

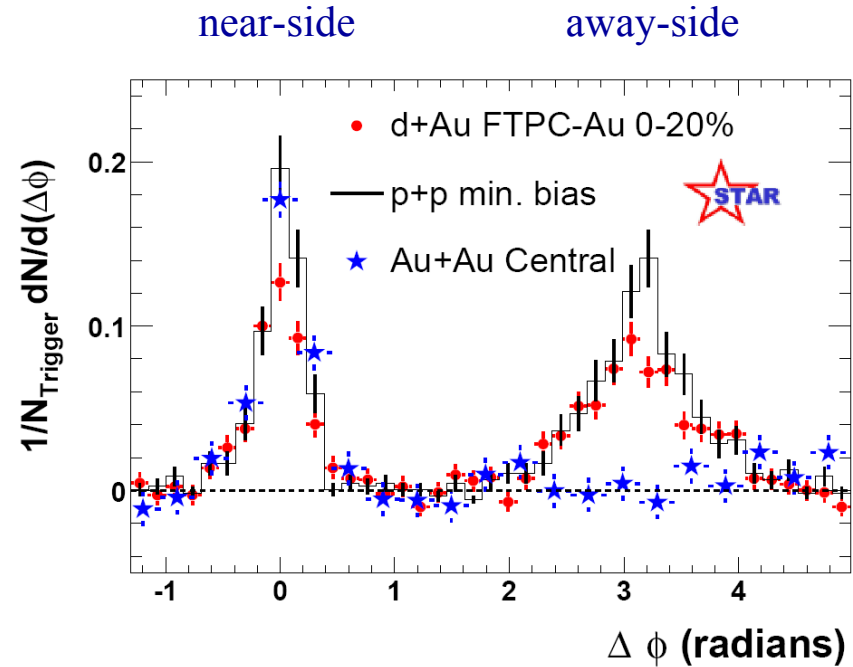
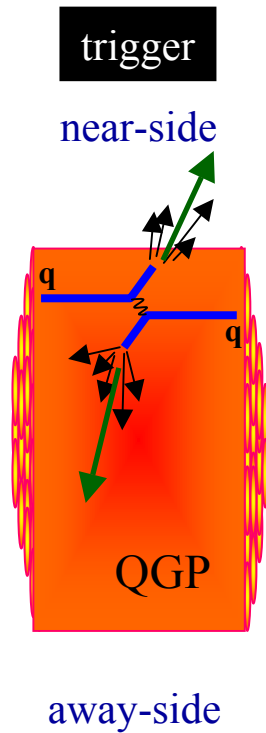
# Dynamics of Unstable Quark-Gluon Plasma

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How to compute  $\hat{q}$  in unstable QGP ?

# Jet quenching @ RHIC



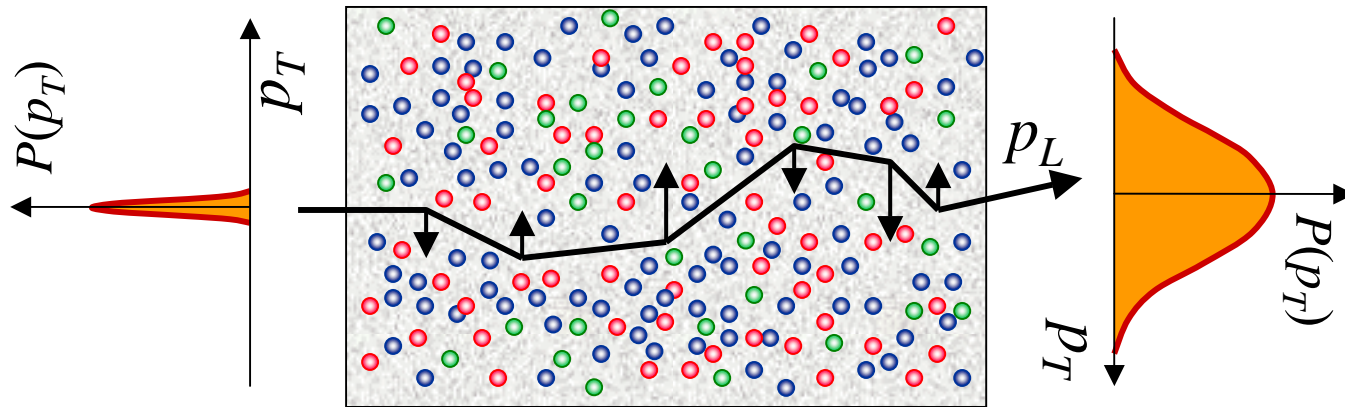
Away-side jet is suppressed  
in central collisions

