Overview of Event-by-Event Fluctuations

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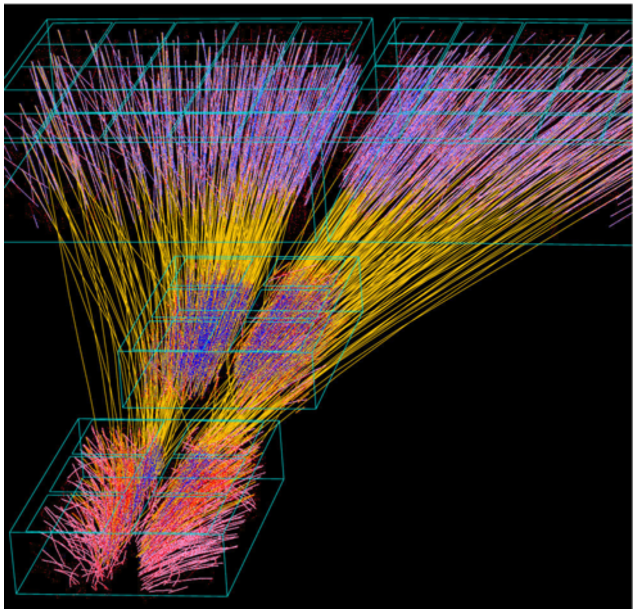
Outline

- Early days of e-b-e physics
- Extensive & intensive quantities
- Fluctuation measures
- p_T fluctuations
- Thermodynamic fluctuations
- Electric charge fluctuations
- Balance functions
- Multiplicity fluctuation
- Elliptic flow fluctuations
- Conclusions & outlook

Large acceptance detectors

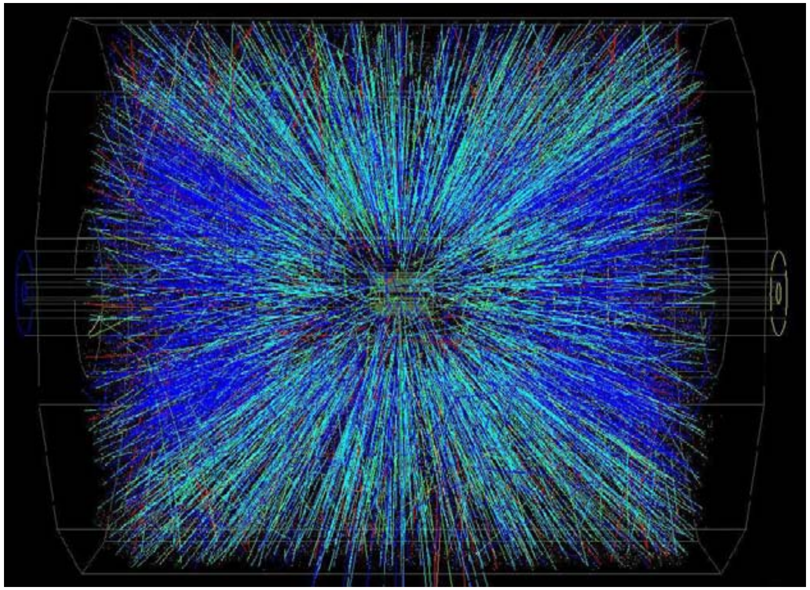
NA49 experiment @ SPS

$\text{Pb}-\text{Pb} @ 158\ A\text{GeV}$



STAR experiment @ RHIC

$\text{Au}-\text{Au} @ \sqrt{s_{NN}} = 200\ \text{GeV}$



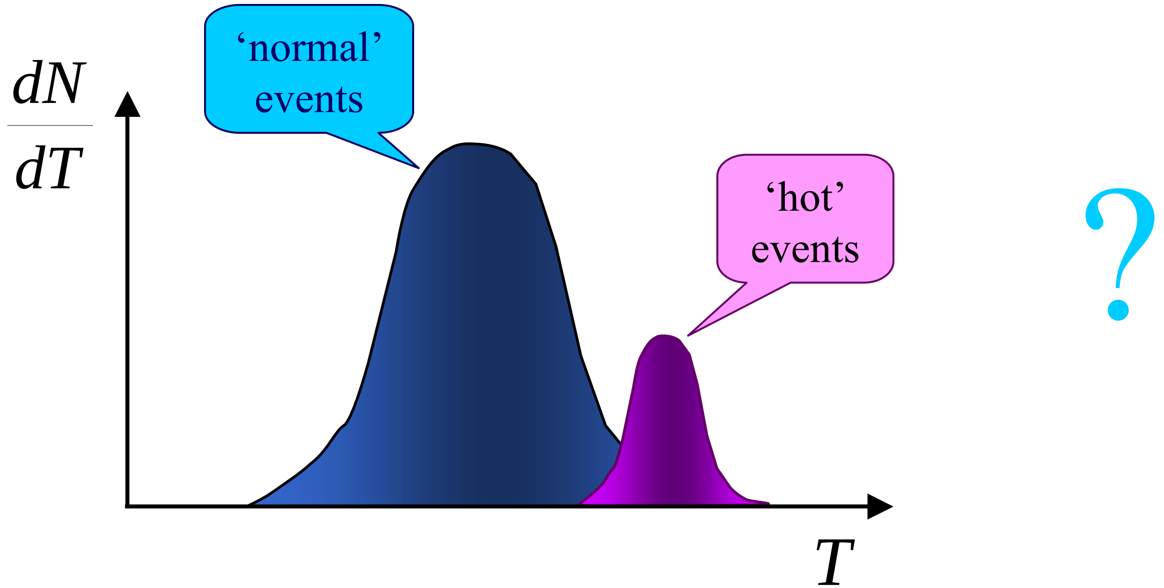
Not single particles but *events* are seen

Early Days Motivation

Event-by-event analysis allows to select 'interesting' events

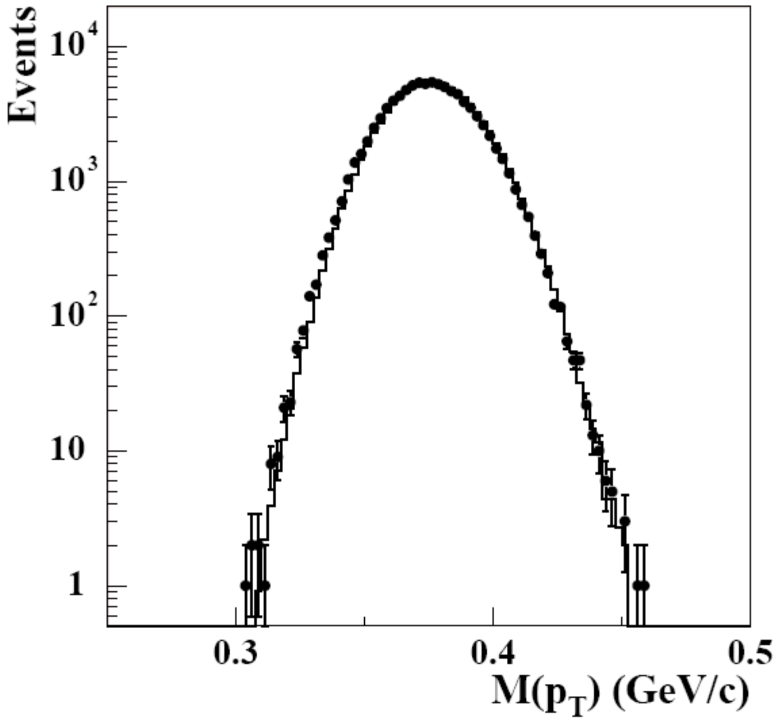
'interesting': QGP, high T, high multiplicity, etc.

