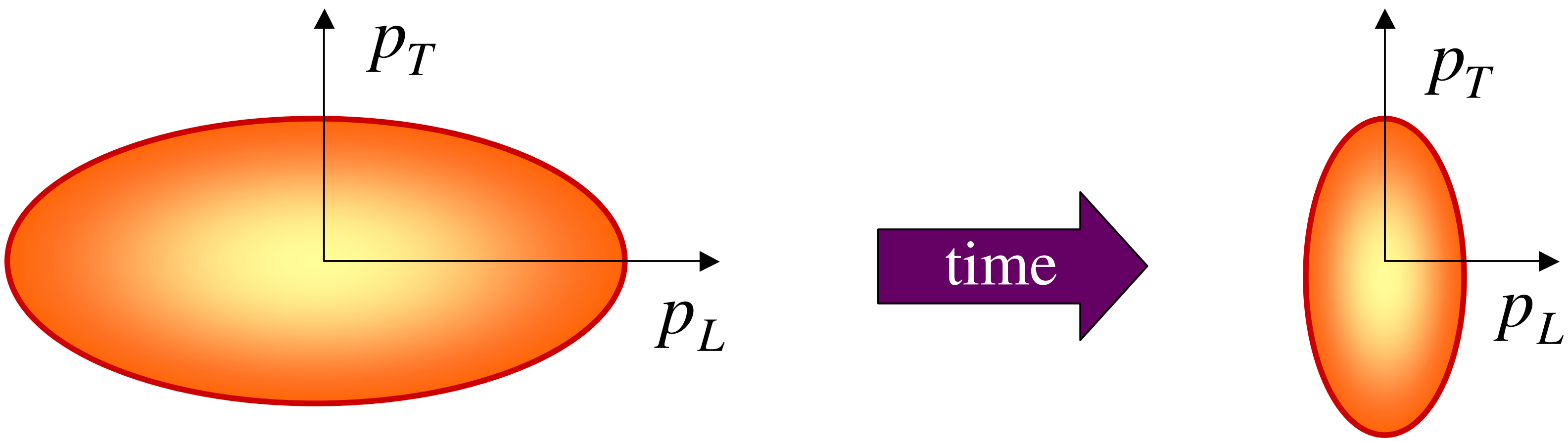


Momentum distribution distribution can monotonously decrease in every direction

Transverse modes are relevant for relativistic nuclear collisions!

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic



CM after 1-st collisions

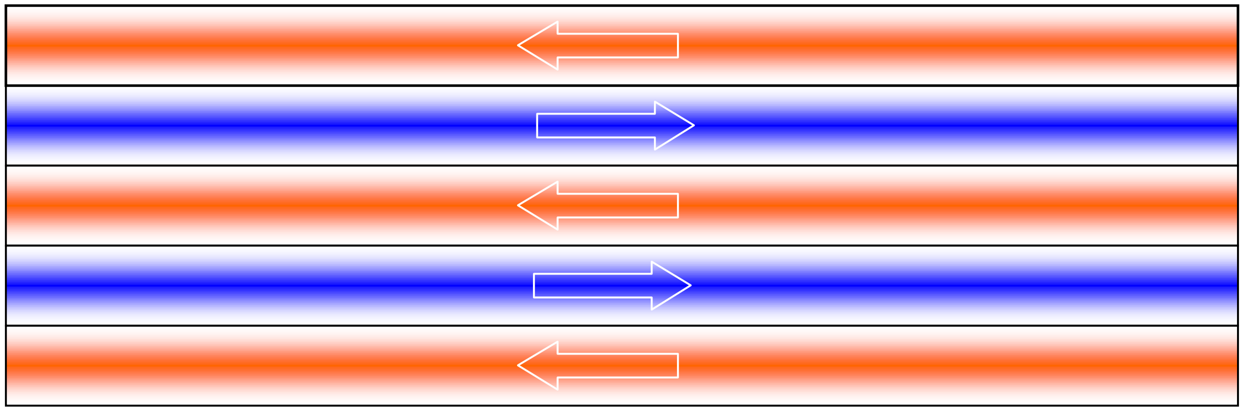
local rest frame

Seeds of instability

$\left\langle j_a^{\mu}(x) \right\rangle = 0$ **but current fluctuations are finite**

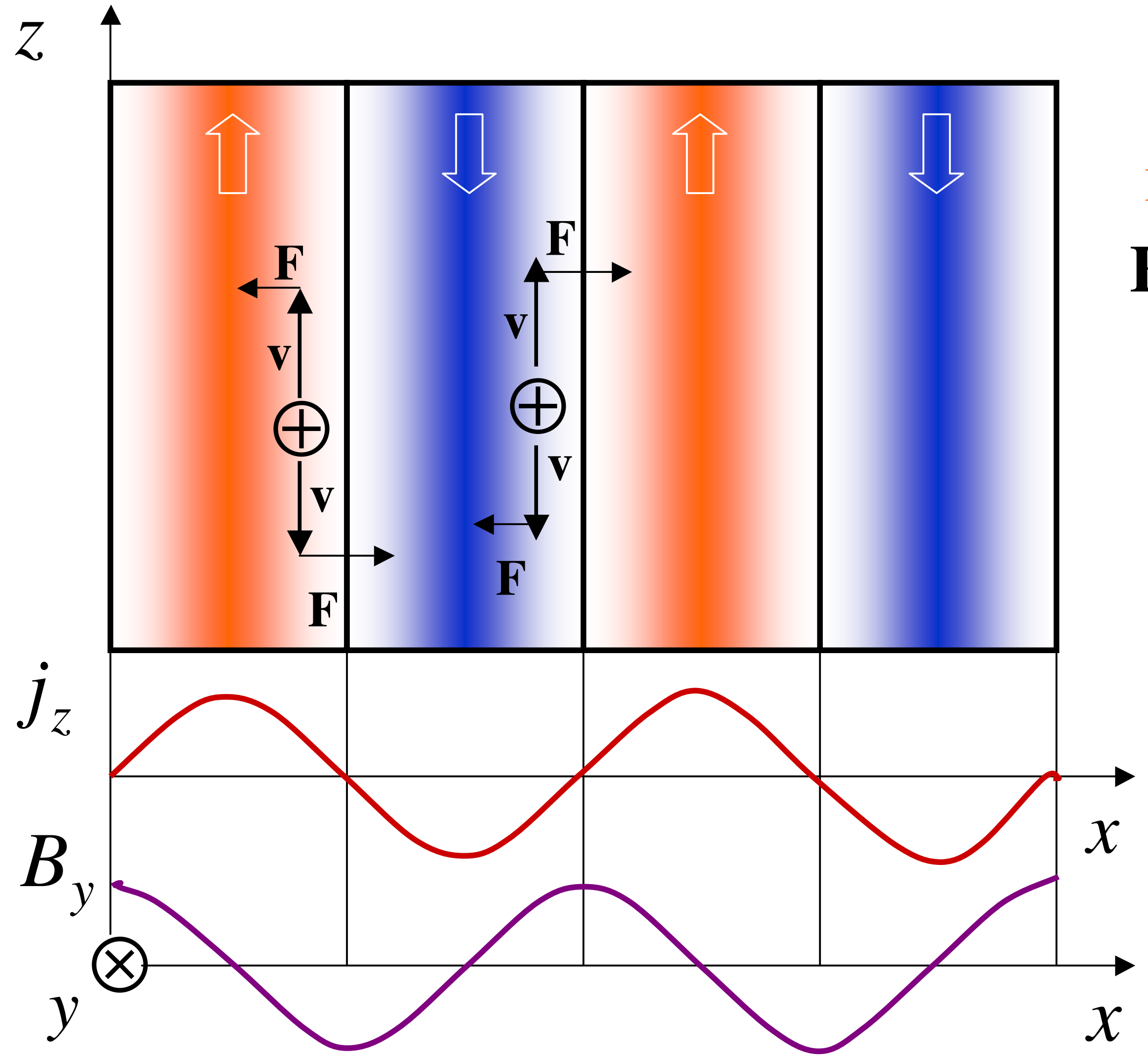
$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j}$$

Dispersion equation

Equation of motion of chromodynamic field A^{μ} in momentum space

$$[k^2 g^{\mu\nu} - k^{\mu}k^{\nu} - \Pi^{\mu\nu}(k)]A_{\nu}(k) = 0$$

gluon self-energy

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^{\mu}k^{\nu} - \Pi^{\mu\nu}(k)] = 0$$

$$k^{\mu} \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with Im$\omega > 0$ $\Rightarrow$ $A^{\mu}(x) \sim e^{\mathrm{Im}\,\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. How to get it?