# Momentum transverse broadening

Radiative energy loss of a fast parton is controlled by

$$\hat{q} \equiv \frac{d\langle \Delta p_T^2(t) \rangle}{dt}$$

Baier, Dokshitzer, Mueller, Peigne & Schiff 1996

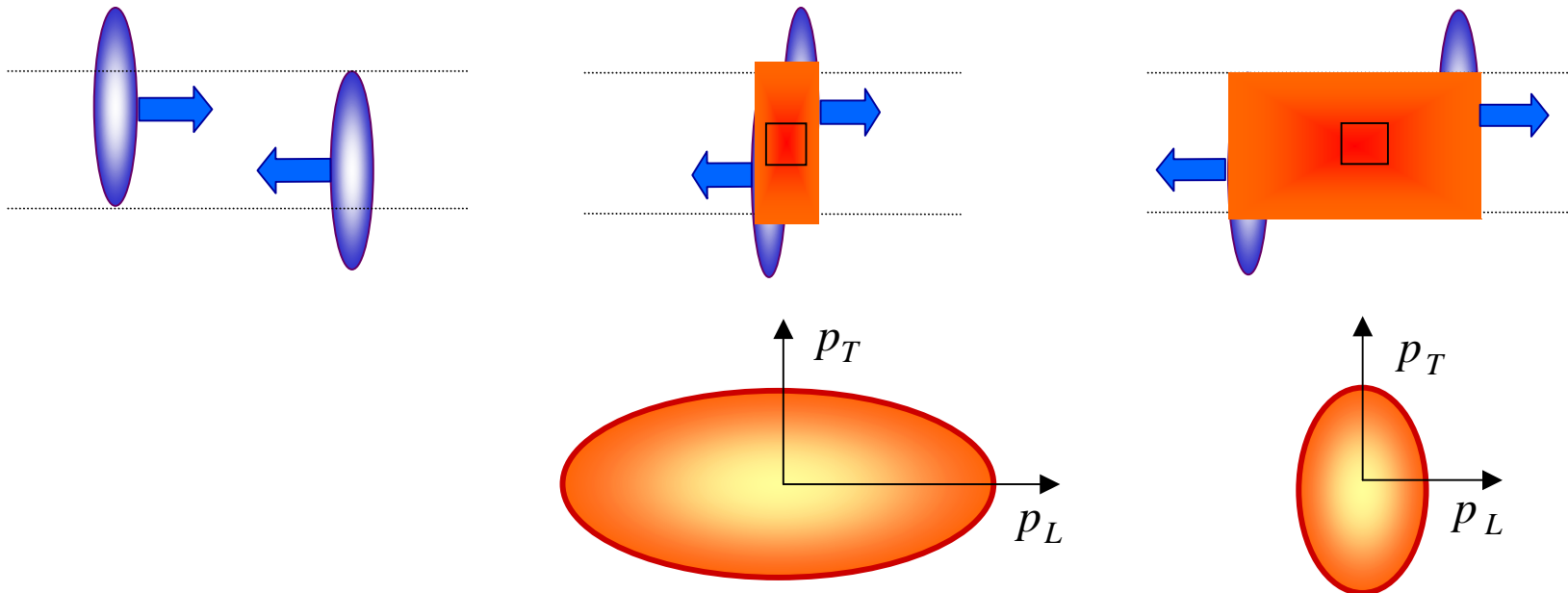


From experiment:  $0.5 \text{ GeV}^2/\text{fm} \leq \hat{q} \leq 15 \text{ GeV}^2/\text{fm}$

$\underbrace{\hspace{10em}}_{\text{pQGP}}$ 
 $\underbrace{\hspace{10em}}_{?}$

Is QGP strongly coupled?