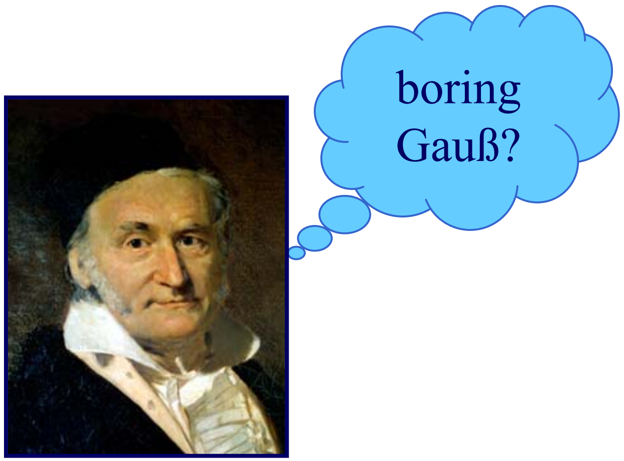
R. Stock ~ '90



‘Interesting’ events not seen

central Pb-Pb @ 158 AGeV





$$M(p_T) \equiv \frac{1}{N} \sum_{i=1}^{N} p_T^i$$

H. Appelshauser et al. [NA49 Collaboration], Phys. Lett. **B459**, 679 (1999)

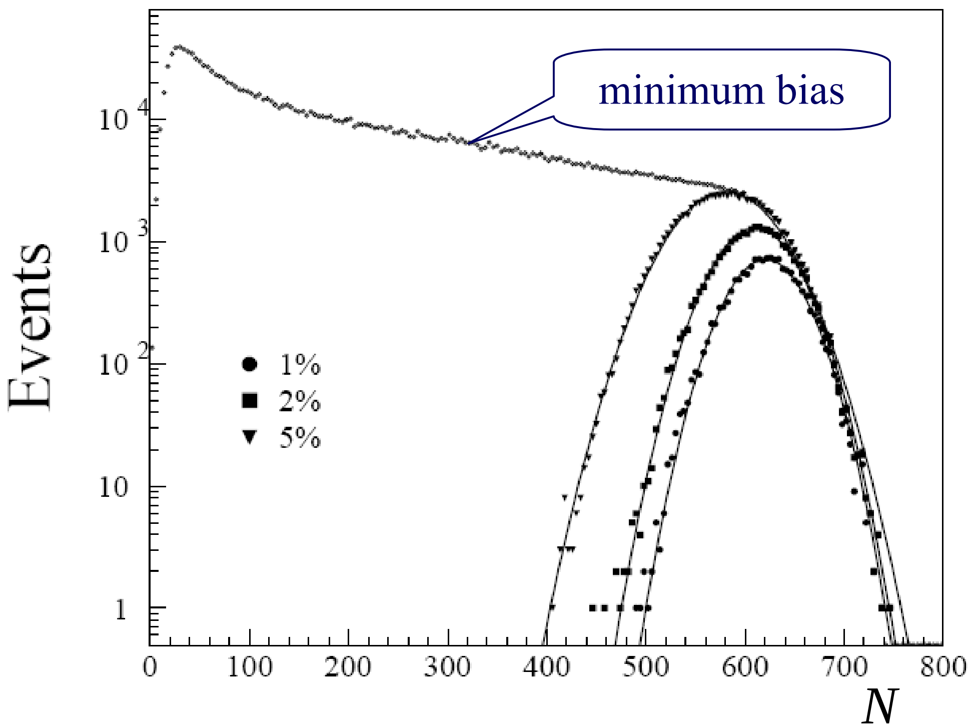
Intensive vs. extensive quantities

	Extensive	Intensive
Thermodynamics	$\sim N, V$ energy, entropy, etc.	$\mathrm{const}(N, V)$ temperature, density, etc.
Heavy-Ion Collisions	$\sim N_{\mathrm{part}}$ multiplicity, energy, etc.	$\mathrm{const}(N_{\mathrm{part}})$ inclusive average p_T, slope of p_T distribution, etc.

N – number of particles
V – volume
N_{part} – number of participants

Multiplicity distributions at different centralities

Pb-Pb @ 158 AGeV

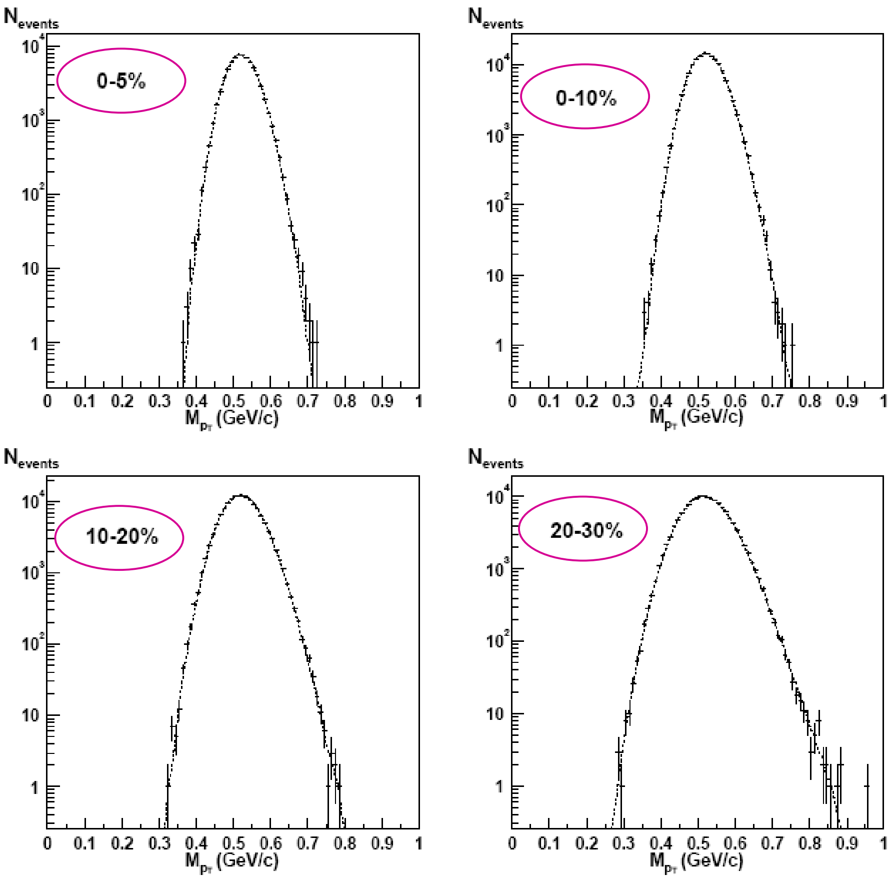


$\langle N \rangle$ & Var(N) strongly depend on trigger conditions

M.M. Aggarwal et al. [WA98 Collaboration], Phys. Rev. **C65**, 054912 (2002)

M_T distributions at different centralities

Au-Au @ $\sqrt{S_{NN}}$ = 130 GeV



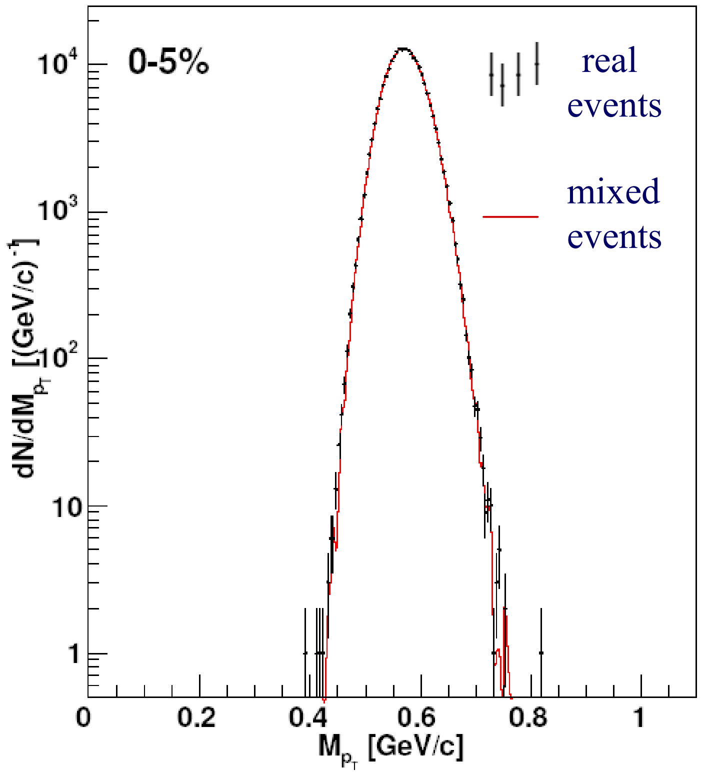
Gaussian width depends on centrality

$$M_{p_T} \equiv \frac{1}{N}\sum_{i=1}^{N} p_T^i$$ not intensive

K. Adcox et al. [PHENIX Collaboration], Phys. Rev. **C66**, 024901 (2002)

Real vs. mixed events

Au-Au @ $\sqrt{s_{NN}} = 200$ GeV



mixed events: no correlations

statistical fluctuations: effect of finite N

Observed fluctuations are dominated by statistical fluctuations

$$M_{p_T} \equiv \frac{1}{N}\sum_{i=1}^{N} p_T^i$$

S.S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett.**93**, 092301 (2004)

Fluctuation measure Φ

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$$

$z \equiv p_T - \overline{p_T}$ one-particle variable

$\overline{\cdots}$ inclusive average $\overline{z} = 0$

$Z \equiv \sum_{i=1}^{N} z^i = \sum_{i=1}^{N} \left(p_T^i - \overline{p_T}\right)$ event variable