Transport theory – transport equations

fundamental:

$$\left(p_\mu D^\mu - g p^\mu F_{\mu\nu}(x)\partial_p^\nu\right) Q(p,x) = C \quad \text{quarks}$$

$$\left(p_\mu D^\mu + g p^\mu F_{\mu\nu}(x)\partial_p^\nu\right) \overline{Q}(p,x) = \overline{C} \quad \text{antiquarks}$$

adjoint:

$$\left(p_\mu \mathcal{D}^\mu - g p^\mu \mathcal{F}_{\mu\nu}(x)\partial_p^\nu\right) G(p,x) = C_g \quad \text{gluons}$$

free streaming — mean-field force — collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$

mean-field generation

collisionless limit: $C = \overline{C} = C_g = 0$

Transport theory - linearization

fluctuation

$$Q(p,x) = Q_0(p) + \delta Q(p,x)$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| \gg |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p)| \gg |\partial_p^\mu \delta Q(p,x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$

$$p_\mu D^\mu \delta \overline{Q}(p,x) + g p^\mu F_{\mu\nu}(x) \partial_p^\nu \overline{Q}_0(p) = 0$$

$$p_\mu \mathcal{D}^\mu \delta G(p,x) - g p^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu G_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p,x) = g\int d^4x'\,\Delta_p(x-x')\,p^\mu F_{\mu\nu}(x)\partial_p^\nu Q_0(p)$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$j^\mu[\delta Q, \delta\overline{Q}, \delta G]$$

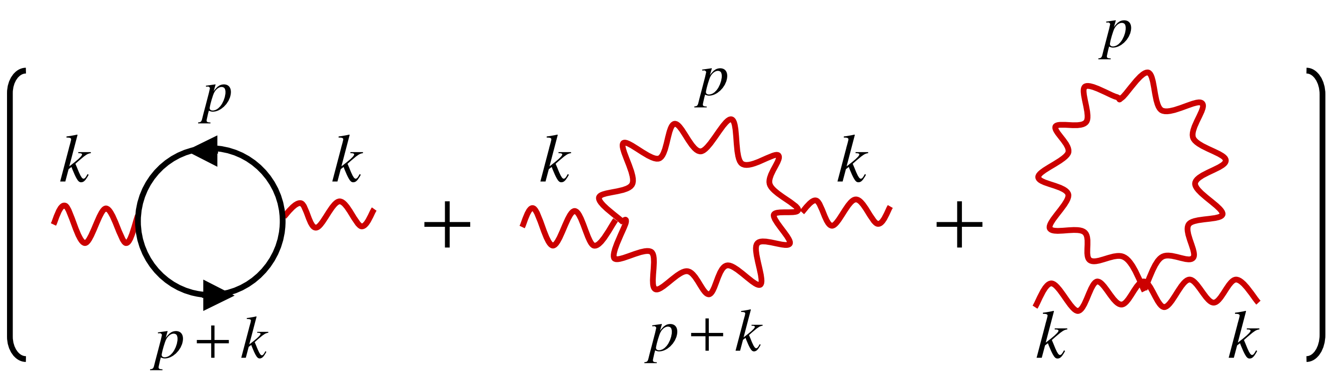
$$j^\mu(k) = \Pi^{\mu\nu}(k)A_\nu(k)$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\right]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$\Pi^{\mu\nu}(k) =$ 

Hard loop approximation: $k^\mu << p^\mu$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\right]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

St. M. & M. Thoma, Phys. Rev. C **62**, 036011 (2000)

Dispersion equation

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(k)$$ **chromodielectric tensor**

$$k^\mu \equiv (\omega, \mathbf{k})$$

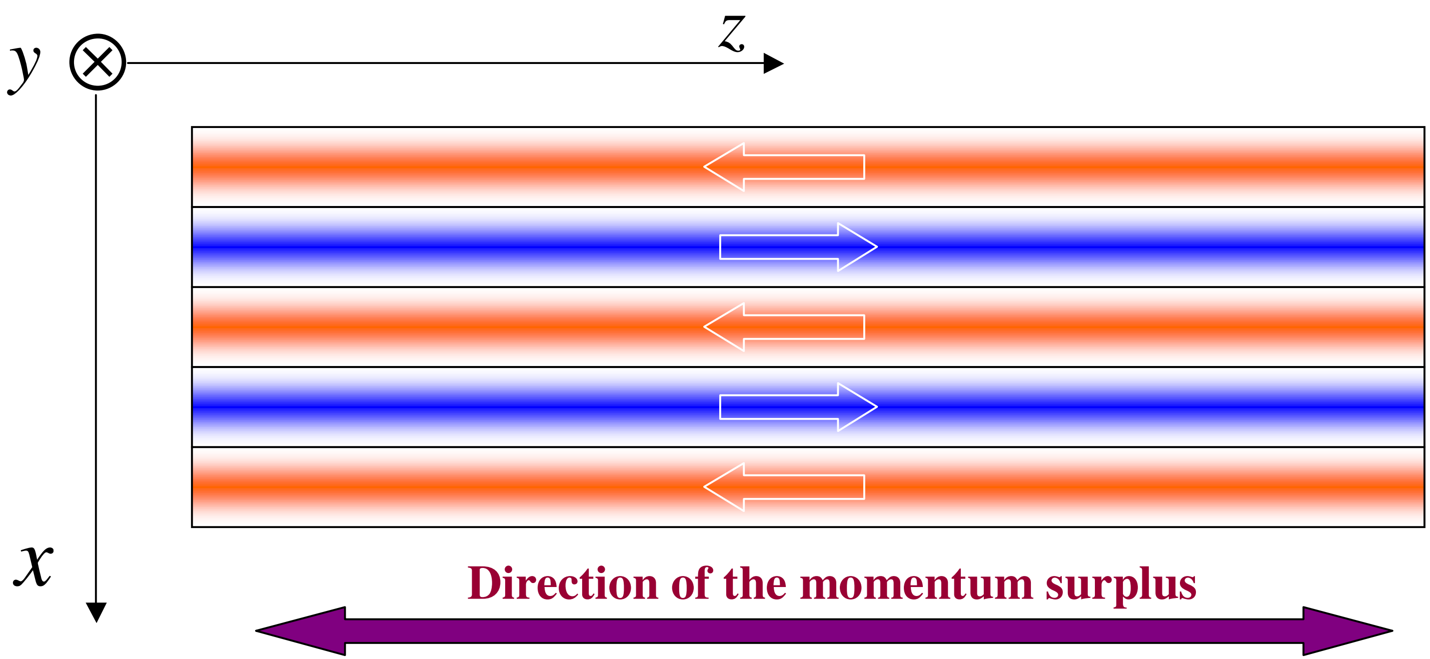
Dispersion equation

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

$$\mathbf{v} \equiv \mathbf{p}/E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0,0,j), \quad \mathbf{E} = (0,0,E), \quad \mathbf{k} = (k,0,0)$$