# Anisotropic QGP @ RHIC



Anisotropic QGP is unstable due to magnetic plasma modes

# Momentum broadening in anisotropic QGP

- ▶ P. Romatschke, Phys. Rev. **C75**, 014901 (2007)
  - ▶ R. Baier and Y. Mehtar-Tani, Phys. Rev. **C78**, 064906 (2008)
- Anisotropic (unstable) QGP was treated as a static medium
- $\hat{q} = \text{const}$

Unstable QGP is generically time dependent

Numerical simulations

$$\hat{q} \sim e^{2\tau}$$

- ▶ A. Dumitru, Y. Nara, B. Schenke & M. Strickland, Phys. Rev. **C78**, 024909 (2008);  
B. Schenke, M. Strickland, A. Dumitru, Y. Nara & C. Greiner, Phys. Rev. **C79**, 034903 (2009)

# Fast parton in QGP

Wong's equation of motion

$$\left\{ \begin{array}{l} \frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau) \\ \frac{dp^\mu(\tau)}{d\tau} = gQ_a(\tau) F_a^{\mu\nu}(x(\tau)) u_\nu(\tau) \\ \frac{dQ_a(\tau)}{d\tau} = -gf^{abc} p_\mu(\tau) A_b^\mu(x(\tau)) Q_c(\tau) \end{array} \right.$$

Initial value problem

Gauge condition

$$p_\mu(\tau) A_b^\mu(x(\tau)) = 0 \Rightarrow Q_a(\tau) = \text{const}$$

Parton travels with constant velocity:  $u^\mu = (\gamma, \gamma \mathbf{v}) = \text{const}$

$$p^\mu(\tau) = p^\mu(0) + gQ_a \int_0^\tau d\tau' F_a^{\mu\nu}(x(\tau')) u_\nu$$

# Langevin problem

$$\langle p^\mu(\tau) p^\nu(\tau) \rangle = p^\mu(0) p^\nu(0) + g^2 C \int_0^\tau d\tau_1 \int_0^\tau d\tau_2 \langle F_a^{\mu\nu}(x(\tau_1)) F_a^{\sigma\rho}(x(\tau_2)) \rangle u_\sigma u_\rho$$

▶  $p^\mu(0) = (E, 0, 0, p_z)$

▶ parton travels with speed of light  
along axis  $z$ :  $\mathbf{v}(t) = \text{const} = (0, 0, 1)$

$$C \equiv \begin{cases} \frac{1}{2N_c} & \text{-- fundamenta l} \\ \frac{N_c}{N_c^2 - 1} & \text{-- adjoint} \end{cases}$$

$$\langle \Delta p_T^2(t) \rangle = \langle p_x^2(t) \rangle + \langle p_y^2(t) \rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \langle (F_a^{\mu 0}(x(t_1)) - F_a^{\mu 3}(x(t_1))) (F_a^{\nu 0}(x(t_2)) - F_a^{\nu 3}(x(t_2))) \rangle$$

$$\begin{aligned} \langle \Delta p_T^2(t) \rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \{ & \langle E_a^x(x(t_1)) E_a^x(x(t_2)) \rangle + \langle E_a^y(x(t_1)) E_a^y(x(t_2)) \rangle - \langle E_a^x(x(t_1)) B_a^y(x(t_2)) \rangle \\ & + \langle E_a^y(x(t_1)) B_a^x(x(t_2)) \rangle - \langle B_a^y(x(t_1)) E_a^x(x(t_2)) \rangle - \langle B_a^x(x(t_1)) E_a^y(x(t_2)) \rangle \\ & + \langle B_a^x(x(t_1)) B_a^x(x(t_2)) \rangle + \langle B_a^y(x(t_1)) B_a^y(x(t_2)) \rangle \} \end{aligned}$$

# How to compute fluctuations in unstable systems?

- Equilibrium methods are not applicable
- We deal with the **initial value** problem

The kinetic theory method by Klimontovich & Silin, Rostoker, Tsytovich, see E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

St. Mrówczyński, Acta Phys. Pol. **B39** (2008) 941 - Electromagnetic Fluctuations  
St. Mrówczyński, Phys. Rev. **D77** (2008) 105022 - Chromodynamic Fluctuations



# Transport equations

fundamental

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C[Q, \bar{Q}, G]$$

quarks

$$p_\mu D^\mu \bar{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}\} = \bar{C}[Q, \bar{Q}, G]$$

antiquarks

adjoint

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu G\} = C_g[Q, \bar{Q}, G]$$

gluons

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \dots], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \bar{Q}, G]$$

mean-field generation

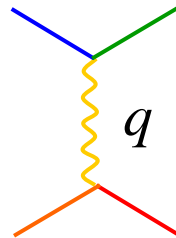
$$\text{collisionless limit: } C = \bar{C} = C_g = 0$$