$\langle \cdots \rangle$ average over events $\langle Z \rangle = 0$

- ✓ $\Phi = 0$ for mixed events (no correlations)
- ✓ Φ strictly intensive

M. Gaździcki & St. Mrówczyński, Z. Phys. **C54,** 127 (1992)

Other measures

$$M_{p_T} \equiv \frac{1}{N}\sum_{i=1}^{N} p_T^i$$ event variable

$$\sigma^2_{p_T,\mathrm{dyn}} \equiv \left\langle \left(M_{p_T} - \left\langle M_{p_T} \right\rangle \right)^2 \right\rangle - \frac{\overline{\left(p_T - \overline{p_T}\right)^2}}{\langle N \rangle}$$

S. A. Voloshin, V. Koch & H.G. Ritter, Phys. Rev. **C60**, 024901 (1999)

$$\Sigma_{p_T} \equiv \mathrm{sgn}(\sigma^2_{p_T,\mathrm{dyn}}) \frac{\sqrt{\left|\sigma^2_{p_T,\mathrm{dyn}}\right|}}{\overline{p_T}}$$

D. Adamova et al. [CERES Collaboration], Nucl. Phys. **A727**, 97 (2003)

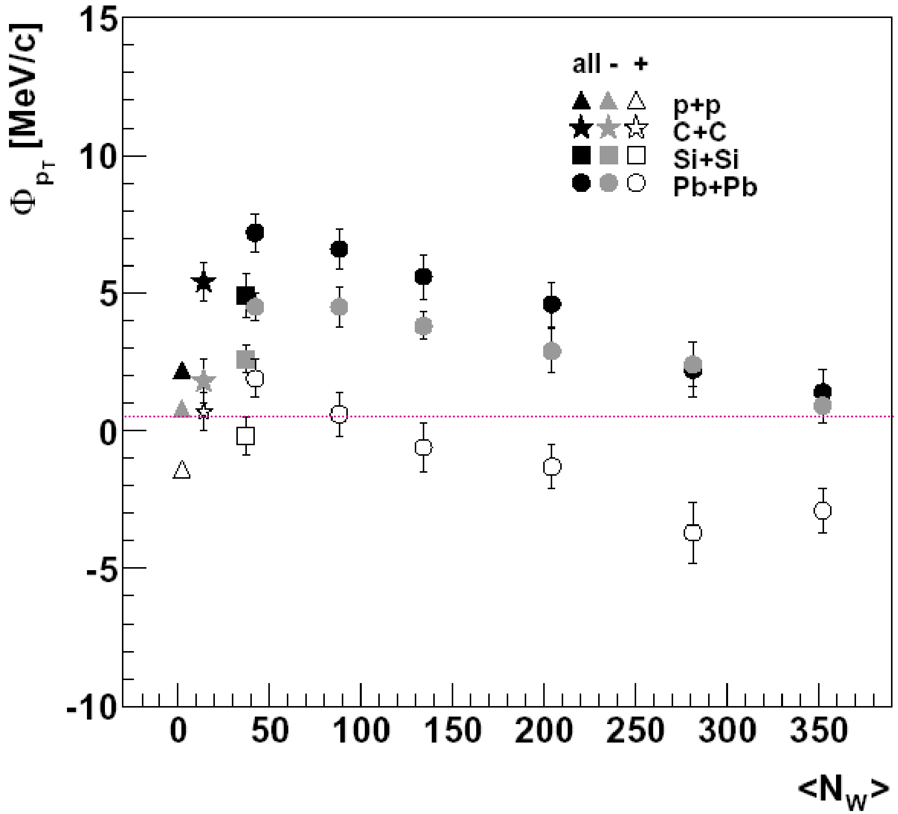
$$\omega \equiv \frac{1}{\left\langle M_{p_T} \right\rangle} \sqrt{\left\langle \left(M_{p_T} - \left\langle M_{p_T} \right\rangle \right)^2 \right\rangle}$$ scaled dispersion

$$F \equiv \frac{\omega_{\mathrm{data}} - \omega_{\mathrm{mixed}}}{\omega_{\mathrm{mixed}}}$$

S.S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. **93**, 092301 (2004)

p_T fluctuations @ SPS

A-A @ 158 AGeV



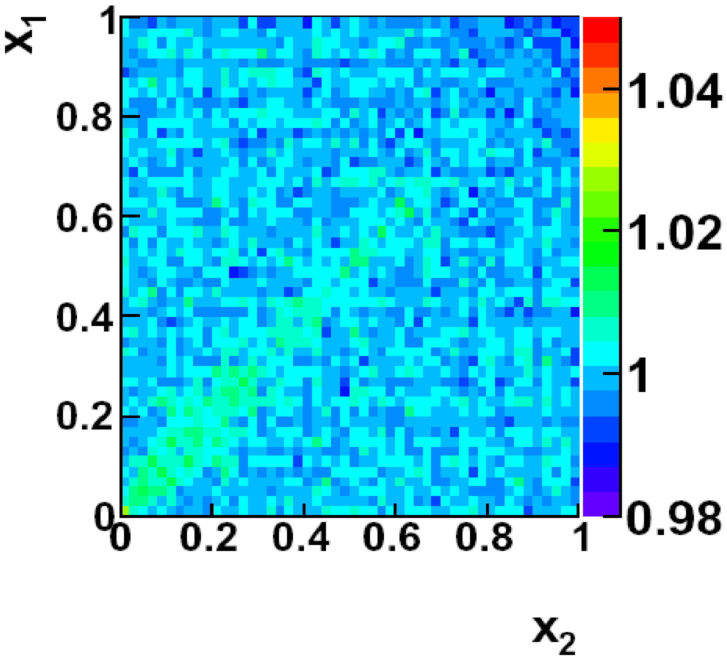
wounded nucleons

similar data by CERES

T. Anticic et al. [NA49 Collaboration], Phys. Rev. **C70**, 034902 (2004)

Differential analysis

Pb-Pb @ 158 AGeV



Bose-Einstein correlations

$$x_{1,2}(p_T) = \int_0^{p_T} dp'_T P(p'_T)$$

$$0 \le x_{1,2}(p_T) \le 1$$

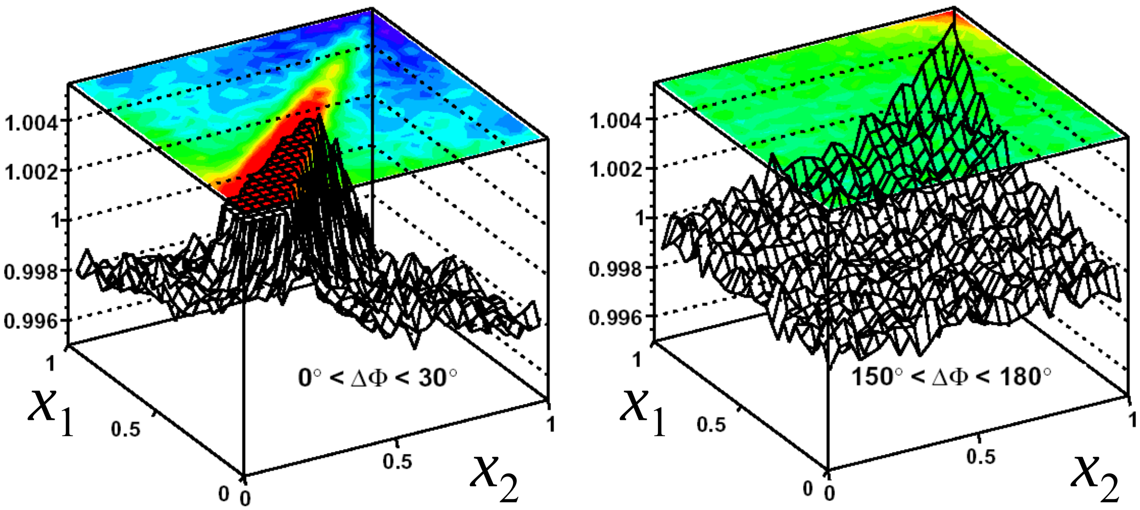
A Białas & M. Gaździcki, Phys. Lett. **B252**, 483 (1990)
T. A. Trainor, arXiv:hep-ph/0001148

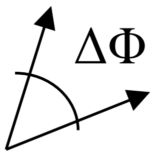
K. Grebieszkow et al. [NA49 Collaboration], PoS **CPOD07**, 022 (2007)

Even more differential analysis

Pb-Au @ 158 AGeV

$$x_{1,2}(p_T) = \int_0^{p_T} dp'_T P(p'_T)$$





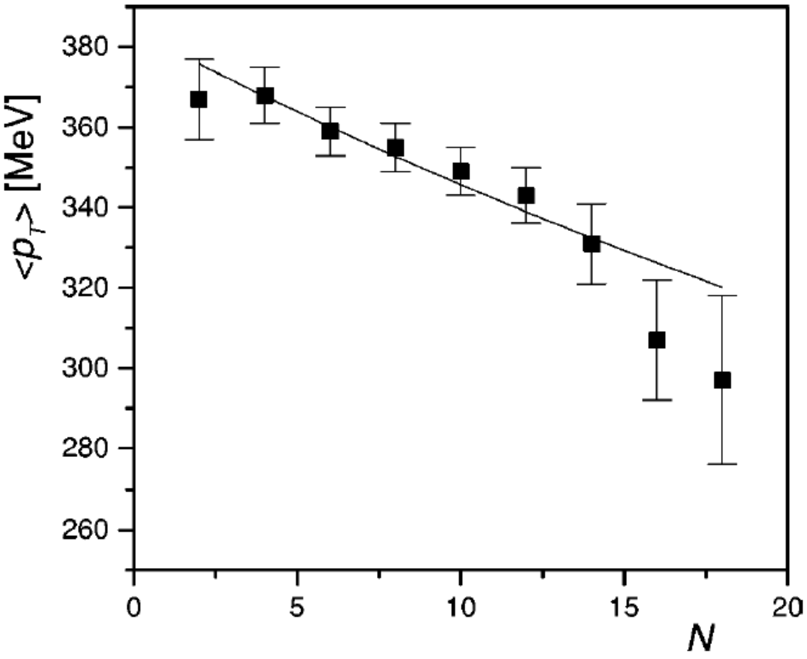
Bose-Einstein correlations

p_T slope fluctuations ?