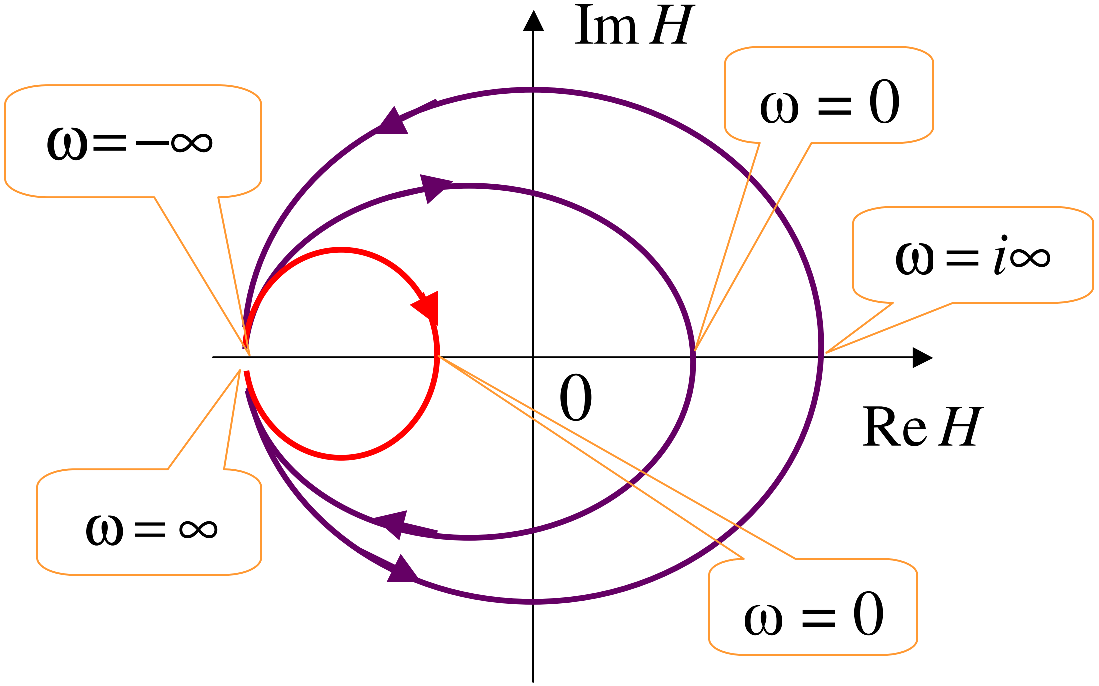
Dispersion equation

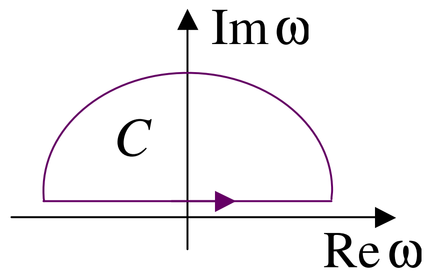
$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

Existence of unstable modes – Penrose criterion

$$H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)$$

$$\oint_C \frac{d\omega}{2\pi i} \frac{1}{H(\omega)} \frac{dH(\omega)}{d\omega} = \begin{cases} \oint_C \frac{d\omega}{2\pi i} \frac{d \ln H(\omega)}{d\omega} = \ln H(\omega)\Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{cases}$$





There are unstable modes if

$$H(\omega = 0) < 0$$

Anisotropy!

Unstable solutions

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho\sigma_\perp^4}{\sigma_\parallel} \frac{1}{(p_\perp^2 + \sigma_\perp^2)^3} e^{-\frac{p_\parallel^2}{2\sigma_\parallel^2}}$$

$$\rho = 6\ \mathrm{fm}^{-3}$$

$$\alpha_s = g^2/4\pi = 0.3$$

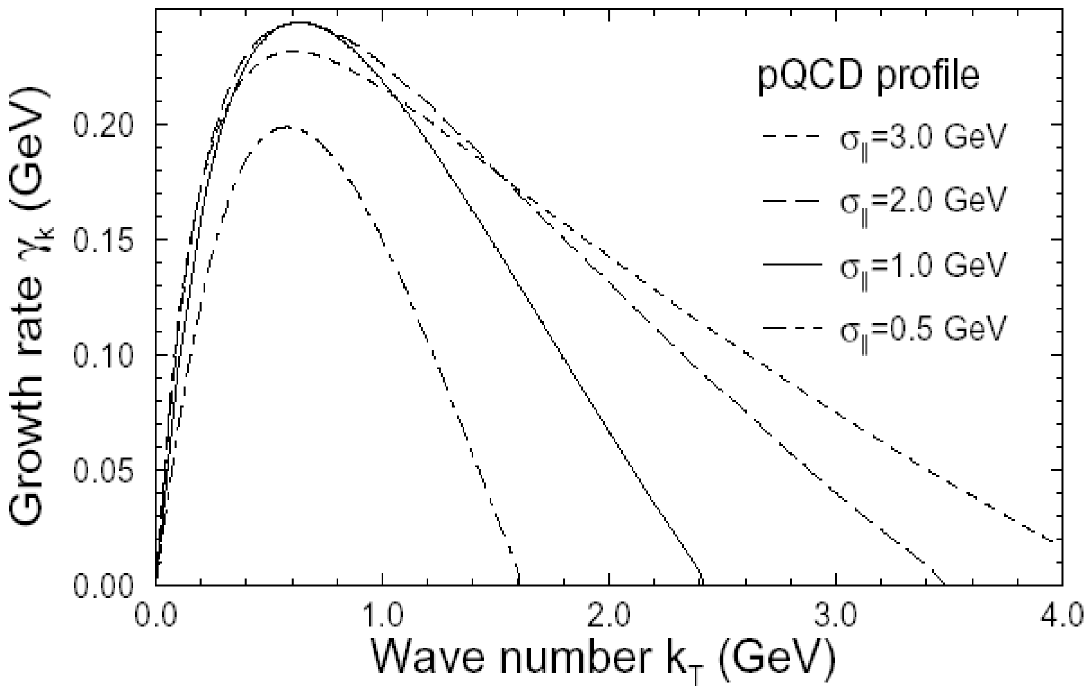
$$\sigma_\perp = 0.3\ \mathrm{GeV}$$

$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

solution

$$\omega(k) = \pm i\,\gamma_k$$

$$0 < \gamma_k \in \Re$$



J. Randrup & St. M., Phys. Rev. C **68**, 034909 (2003)

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A^\mu_a = A^\mu_a(t, z)$$

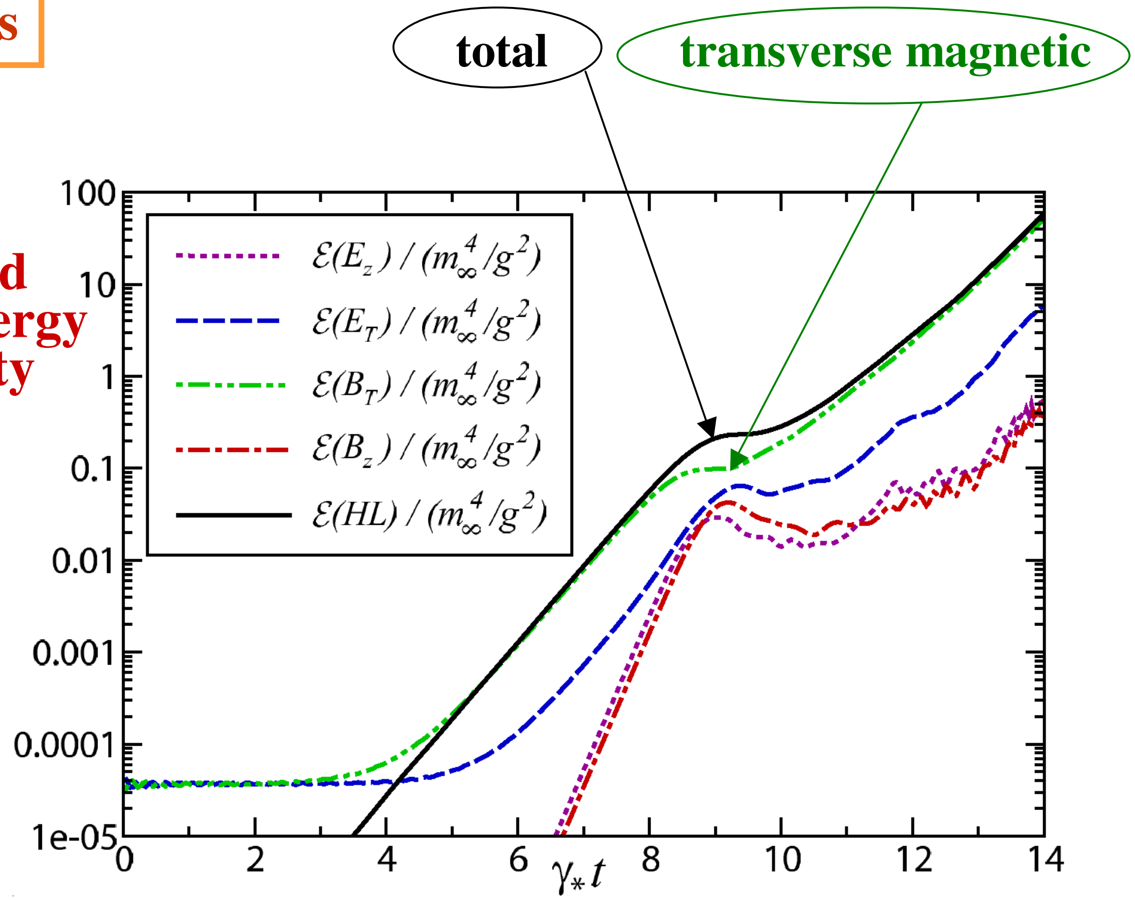
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\mathrm{iso}}(|\,\mathbf{p}\,| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\, p^2 \frac{df_{\mathrm{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