# Time scale of collisional processes

Time scale of processes driven by parton-parton scattering

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g) T}$$

$$t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g) T}$$



hard scattering:  $q \sim T$

soft scattering:  $q \sim gT$

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g T}$$

$$g^2 \ll 1 \Rightarrow t_{\text{hard}} \gg t_{\text{soft}} \gg t_{\text{collec}}$$

The instabilities are fast if QGP is weakly coupled

# Small fluctuations

The distribution function of quarks

fluctuation

$$Q(t, \mathbf{r}, \mathbf{p}) = Q_0(\mathbf{p}) + \delta Q(t, \mathbf{r}, \mathbf{p})$$

stationary colorless state  $Q_0^{ij}(\mathbf{p}) = \delta^{ij} n(\mathbf{p})$

$$|Q_0(\mathbf{p})| \gg |\delta Q(t, \mathbf{r}, \mathbf{p})|, \quad |\nabla_p Q_0(\mathbf{p})| \gg |\nabla_p \delta Q(t, \mathbf{r}, \mathbf{p})|$$

$$\mathbf{E}(t, \mathbf{r}), \mathbf{B}(t, \mathbf{r}), A^0(t, \mathbf{r}), \mathbf{A}(t, \mathbf{r}) \sim \delta Q(t, \mathbf{r}, \mathbf{p})$$

quarks only, inclusion of antiquarks and gluons:  $n(\mathbf{p}) \rightarrow n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$

# Linearized equations

Transport equation

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \delta Q(t, \mathbf{r}, \mathbf{p}) - g (\mathbf{E}(t, \mathbf{r}) + \mathbf{v} \times \mathbf{B}(t, \mathbf{r})) \nabla_p n(\mathbf{p}) = 0$$

Yang-Mills (Maxwell) equations

$$\begin{aligned} \nabla \cdot \mathbf{E}(t, \mathbf{r}) &= \rho(t, \mathbf{r}), & \nabla \cdot \mathbf{B}(t, \mathbf{r}) &= 0, \\ \nabla \times \mathbf{E}(t, \mathbf{r}) &= -\frac{\partial \mathbf{B}(t, \mathbf{r})}{\partial t}, & \nabla \times \mathbf{B}(t, \mathbf{r}) &= \mathbf{j}(t, \mathbf{r}) + \frac{\partial \mathbf{E}(t, \mathbf{r})}{\partial t} \end{aligned}$$

$$\left\{ \begin{aligned} \rho_a(t, \mathbf{r}) &= -g \int \frac{d^3 p}{(2\pi)^3} \text{Tr} [\tau^a \delta Q(t, \mathbf{r}, \mathbf{p})], \\ \mathbf{j}_a(t, \mathbf{r}) &= -g \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} \text{Tr} [\tau^a \delta Q(t, \mathbf{r}, \mathbf{p})], \end{aligned} \right.$$

gauge dependence  
discussed *a posteriori*

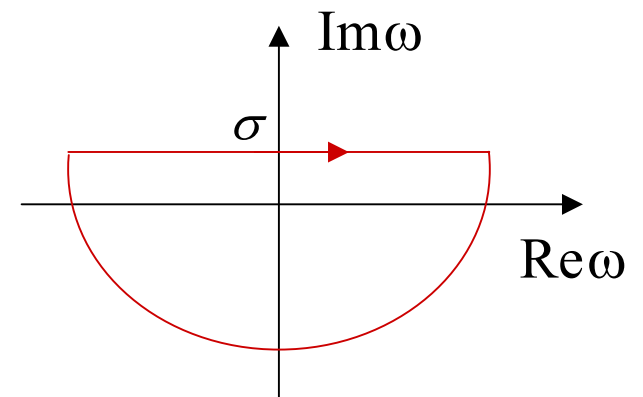
# Initial value problem

$$\delta Q(t = 0, \mathbf{r}, \mathbf{p}) = \delta Q_0(\mathbf{r}, \mathbf{p}),$$
$$\mathbf{E}(t = 0, \mathbf{r}, \mathbf{p}) = \mathbf{E}_0(\mathbf{r}, \mathbf{p}), \quad \mathbf{B}(t = 0, \mathbf{r}, \mathbf{p}) = \mathbf{B}_0(\mathbf{r}, \mathbf{p})$$