D. Adamova et al. [CERES Collaboration], arXiv:0803.2407 [nucl-ex].

$p_T - N$ correlations

p-p @ 205 GeV



multiplicity dependent slope

$$T_N = T + \delta T \left(1 - \frac{N}{\langle N \rangle} \right)$$

$$\Phi \approx \sqrt{2} \, \frac{(\delta T)^2}{T} \, \frac{\mathrm{Var}(N)}{\langle N \rangle}$$

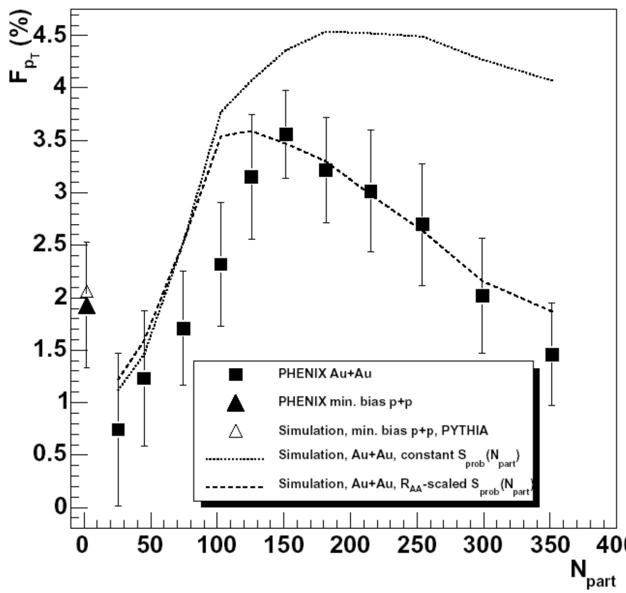
St. Mrówczyński, M. Rybczyński & Z. Włodarczyk, Phys. Rev. **C70,** 054906 (2004)

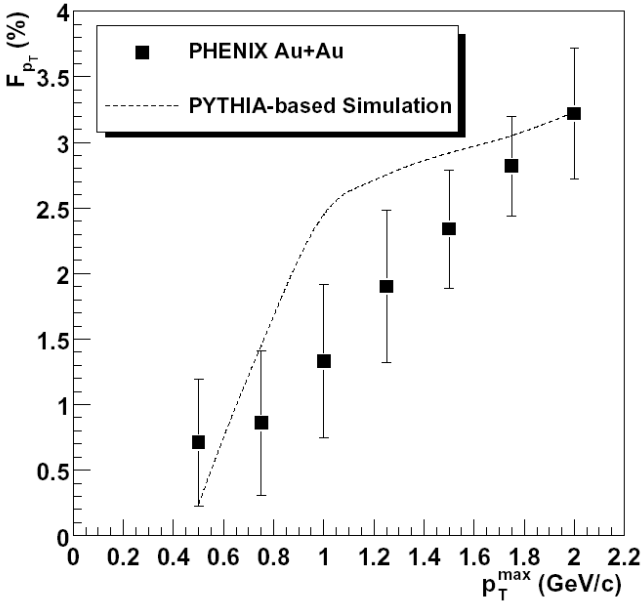
T. Kafka et al. Phys. Rev. **D16**, 1261 (1977)

p_T fluctuations @ RHIC

Au-Au @ $\sqrt{S_{NN}}$ = 200 GeV

similar data by STAR





$$F \equiv \frac{\omega_{\text{data}} - \omega_{\text{mixed}}}{\omega_{\text{mixed}}}$$

jets ?

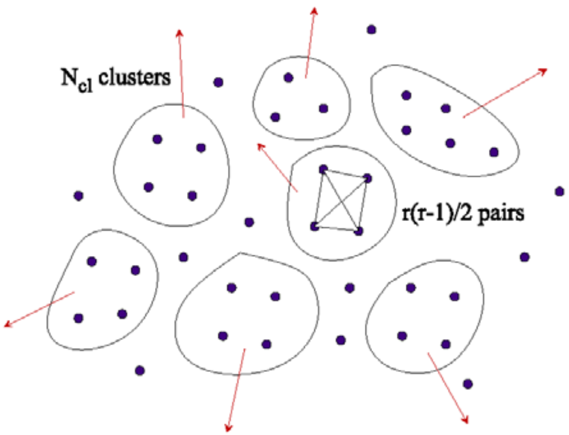
S.S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. **93**, 092301 (2004)

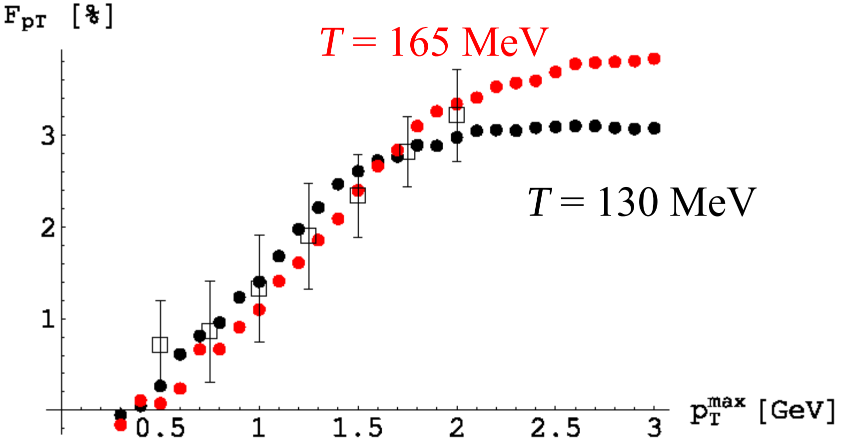
p_T fluctuations @ RHIC

$$\sigma^2_{p_T,\mathrm{dyn}} \sim \frac{1}{\langle N \rangle}$$

$$\Phi(p_T) \approx \mathrm{const}(N_{\mathrm{part}})$$

$\Rightarrow$





clusters ?

W. Broniowski, P. Bozek, W. Florkowski & B. Hiller, PoS **CFRNC2006**, 020 (2006)

Thermodynamic fluctuations

Temperature fluctuations

$$\langle T^2 \rangle - \langle T \rangle^2 = \frac{\langle T \rangle^2}{C_v} \qquad C_v \equiv T \left(\frac{\partial S}{\partial V} \right)_{V,\langle N \rangle}$$

heat capacity ?