Scaled field energy density



Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Growth of instabilities – 1+1 numerical simulations

Classical system of colored particles & fields

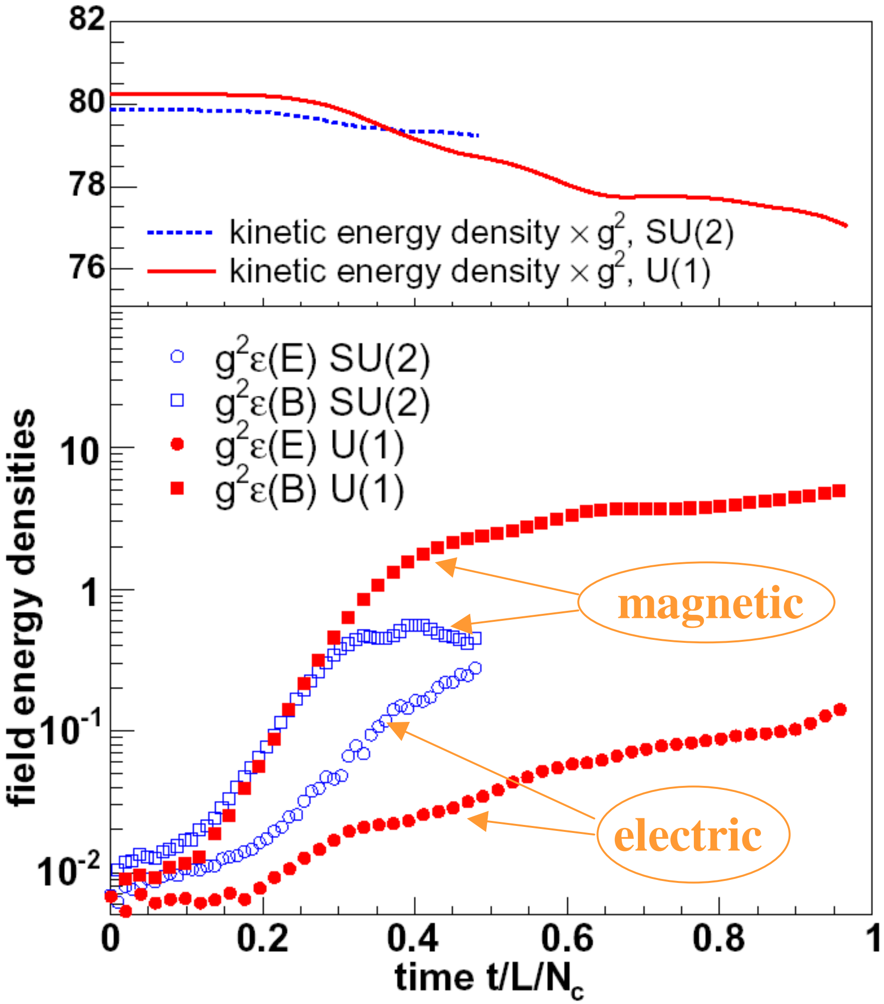
initial fields: **Gaussian noise as in Color Glass Condensate**

initial anisotropic particle distribution:

$$f_0(\mathbf{p},\mathbf{x}) \sim \delta(p_x)\, e^{-\frac{\sqrt{p_y^2+p_z^2}}{p_{\rm hard}}}$$

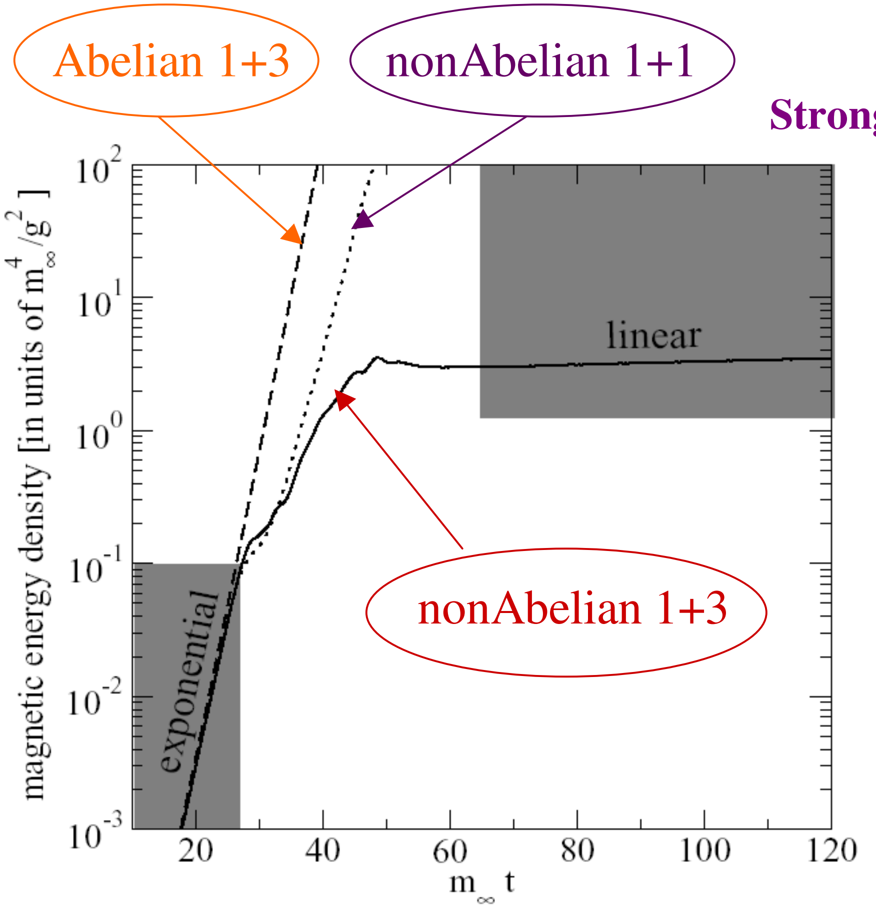
$$p_{\rm hard} = 10\,\mathrm{GeV}$$

$$L = 40\,\mathrm{fm} \qquad \rho = 10\,\mathrm{fm}^{-3}$$



A. Dumitru & Y. Nara, hep-ph/0503121

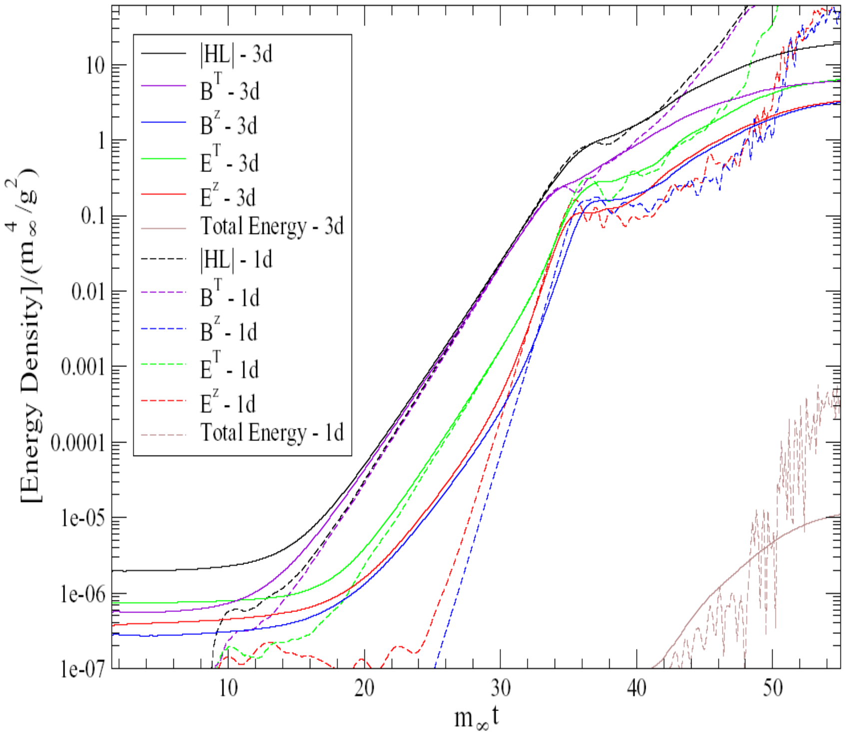
Growth of instabilities – 1+3 numerical simulations