One-sided Fourier transformations

$$\left\{ \begin{array}{l} f(\omega, \mathbf{k}) = \int_0^{\infty} dt \int d^3 r e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} f(t, \mathbf{r}) \\ f(t, \mathbf{r}) = \int_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} f(\omega, \mathbf{k}) \end{array} \right.$$

$$0 < \sigma \in \mathbb{R}$$



# Transformed linear equations

Transport equation

$$-i(\omega - \mathbf{v} \cdot \mathbf{k})\delta Q(\omega, \mathbf{k}, \mathbf{p}) - g(\mathbf{E}(\omega, \mathbf{k}) + \mathbf{v} \times \mathbf{B}(\omega, \mathbf{k}))\nabla_{\mathbf{p}} n(\mathbf{p}) = \delta Q_0(\mathbf{k}, \mathbf{p})$$

Yang-Mills (Maxwell) equations

$$\begin{aligned}i\mathbf{k} \cdot \mathbf{E}(\omega, \mathbf{k}) &= \rho(\omega, \mathbf{k}), & i\mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) &= 0, \\i\mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) &= i\omega\mathbf{B}(\omega, \mathbf{k}) + \mathbf{B}_0(\mathbf{k}), \\i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) &= \mathbf{j}(\omega, \mathbf{k}) - i\omega\mathbf{E}(\omega, \mathbf{k}) - \mathbf{E}_0(\mathbf{k})\end{aligned}$$

$$\left\{ \begin{aligned}\rho_a(\omega, \mathbf{k}) &= -g \int \frac{d^3 p}{(2\pi)^3} \text{Tr} [\tau^a \delta Q(\omega, \mathbf{k}, \mathbf{p})], \\ \mathbf{j}_a(\omega, \mathbf{k}) &= -g \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} \text{Tr} [\tau^a \delta Q(\omega, \mathbf{k}, \mathbf{p})],\end{aligned}\right.$$

# Solution

$$\left[ -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k}) \right] E^j(\omega, \mathbf{k}) = -g\omega \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{v} \cdot \mathbf{k}} \delta Q_0(\mathbf{k}, \mathbf{p})$$

$$-i \frac{g^2}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{v} \cdot \mathbf{k}} \frac{\mathbf{v} \times \mathbf{B}_0(\mathbf{k})}{\omega} \cdot \nabla_p n(\mathbf{p}) + i\omega E_0^i(\mathbf{k}) - i(\mathbf{k} \times \mathbf{B}_0(\mathbf{k}))^i$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

Isotropic system

$$\varepsilon^{ij}(\omega, \mathbf{k}) \equiv \varepsilon_L(\omega, \mathbf{k}) \frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega, \mathbf{k}) \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right)$$

$$\left( \Sigma^{-1} \right)^{ij}(\omega, \mathbf{k}) = \frac{1}{\omega^2 \varepsilon_L(\omega, \mathbf{k})} \frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2} \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right)$$

# Fluctuations of E field

The solution

$$E^i(\omega, \mathbf{k}) = (\Sigma^{-1})^{ij}(\omega, \mathbf{k}) [\dots \delta Q_0(\mathbf{k}, \mathbf{p}) + \dots \mathbf{E}_0(\mathbf{k}) + \dots \mathbf{B}_0(\mathbf{k})]^j$$

The correlation function

$$\begin{aligned} \langle E^i(\omega, \mathbf{k}) E^j(\omega', \mathbf{k}') \rangle &= (\Sigma^{-1})^{ik}(\omega, \mathbf{k}) (\Sigma^{-1})^{jl}(\omega', \mathbf{k}') [\dots \langle \delta Q_0(\mathbf{k}, \mathbf{p}) \delta Q_0(\mathbf{k}', \mathbf{p}') \rangle \\ &+ \dots \langle \delta Q_0(\mathbf{k}, \mathbf{p}) E_0^m(\mathbf{k}') \rangle + \dots \langle \delta Q_0(\mathbf{k}, \mathbf{p}) B_0^m(\mathbf{k}') \rangle \\ &+ \dots \langle E_0^m(\mathbf{k}) E_0^n(\mathbf{k}') \rangle + \dots \langle E_0^m(\mathbf{k}) B_0^n(\mathbf{k}') \rangle \\ &+ \dots \langle B_0^m(\mathbf{k}) B_0^n(\mathbf{k}') \rangle]^{kl} \end{aligned}$$