L. Stodolsky, Phys. Rev. Lett. **75**, 1044 (1995); E. V. Shuryak, Phys. Lett. **B423**, 9 (1998)

Multiplicity fluctuations

$$\langle N^2 \rangle - \langle N \rangle^2 = \frac{T \langle N \rangle^2}{V^2 \kappa} \qquad \kappa \equiv - \left(\frac{\partial p}{\partial V} \right)_{T,\langle N \rangle}$$

compressibility

St. Mrówczyński, Phys. Lett. **B430**, 9 (1998)

Electric charge fluctuations

$$\langle Q^2 \rangle - \langle Q \rangle^2 = TV \chi_Q \qquad \chi_Q \equiv - \frac{1}{V} \left(\frac{\partial^2 F}{\partial \mu_Q^2} \right)_{T,V}$$

electric charge susceptibility

S. Jeon & V. Koch, Phys. Rev. Lett. **85**, 2076 (2000);
M. Asakawa, U.W. Heinz & B. Muller, Phys. Rev. Lett. **85**, 2072 (2000)

Electric charge fluctuations

Ideal classical gas of charged particles

$$Q = q(N_+ - N_-) \qquad \delta Q \equiv Q - \langle Q \rangle \qquad \delta N_\pm \equiv N_\pm - \langle N_\pm \rangle$$

$$\langle \delta Q^2 \rangle = q^2 \langle (\delta N_+ - \delta N_-)^2 \rangle = q^2 \left(\langle \delta N_+^2 \rangle + \langle \delta N_+^2 \rangle - 2 \langle \delta N_+ \delta N_- \rangle \right)$$

$$\langle \delta N_+ \delta N_- \rangle = 0 \qquad \langle \delta N_\pm^2 \rangle = \langle N_\pm \rangle \qquad \langle N \rangle \equiv \langle N_+ \rangle + \langle N_- \rangle$$

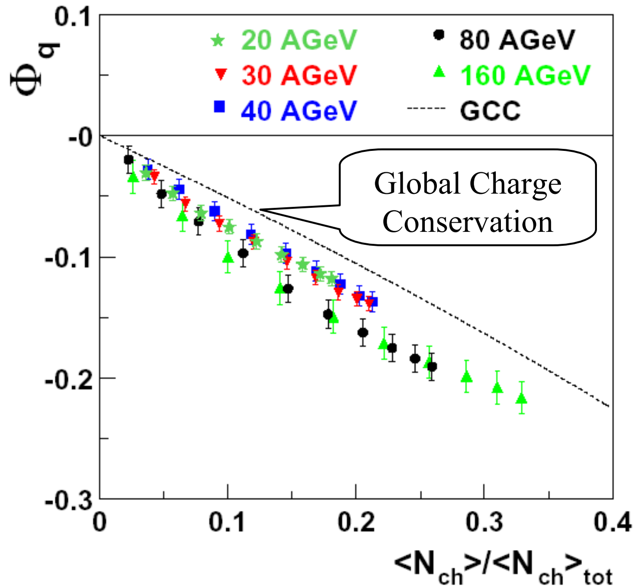
$$\frac{\langle \delta Q^2 \rangle}{\langle N \rangle} = q^2 \quad \Longrightarrow \quad \frac{\langle \delta Q^2 \rangle}{S} = \begin{cases} \frac{1}{6} & \text{pion gas} \\ \frac{1}{24} & \text{QGP} \end{cases}$$

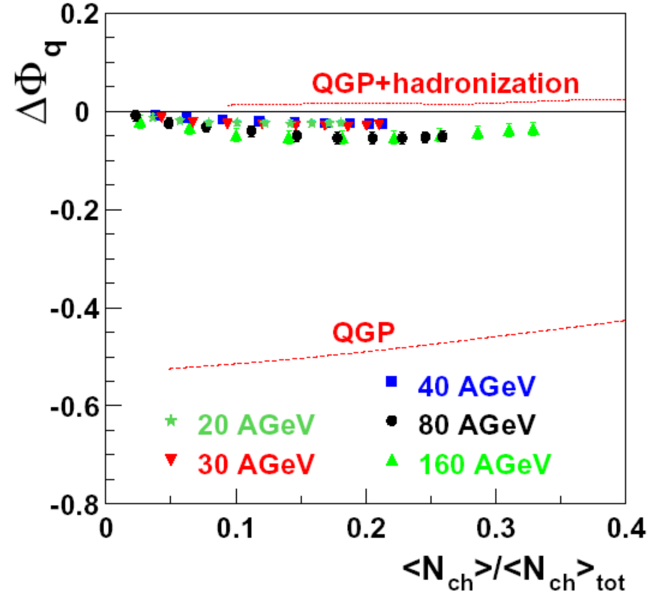
$S \sim \langle N \rangle$ conserved

S. Jeon & V. Koch, *Quark-Gluon Plasma 3*, eds. R. Hwa & X-N. Wang (World Scientific, Singapore, 2004)

Electric charge fluctuations @ SPS

Pb-Pb





$$\Phi_q \equiv \sqrt{\frac{\langle Z \rangle^2}{\langle N \rangle}} - \sqrt{\overline{z}^2}$$

$$\Phi_q^{GCC} = \sqrt{1 - \frac{\langle N_{\mathrm{ch}} \rangle}{\langle N_{\mathrm{ch}} \rangle_{\mathrm{tot}}}} - 1$$

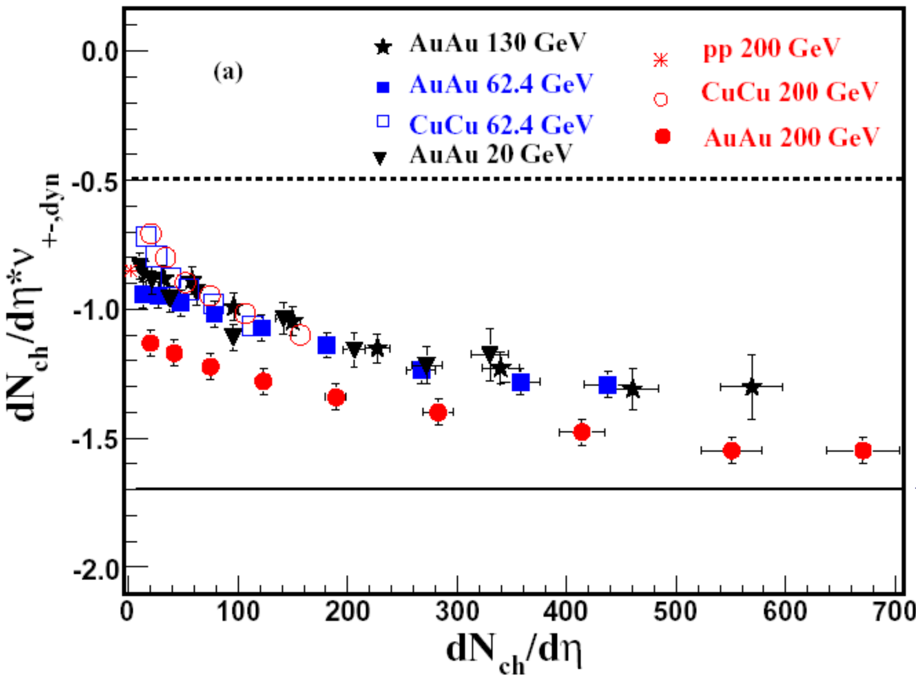
$$\Delta\Phi_q \equiv \Phi_q - \Phi_q^{GCC}$$

$$z \equiv q - \overline{q} \qquad Z \equiv \sum_{i=1}^{N} z^i$$

Global Charge Conservation

C. Alt et al. [NA49 Collaboration], Phys. Rev. **C70**, 064903 (2004)

Electric charge fluctuations @ RHIC



charge conservation effect

hadron resonance gas

QGP

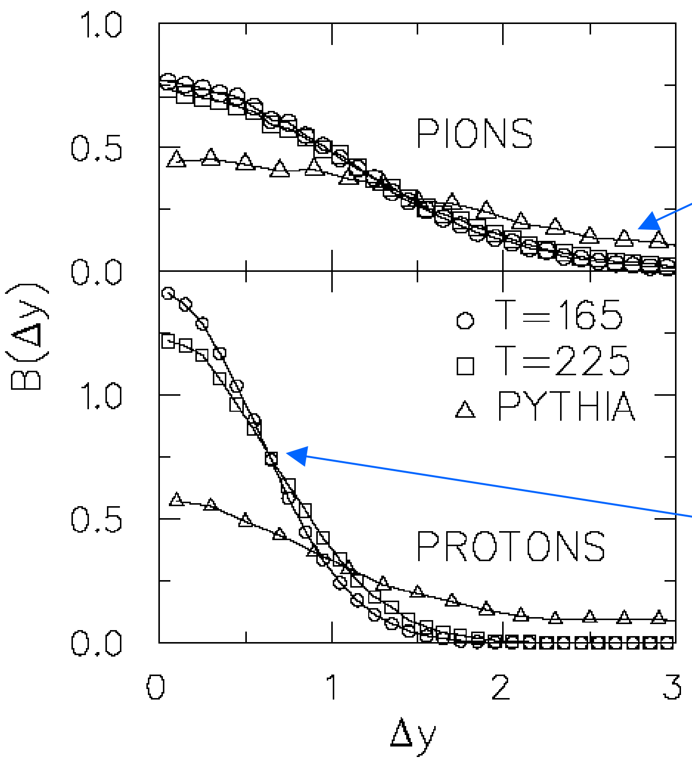
QGP electric fluctuations
are not seen in the final state