P. Arnold, G.D. Moore & L.G. Yaffe, hep-ph/0505212

SU(2) Hard Loop Dynamics

Strongly anisotropic particle's momentum distribution



A. Rebhan, P. Romatschke & M.Strickland, hep-ph/0505261

Abelanization

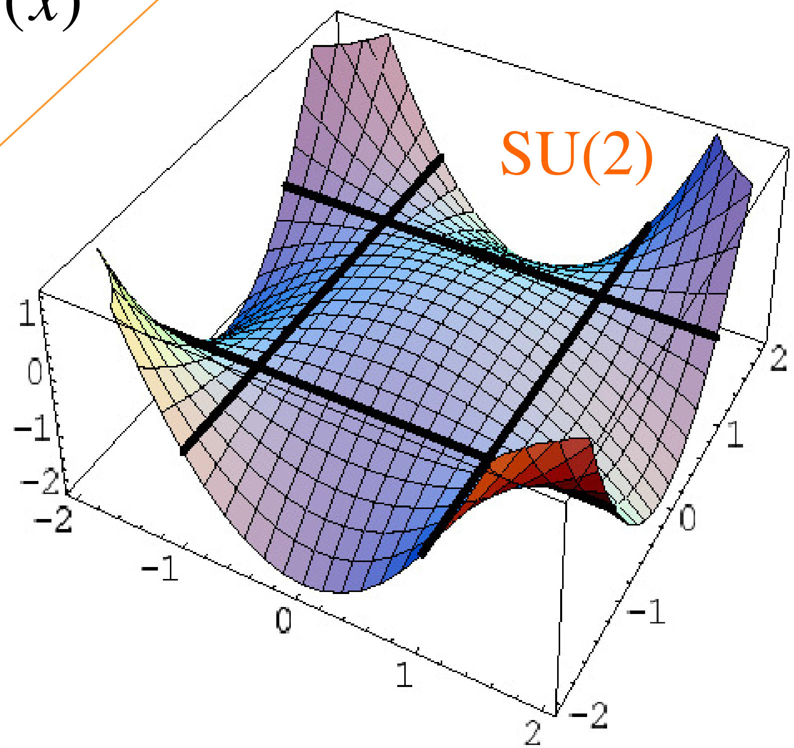
$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2 \mathbf{A}^a \cdot \mathbf{A}^a + \frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

the gauge $A_0^a = 0, \quad A_i^a(t,x,y,z) = A_i^a(x)$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} \mathbf{B}^a \mathbf{B}^a$$

$$= -\frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a + \frac{g}{2} f_{abc} \mathbf{A}^b \times \mathbf{A}^c$$



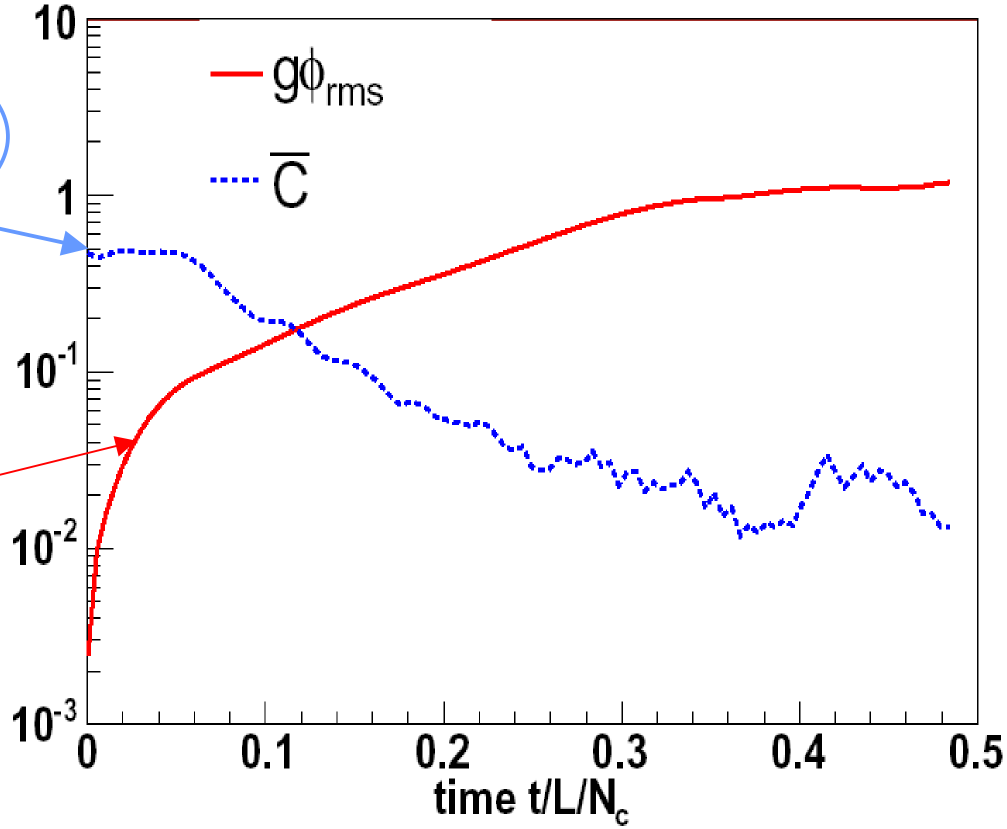
P. Arnold & J. Lenaghan, Phys. Rev. D **70**, 114007 (2004)

Abelanization – 1+1 numerical simulations

Classical system of colored particles & fields

$$\overline{C} \equiv \int_0^L \frac{dx}{L} \frac{\sqrt{\mathrm{Tr}((i[A_y, A_z])^2)}}{\mathrm{Tr}[\mathbf{A}^2]}$$

$$\phi_{rms} \equiv \sqrt{\int_0^L \frac{dx}{2L} \mathrm{Tr}[\mathbf{A}^2]}$$



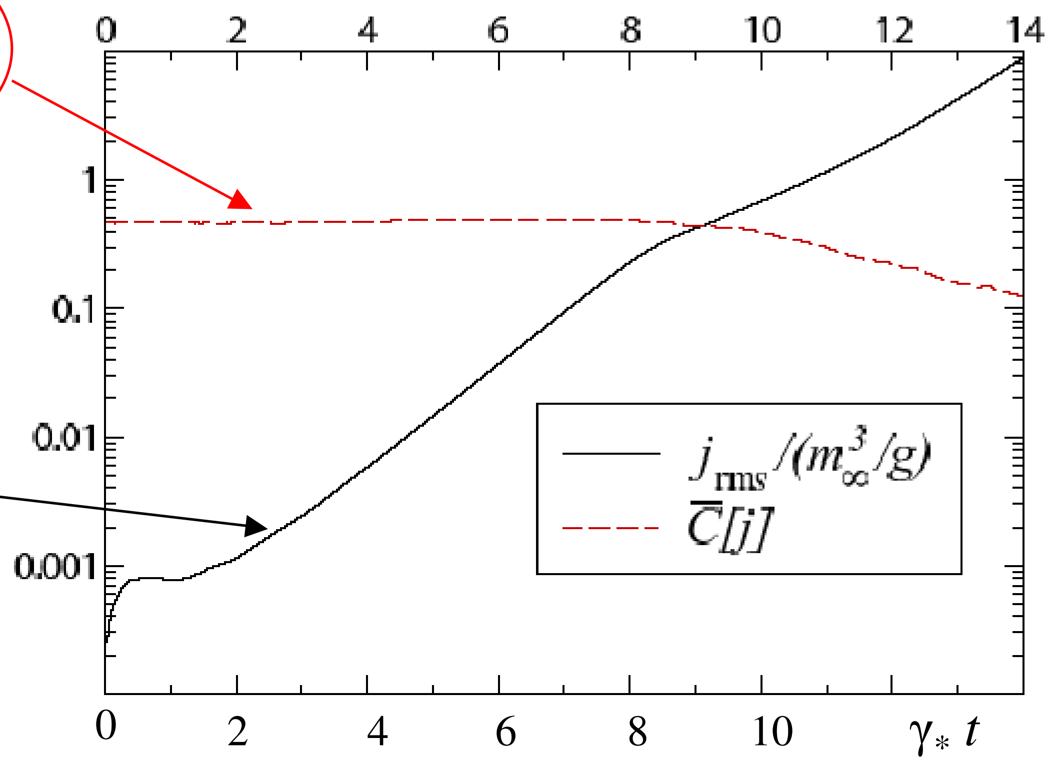
A. Dumitru & Y. Nara, hep-ph/0503121

Abelanization – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

$$\overline{C} \equiv \int_0^L \frac{dz}{L} \frac{\sqrt{\mathrm{Tr}((i[j_x, j_y])^2)}}{\mathrm{Tr}[\mathbf{j}^2]}$$

$$j_{\mathrm{rms}} \equiv \sqrt{\int_0^L \frac{dz}{L} 2\,\mathrm{Tr}[\mathbf{j}^2]}$$