$\langle \dots \rangle$  - statistical ensemble average



## **B, $\rho$ , j are given by E**

From Maxwell equations

$$\mathbf{B}(\omega, \mathbf{k}) = \frac{\mathbf{k}}{\omega} \times \mathbf{E}(\omega, \mathbf{k}) + \frac{i}{\omega} \mathbf{B}_0(\mathbf{k})$$

$$\rho(\omega, \mathbf{k}) = i\mathbf{k} \cdot \mathbf{E}(\omega, \mathbf{k})$$

$$\mathbf{j}(\omega, \mathbf{k}) = i\omega\mathbf{E}(\omega, \mathbf{k}) - i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) + \mathbf{E}_0(\mathbf{k})$$

## Initial values

Using Maxwell equations

$\mathbf{E}_0(\mathbf{k}), \mathbf{B}_0(\mathbf{k}), \rho_0(\mathbf{k}), \mathbf{j}_0(\mathbf{k})$  can be expressed through  $\delta Q_0(\mathbf{k}, \mathbf{p})$

# Initial fluctuations

color indices  $i, j, k, l = 1, 2, \dots, N_c$

$$\langle \delta Q_0^{ij}(\mathbf{r}, \mathbf{p}) \delta Q_0^{kl}(\mathbf{r}', \mathbf{p}') \rangle = ?$$

Assumption

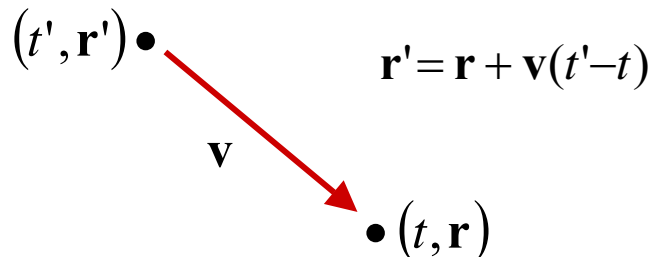
The initial fluctuations are given by  $\langle \delta Q^{ij}(t=0, \mathbf{r}, \mathbf{p}) \delta Q^{kl}(t'=0, \mathbf{r}', \mathbf{p}') \rangle_{\text{free}}$

colorless state

$$\delta Q^{ij}(t, \mathbf{r}, \mathbf{p}) \equiv Q^{ij}(t, \mathbf{r}, \mathbf{p}) - \langle Q^{ij}(t, \mathbf{r}, \mathbf{p}) \rangle = Q^{ij}(t, \mathbf{r}, \mathbf{p}) - \delta^{ij} n(\mathbf{p})$$

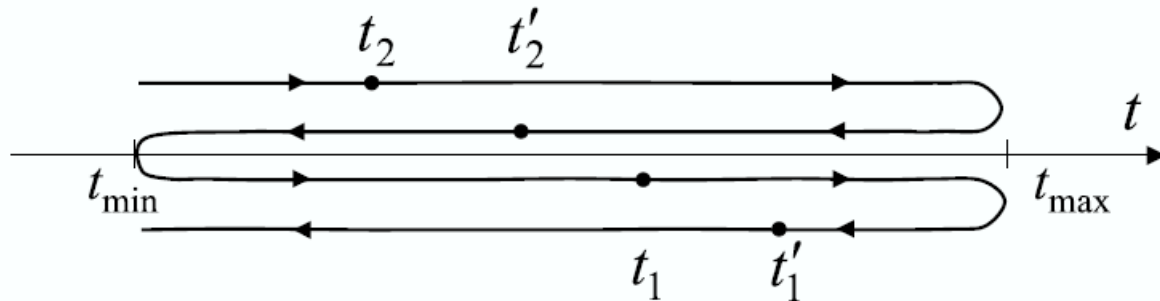
Classical limit

$$\langle \delta Q^{ij}(t, \mathbf{r}, \mathbf{p}) \delta Q^{kl}(t', \mathbf{r}', \mathbf{p}') \rangle_{\text{free}} = \delta^{il} \delta^{jk} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') (2\pi)^3 \delta^{(3)}(\mathbf{r}' - \mathbf{r} - \mathbf{v}(t' - t)) n(\mathbf{p})$$