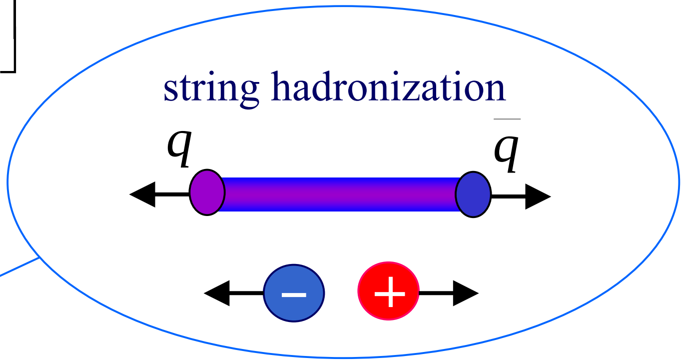
$$\nu_{+-,\mathrm{dyn}} \equiv \frac{\langle N_+(N_+-1)\rangle}{\langle N_+\rangle^2} + \frac{\langle N_-(N_--1)\rangle}{\langle N_-\rangle^2} - 2\frac{\langle N_-N_+\rangle}{\langle N_-\rangle\langle N_+\rangle}$$

B.I. Abelev et al. [STAR Collaboration], arXiv:0807.3269 [nucl-ex]

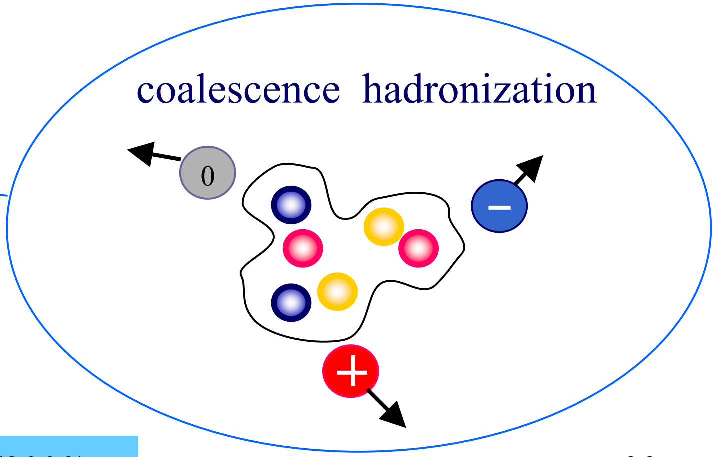
Balance functions

$$B(\Delta y) \equiv \frac{1}{2}\left[\frac{N_{+-}(\Delta y) - N_{--}(\Delta y)}{N_{-}(\Delta y)} + \frac{N_{-+}(\Delta y) - N_{++}(\Delta y)}{N_{+}(\Delta y)}\right]$$





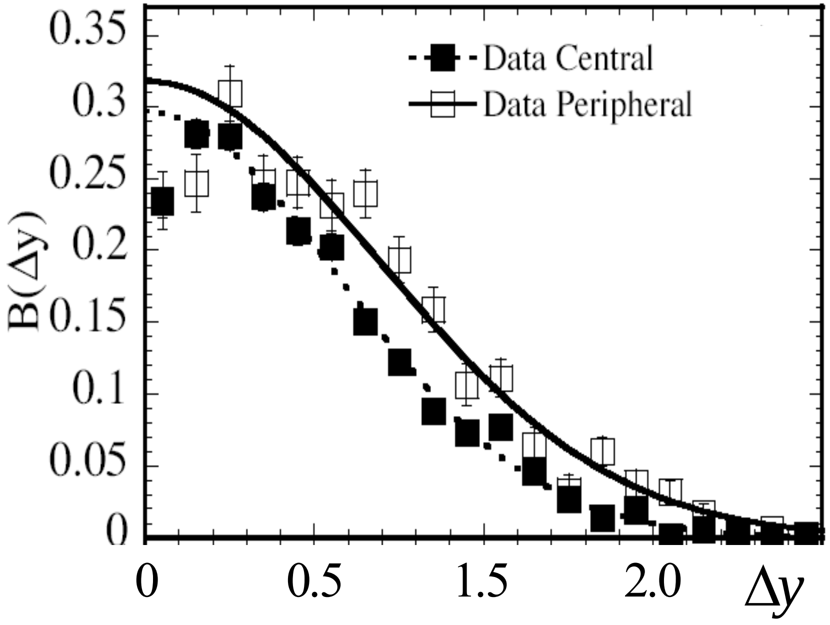
A. Bialas, Phys. Lett. **B579**, 31 (2004)



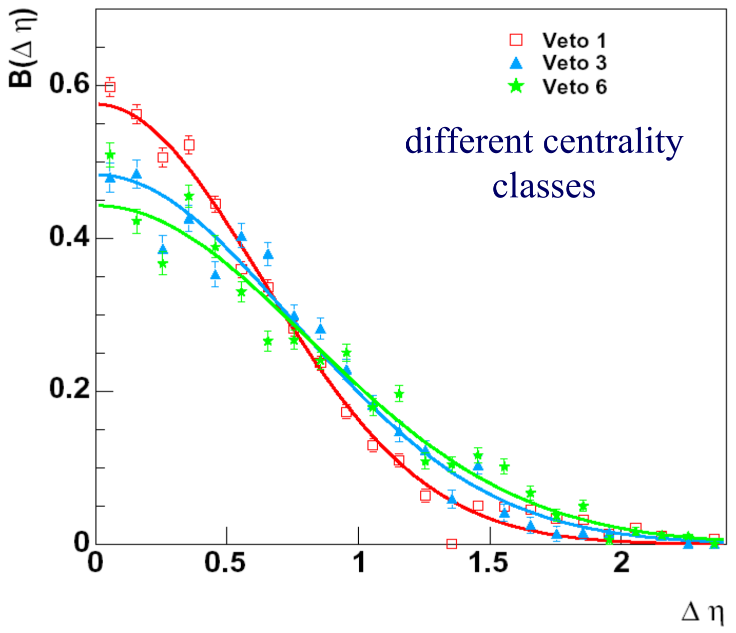
S.A. Bass, P. Danielewicz & S. Pratt, Phys. Rev. Lett. **85**, 2689 (2000)