A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

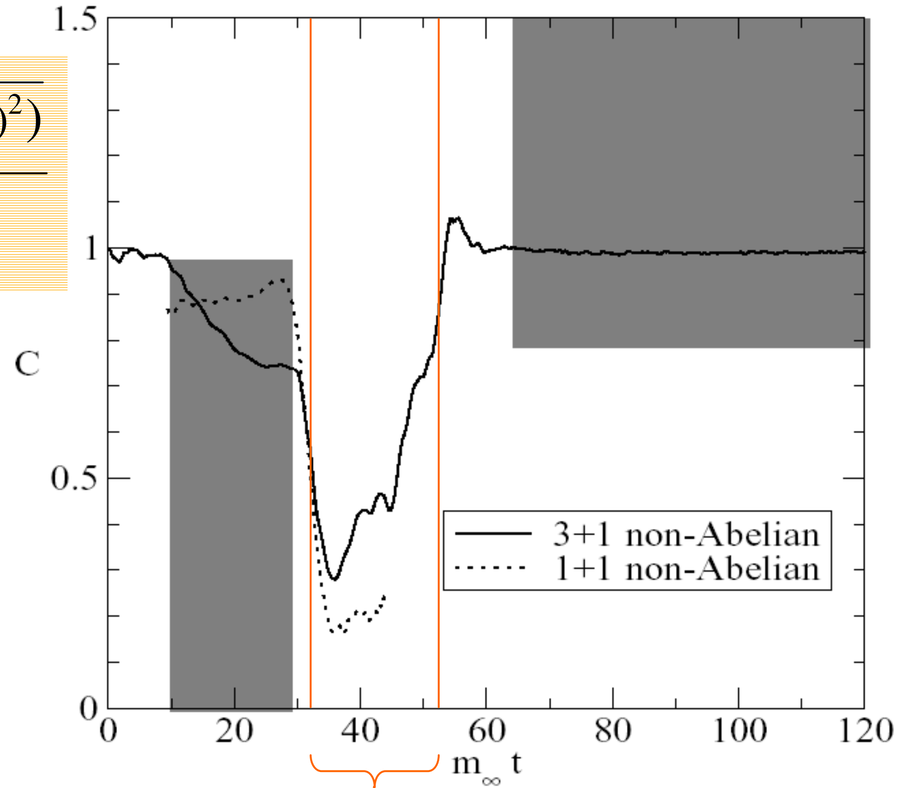
Abelanization – 1+3 numerical simulations

SU(2) Hard Loop Dynamics

$$C \equiv \frac{3}{\sqrt{2}} \frac{\int \frac{d^3x}{V} \sqrt{\mathrm{Tr}((i[j_x, j_y])^2 + (i[j_y, j_z])^2 + (i[j_z, j_x])^2)}}{\int \frac{d^3x}{V} \mathrm{Tr}(\mathbf{j}^2)}$$

$$A_i^a \sim e^{\gamma t}$$

$$A_i^a \sim \frac{k_{\mathrm{field}}}{g} << \frac{p_{\mathrm{hard}}}{g}$$

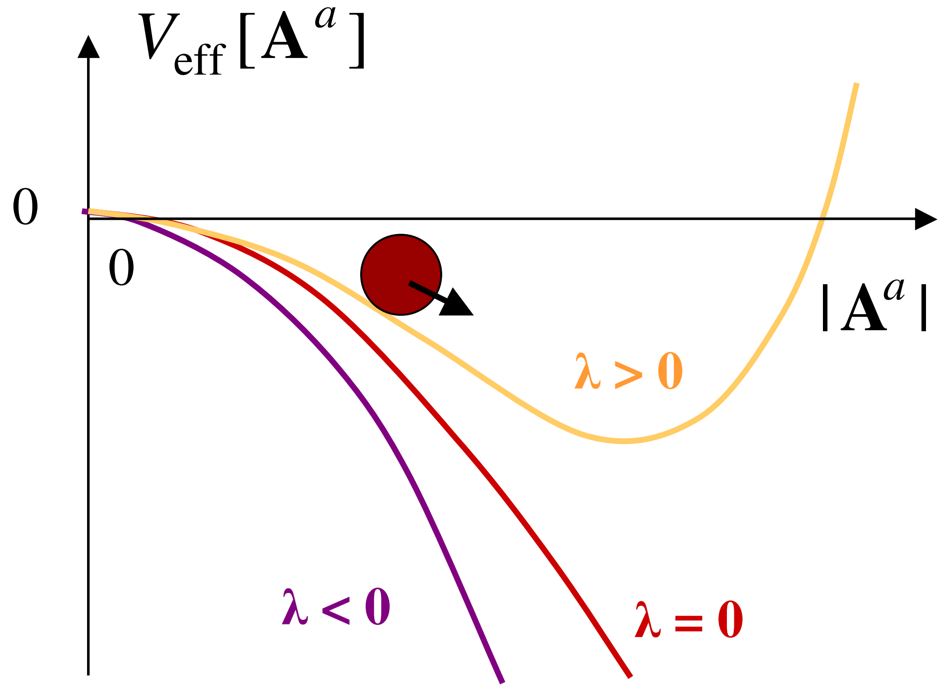


P. Arnold, G.D. Moore & L.G. Yaffe, hep-ph/0505212

Beyond Hard Loop level

$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2\,\mathrm{Tr}[\mathbf{A}^2] + \lambda\,\mathrm{Tr}[\mathbf{A}^4] + \cdots$$