## Fluctuations of free distribution functions cont.

$$\langle \varphi_j^*(x'_1) \varphi_i(x_1) \varphi_l^*(x'_2) \varphi_k(x_2) \rangle = \langle T_c \left( \varphi_j^*(x'_1) \varphi_i(x_1) \varphi_l^*(x'_2) \varphi_k(x_2) \right) \rangle$$



Wick theorem (lowest order)

$$\begin{aligned} \langle T_c \left( \varphi_j^*(x'_1) \varphi_i(x_1) \varphi_l^*(x'_2) \varphi_k(x_2) \right) \rangle &= \langle T_c \left( \varphi_j^*(x'_1) \varphi_i(x_1) \right) \rangle \langle T_c \left( \varphi_l^*(x'_2) \varphi_k(x_2) \right) \rangle \\ &\quad + \langle T_c \left( \varphi_j^*(x'_1) \varphi_k(x_2) \right) \rangle \langle T_c \left( \varphi_l^*(x'_2) \varphi_i(x_1) \right) \rangle \end{aligned}$$

$$\begin{aligned} \langle \varphi_j^*(x'_1) \varphi_i(x_1) \varphi_l^*(x'_2) \varphi_k(x_2) \rangle &= \langle \varphi_j^*(x'_1) \varphi_i(x_1) \rangle \langle \varphi_l^*(x'_2) \varphi_k(x_2) \rangle \\ &\quad + \langle \varphi_j^*(x'_1) \varphi_k(x_2) \rangle \langle \varphi_i(x_1) \varphi_l^*(x'_2) \rangle \end{aligned}$$

# Fluctuations in isotropic (stable) system

$$\langle E_a^i(\omega, \mathbf{k}) E_b^j(\omega', \mathbf{k}') \rangle = \frac{g^2}{2} \delta^{ab} (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \int \frac{d^3 p}{(2\pi)^3} n(\mathbf{p}) F(\omega, \mathbf{k}, \omega', \mathbf{k}', \mathbf{p})$$

colorless background

translational invariance

$F(\omega, \mathbf{k}, \omega', \mathbf{k}', \mathbf{p})$  has poles at:

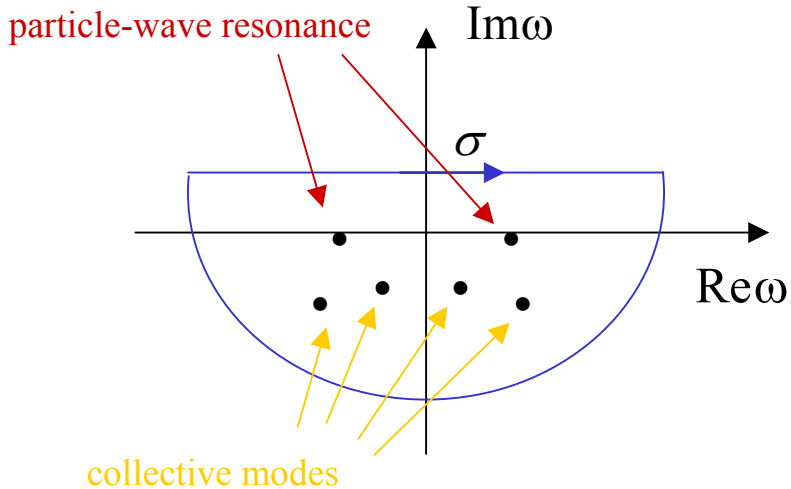
particle-wave resonance  $\left\{ \begin{array}{l} \omega - \mathbf{v} \cdot \mathbf{k} = 0 \\ \omega' - \mathbf{v}' \cdot \mathbf{k}' = 0 \end{array} \right.$

collective longitudinal modes  $\left\{ \begin{array}{l} \varepsilon_L(\omega, \mathbf{k}) = 0 \\ \varepsilon_L(\omega', \mathbf{k}') = 0 \end{array} \right.$

collective transverse modes  $\left\{ \begin{array}{l} \omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2 = 0 \\ \omega'^2 \varepsilon_T(\omega', \mathbf{k}') - \mathbf{k}'^2 = 0 \end{array} \right.$

# Fluctuations in isotropic (stable) system

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega'}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} e^{-i(\omega t + \omega' t' - \mathbf{k}\mathbf{r} - \mathbf{k}'\mathbf{r}')} \times \langle E_a^i(\omega, \mathbf{k}) E_b^j(\omega', \mathbf{k}') \rangle$$