Experimental balance functions

Au-Au @ $\sqrt{S_{NN}}$ = 130 GeV



Pb-Pb @ 158 AGeV

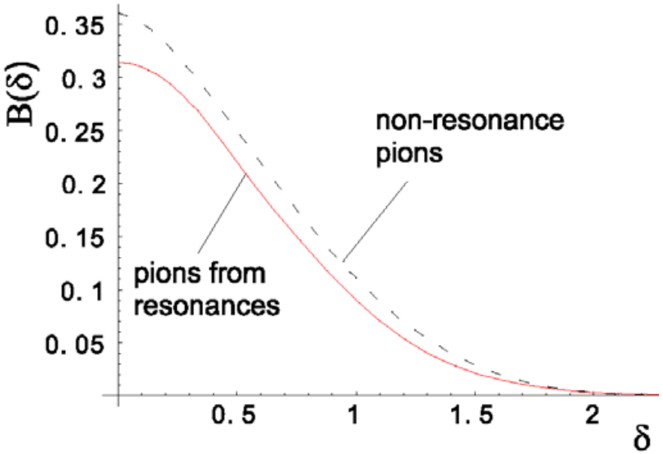


J. Adams et al. [STAR Collaboration], Phys Rev Lett **90**, 172301 (2003)

C. Alt et al. [NA49 Collaboration], Phys. Rev. **C71**, 034903 (2005)

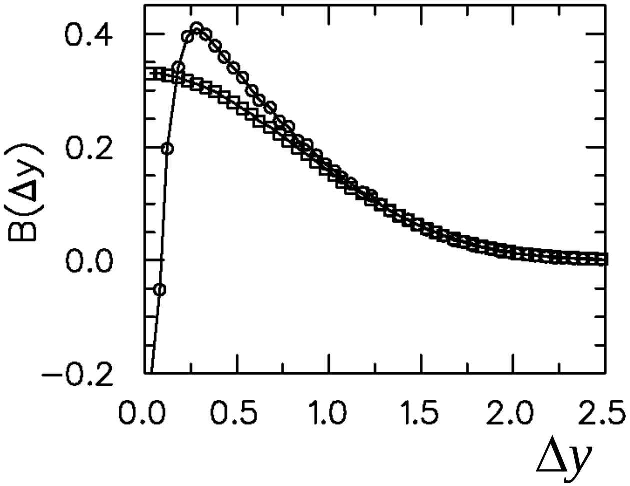
Interpretation of balance functions

- **effect of resonances**



P. Bożek, W. Broniowski & W. Florkowski, Acta Phys. Hung. **A22**, 149 (2005)

- **role of final state interactions**



S. Pratt & S. Cheng, Phys. Rev. **C68**, 014907 (2003)

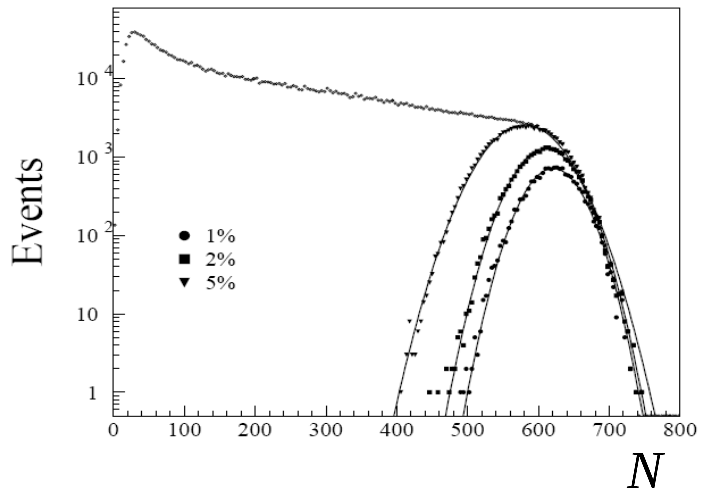
- **effect of flow**

$B(\Delta y) \longrightarrow B(\varphi)$

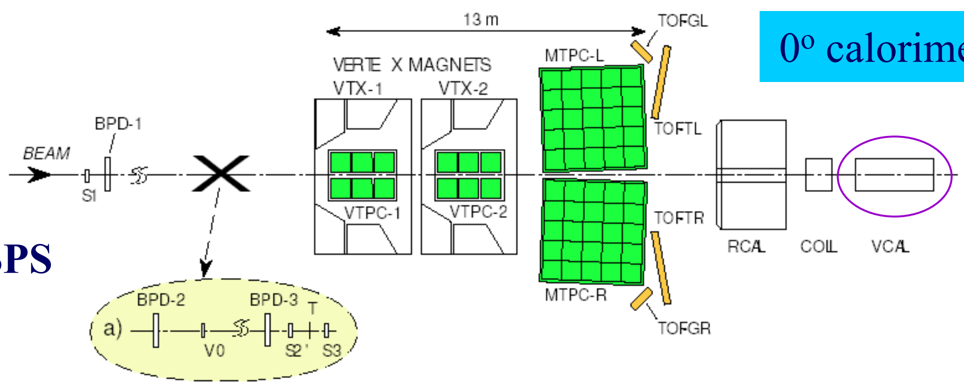
P. Bozek, Phys. Lett. **B609**, 247 (2005)

Multiplicity fluctuations

Pb-Pb @ 158 AGeV



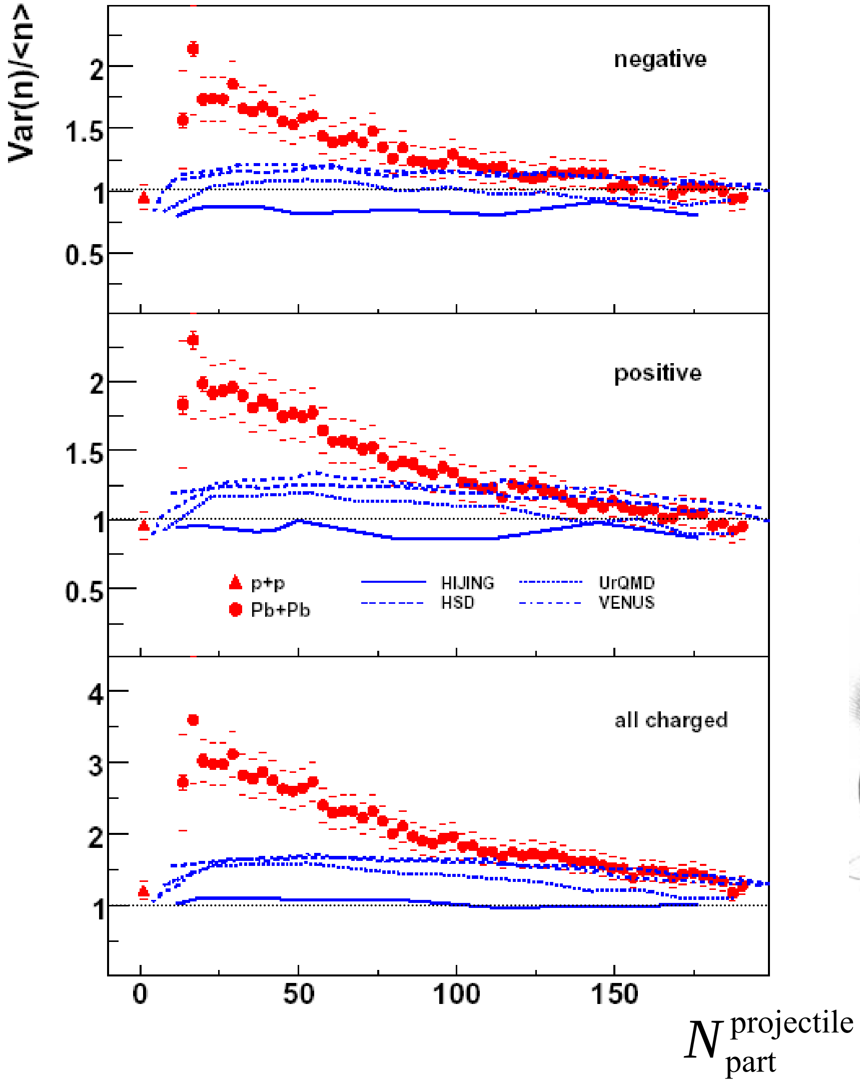
M.M. Aggarwal et al. [WA98 Collaboration], Phys. Rev. **C65**, 054912 (2002)



0° calorimeter

NA49 experiment @ SPS

Multiplicity fluctuations at fixed projectile N_{part}

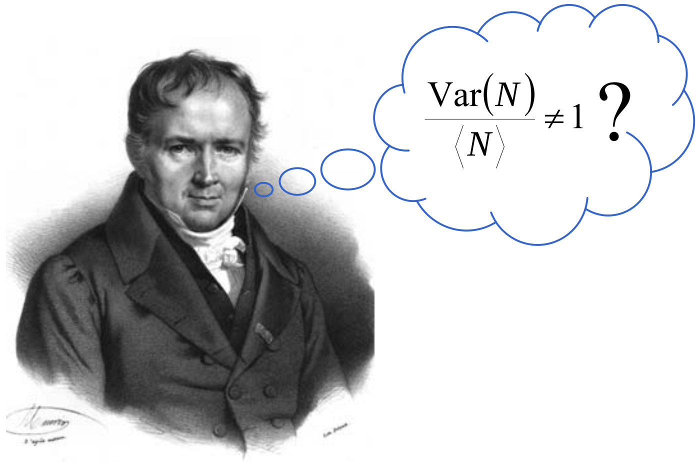


A-A @ 158 AGeV

$N_{part}^{projectile}$ fixed by 0° calorimeter

$$\frac{\mathrm{Var}(N)}{\langle N \rangle} = 1 \quad \text{for Possion}$$

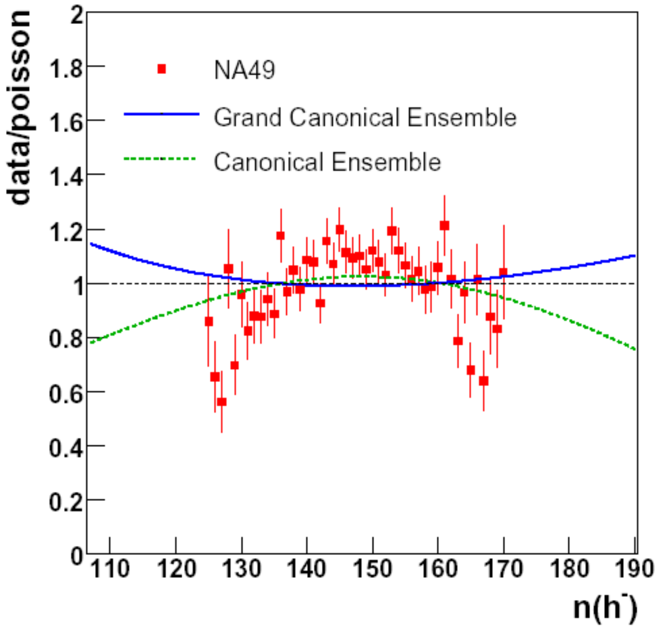
similar data by WA98 & PHENIX



C. Alt et al. [NA49 Collaboration], Phys. Rev. **C75**, 064904 (2007)

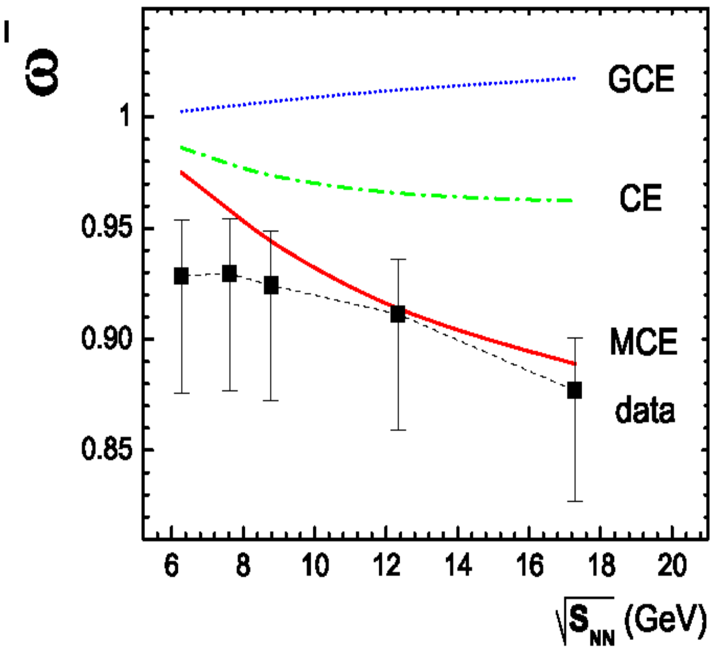
Multiplicity fluctuations in very central collisions