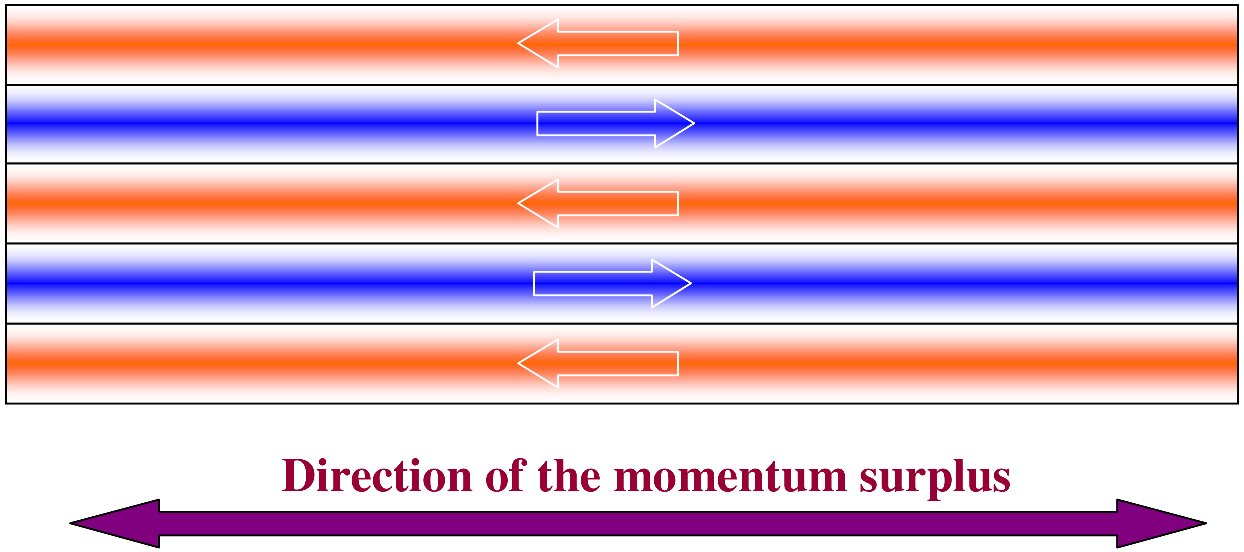
hard-loop term

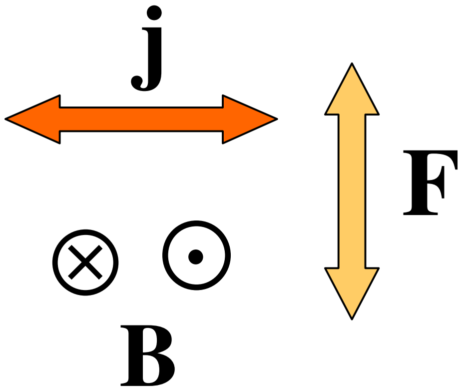


λ depends on the momentum distribution of particles

C. Manuel & St. M., hep-ph/0504156

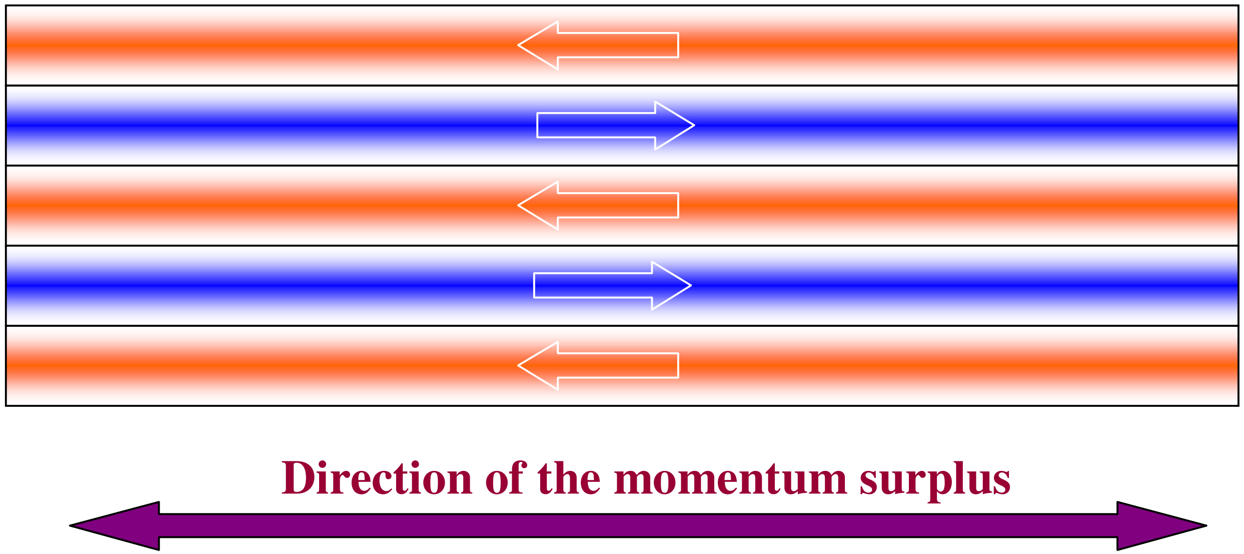
Isotropization - particles

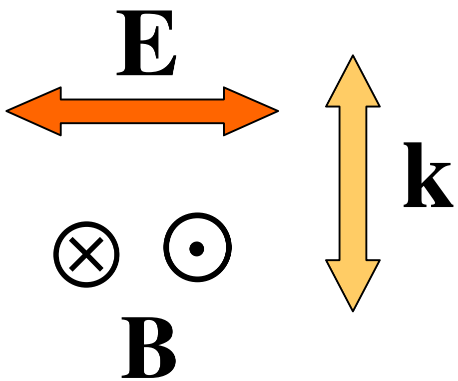




$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields



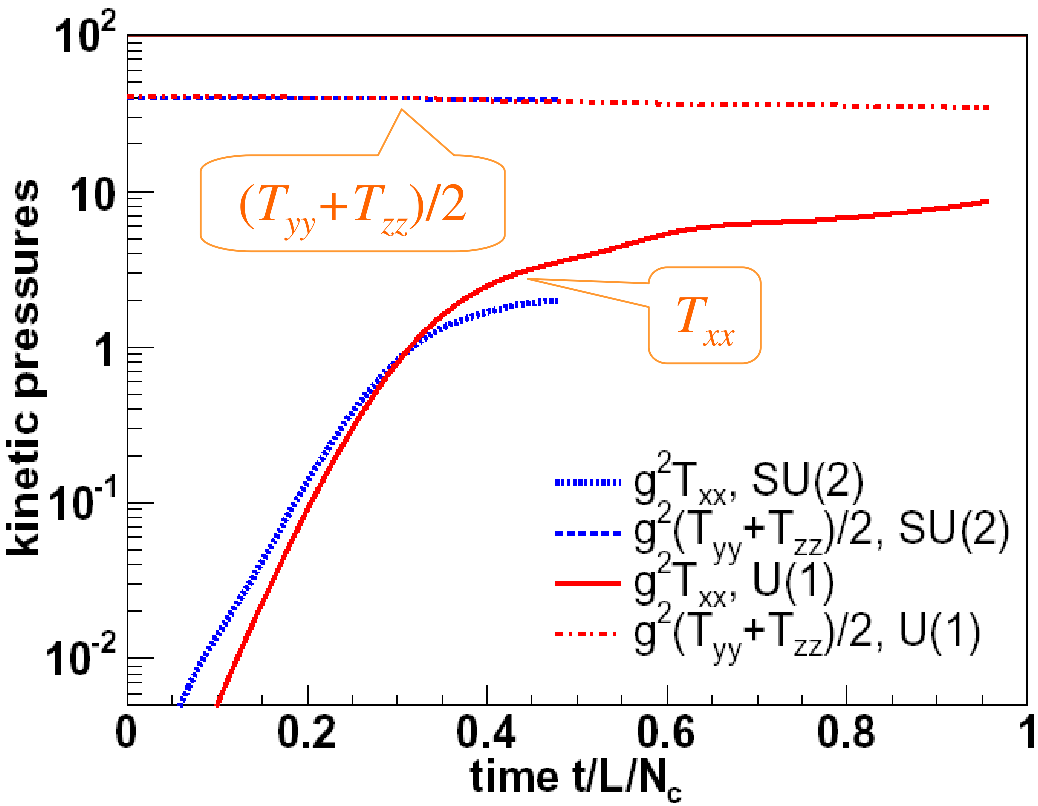


$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^{a} \times \mathbf{E}^{a} \sim \mathbf{k}$$

Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$



Isotropy:

$$T_{xx} = (T_{yy} + T_{zz})/2$$

A. Dumitru & Y. Nara, hep-ph/0503121

Isotropization vs. equilibration

Three comments:

Isotropization is a mean-field phenomenon which is not associated with the entropy production.

Collisions are needed for equilibration.

After the stage of instabilities, the system is in prequilibrium.

Collective flow in preequilibrium

Two comments:

Elliptic flow starts in prequilibrium stage.

Approximate hydrodynamics requires not equilibrium but merely isotropic momentum distribution.

Hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0$$

Energy-momentum tensor of ideal fluid

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p\, g^{\mu\nu}$$

Equation of state?

P. Arnold, J. Lenaghan, G.D. Moore and L.G. Yaffe, Phys. Rev. Lett. **94**, 072302 (2005)

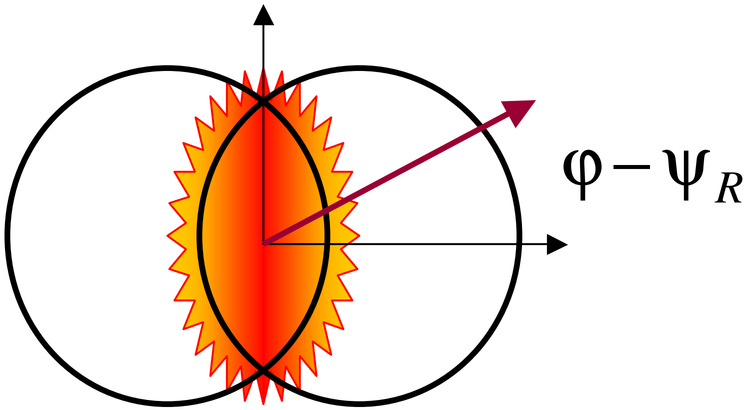
Equilibrium vs. preequilibrium

Q: How to distinguish equilibrium form preequilibrium collective flow?