Pb-Pb @ 158 AGeV



C. Alt et al. [NA49 Collaboration], arXiv:0712.3216 [nucl-ex]

Fluctuations in various statistical ensembles

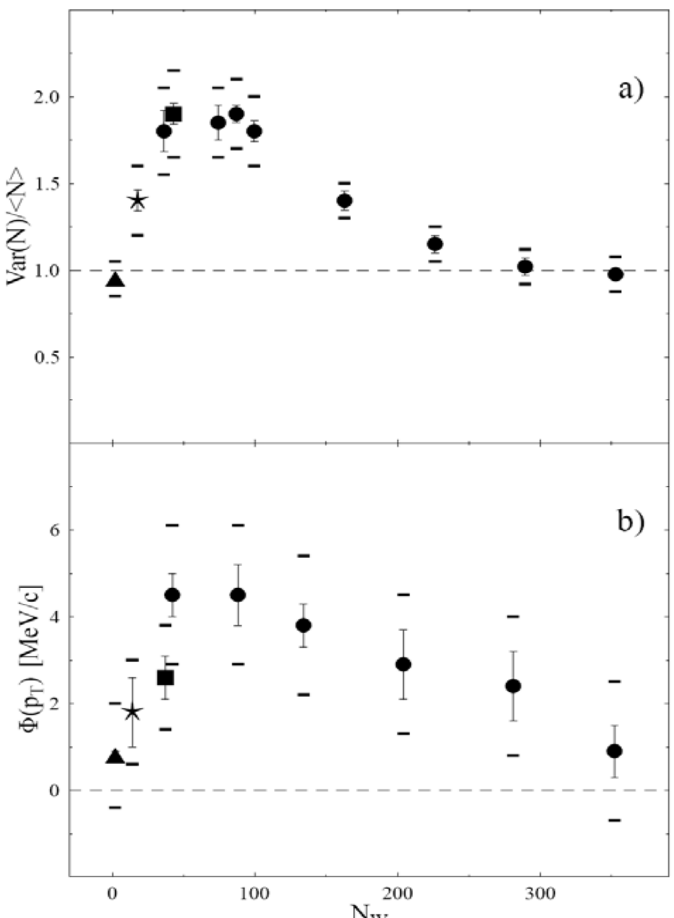


V.V. Begun, M. Gaździcki, M.I. Gorenstein, M. Hauer, V.P. Konchakovski & B.Lungwitz, Phys. Rev. **C76**, 024902 (2007)

$$\omega \equiv \mathrm{Var}(N)/\langle N \rangle$$

Multiplicity vs. p_T fluctuations

Pb-Pb @ 158 AGeV



wounded nucleons

multiplicity dependent slope

$$T_N = T + \delta T \left(1 - \frac{N}{\langle N \rangle} \right)$$

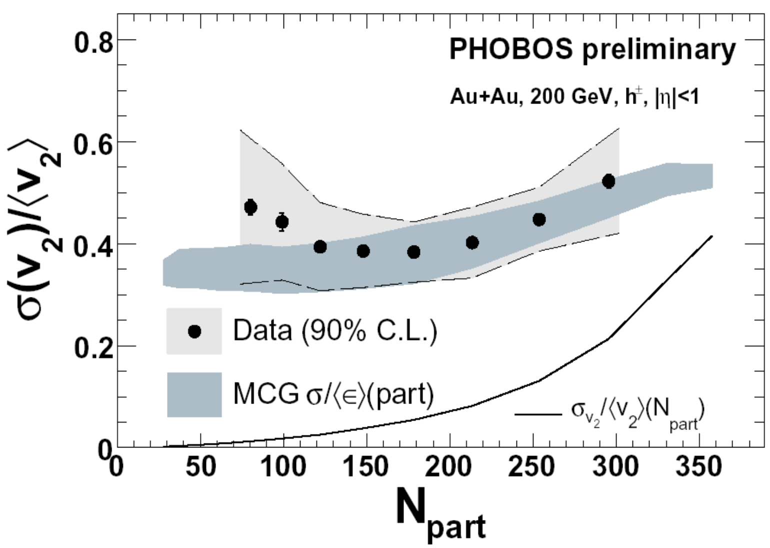
$$\Phi \approx \sqrt{2} \frac{(\delta T)^2}{T} \frac{\mathrm{Var}(N)}{\langle N \rangle}$$

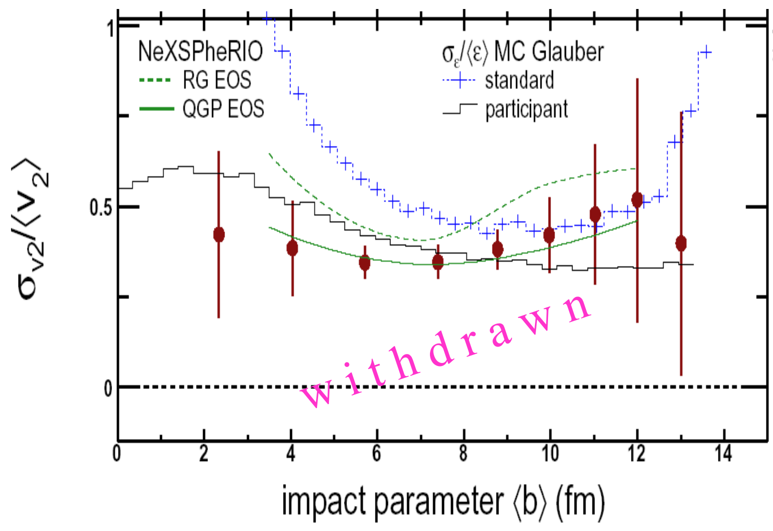
p_T fluctuations are coupled to multiplicity fluctuations

St. Mrówczyński, M. Rybczyński & Z. Włodarczyk, Phys. Rev. **C70,** 054906 (2004)

Elliptic flow fluctuations

Au-Au @ $\sqrt{S_{NN}}$ = 200 GeV





B. Alver et al. [PHOBOS Collaboration], J. Phys. G34, S907 (2007)

P. Sorensen [STAR Collaboration], J.Phys. **G34**, S897 (2007)

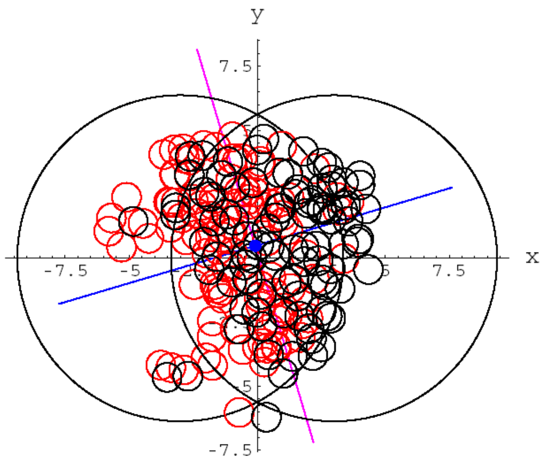
Upper limit only!
v_2 fluctuations and non-flow effects cannot be disentangled!

Eccentricity & v_2 fluctuations

Geometric fluctuations only

$$v_2 \sim \varepsilon \quad \Rightarrow \quad \frac{\sigma_{v_2}}{\langle v_2 \rangle} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle}$$

M. Miller & R. Snellings, arXiv:nucl-ex/0312008



W. Broniowski, P. Bożek & M. Rybczyński, Phys. Rev. **C76**, 054905 (2007)

No dynamic fluctuations?

St.Mrówczyński & E.V. Shuryak, Acta Phys. Pol. **B34**, 4241 (2003)

Conclusions & outlook

- Some interesting but no spectacular e-b-e results
- Dynamic fluctuations usually small
- Measurements in large acceptance needed
- Dedicated detectors desirable (NA61)
- Realistic modeling needed