A: Look for flow fluctuations.

Elliptic flow

$$P(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos(n(\varphi - \psi_R))\right]$$



$$v_2 = \langle \cos(2(\varphi - \psi_R)) \rangle$$

Elliptic flow fluctuations

$$\langle v_2 \rangle = \frac{\left\langle \overline{\cos(2(\varphi - \psi_E))} \right\rangle}{\langle R \rangle}$$

$$R \equiv \cos(\psi_R - \psi_E)$$

$\overline{\cdots}$ averaging over particles from a single event

$\langle \cdots \rangle$ averaging over events

$$\mathrm{Var}(v_2) \equiv \left\langle v_2^2 \right\rangle - \left\langle v_2 \right\rangle^2$$

$$\mathrm{Var}(v_2) = \frac{1}{\langle R \rangle^2} \left(\left\langle \overline{\cos(2(\varphi - \psi_E))}^2 \right\rangle - \left\langle \overline{\cos(2(\varphi - \psi_E))} \right\rangle^2 \right)$$

St. M. and E.V. Shuryak, Acta Phys. Pol. **B34**, 4241 (2003)

Statistical noise

Due to the finite particle multiplicity

$$\mathrm{Var}(v_2) = \frac{1}{2\langle R\rangle^2 \langle N\rangle} + \langle v_2\rangle^2 \frac{\mathrm{Var}(R)}{\langle R\rangle^2}$$

N number of particles used to compute $\cos(\varphi-\psi_E)$

M number of particles used to determine ψ_R

$\psi_R - \psi_E \to 0$

$\langle(\psi_R - \psi_E)^2\rangle \sim \frac{1}{\langle M\rangle}$

$$\langle R\rangle^2 = \langle \cos(2(\psi_R - \psi_E))\rangle^2 \approx \left(1 - 2\langle(\psi_R - \psi_E)^2\rangle\right)^2 \approx \left(1 - \frac{2\alpha}{\langle M\rangle}\right)^2 \approx 1 - \frac{4\alpha}{\langle M\rangle}$$

$$\langle R^2\rangle = \langle \cos^2(2(\psi_R - \psi_E))\rangle \approx 1 - 4\langle(\psi_R - \psi_E)^2\rangle \approx 1 - \frac{4\alpha}{\langle M\rangle}$$

$$\mathrm{Var}(R) \sim \frac{1}{\langle M\rangle^2} << \frac{1}{\langle N\rangle}$$

Statistical noise

$$\delta v_2 = \frac{1}{\langle R \rangle \sqrt{2\langle N \rangle}}$$

$$\delta v_2 \equiv \sqrt{\mathrm{Var}(v_2)}$$

$$\langle R \rangle \sim 1 \quad \& \quad \langle N \rangle \sim 10^3 \quad \Rightarrow \quad \delta v_2 \sim 10^{-2}$$

Fluctuations due to b - variation

$$\delta v_2 = \frac{d\langle v_2 \rangle}{db}\delta b$$

$$b \to N_p \qquad N_p = 2Z\left(1 - \frac{b}{b_{max}}\right)$$

$$\delta v_2 \approx 8\times 10^{-4}\,\delta N_p$$

for b=5 fm where $v_2 = 0.03$ in Au-Au

$$\delta N_p \sim 10 \quad \Rightarrow \quad \delta v_2 \sim 10^{-2}$$

At maximum flow:

$$\frac{d\langle v_2 \rangle}{db} = 0 \quad \Rightarrow \quad \delta v_2 = 0$$

Elliptic flow fluctuations

Statistical noise & b variation are under control

Dynamical elliptic flow fluctuations seem to be measurable

Integrated azimuthal fluctuations & Φ-measure

- single particle's variable $z = \varphi - \overline{\varphi}$ $\qquad \overline{\ldots}$ inclusive averaging

- event's variable $Z = \sum_{i=1}^{N} (\varphi_i - \overline{\varphi})$ $\qquad \langle \cdots \rangle$ averaging over events

$$\langle Z \rangle = 0$$

- Φ-measure

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$$

$\Phi = 0$ no correlations

M. Gaździcki and St. M., Z. Phys. **C54**, 127 (1992)

Φ-measure of flow fluctuations

$$P_{\mathrm{ev}}(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos(n(\varphi - \psi_n))\right]$$

$$\Phi \approx \frac{3}{2\pi^2}\langle N\rangle \begin{cases} \left\langle \sum_n \left(\frac{v_n}{n}\right)^2 \right\rangle & \text{for} \quad \langle \psi_n \psi_m \rangle = \langle \psi_n \rangle \langle \psi_m \rangle, \; n \neq m \\ \left\langle \left(\sum_n \frac{v_n}{n} \right)^2 \right\rangle & \text{for} \quad n\psi_n + \alpha_n = \psi_R \end{cases}$$

no flow fluctuations!

$\langle N \rangle = 10^3, \quad v_1 = v_2 = 0.03, \quad v_n = 0, \, n \geq 3$

$$0.34 \leq \Phi \leq 0.62$$

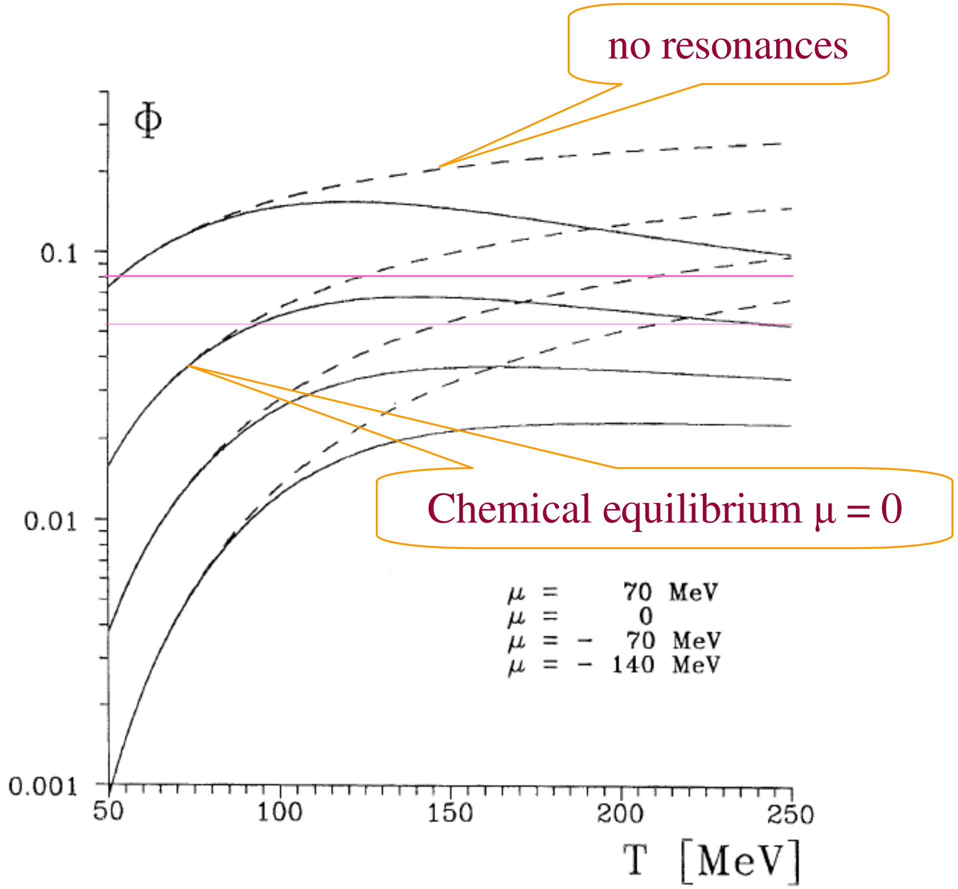
Non-flow azimuthal fluctuations

Bose-Einstein correlations

$$\Phi = \frac{\pi}{\sqrt{3}}\left(\sqrt{\frac{\tilde{\rho}}{\rho}} - 1\right)$$

$$\rho = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(E-\mu)} - 1}$$

$$\tilde{\rho} = \int \frac{d^3p}{(2\pi)^3} \frac{e^{\beta(E-\mu)}}{(e^{\beta(E-\mu)} - 1)^2}$$



Conclusion

Azimuthal fluctuations can tell us whether the elliptic flow is generated in the fully equilibrated sQGP or in the prequilibrium pQGP isotropized by instabilities