$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle \sim f(\mathbf{r} - \mathbf{r}')$$

$$\langle E_a^i(\omega, \mathbf{k}) E_b^j(\omega', \mathbf{k}') \rangle \sim \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle = \left( \begin{array}{c} \text{collective} \\ \text{modes} \end{array} \right) \left( e^{-\gamma t} \text{ or } e^{-\gamma' t'} \right) + \left( \begin{array}{c} \text{particle-wave} \\ \text{resonance} \end{array} \right) f(t - t')$$

$$\gamma \equiv \text{Im } \omega > 0$$

# Fluctuations in equilibrium system

Long time limit

$$t, t' \rightarrow \infty \quad \langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle_\infty = f(t'-t, \mathbf{r}'-\mathbf{r})$$

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}'))} \langle E_a^i E_b^j \rangle_{\omega, \mathbf{k}}$$

fluctuation spectrum

Fluctuation dissipation relation

$$\langle E_a^i E_b^j \rangle_{\omega, \mathbf{k}} = 2\delta^{ab} T \omega^3 \left[ \frac{k^i k^j}{\mathbf{k}^2} \frac{\text{Im } \varepsilon_L(\omega, \mathbf{k})}{|\omega^2 \varepsilon_L(\omega, \mathbf{k})|^2} + \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\text{Im } \varepsilon_T(\omega, \mathbf{k})}{|\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2|^2} \right]$$

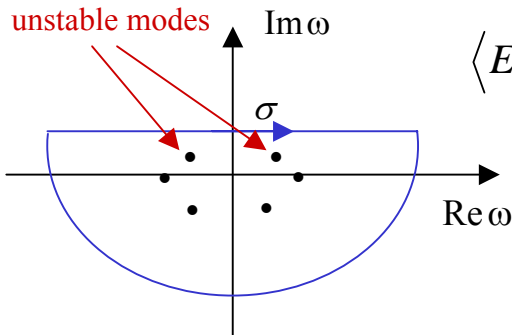
$$\langle B_a^i B_b^j \rangle_{\omega, \mathbf{k}} = 2\delta^{ab} T \omega (\mathbf{k}^2 \delta^{ij} - k^i k^j) \frac{\text{Im } \varepsilon_T(\omega, \mathbf{k})}{|\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2|^2}$$

# Fluctuations in unstable systems

## Two-stream system

$$n(\mathbf{p}) = (2\pi)^3 n [\delta^{(3)}(\mathbf{p} - \mathbf{q}) + \delta^{(3)}(\mathbf{p} + \mathbf{q})]$$

Longitudinal electric field:  $\omega_+(\mathbf{k})$  - stable mode,  $\omega_-(\mathbf{k})$  - unstable mode



$$\begin{aligned} \langle E_a^i(\omega, \mathbf{k}) E_b^i(\omega', \mathbf{k}') \rangle &= \frac{g^2}{2} \delta^{ab} (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{\mathbf{k} \cdot \mathbf{k}'}{\mathbf{k}^2 \mathbf{k}'^2} \\ &\times \frac{1}{\varepsilon_L(\omega, \mathbf{k})} \frac{1}{\varepsilon_L(\omega', \mathbf{k}')} \int \frac{d^3 p}{(2\pi)^3} \frac{n(\mathbf{p})}{(\omega - \mathbf{v} \cdot \mathbf{k})(\omega' - \mathbf{v}' \cdot \mathbf{k}')} \end{aligned}$$

broken time translational invariance

$$\begin{aligned} \langle E_a^i(t, \mathbf{r}) E_b^i(t', \mathbf{r}') \rangle_{\text{unstable}} &= \frac{g^2}{2} \delta^{ab} n \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2} \frac{1}{(\omega_+^2 - \omega_-^2)^2} \frac{(\gamma_k^2 + (\mathbf{k}\mathbf{u})^2)^2}{\gamma_k^2} \\ &\times \left[ (\gamma_k^2 + (\mathbf{k}\mathbf{u})^2) \cosh(\gamma_k(t+t')) + (\gamma_k^2 - (\mathbf{k}\mathbf{u})^2) \cosh(\gamma_k(t-t')) \right] \end{aligned}$$

$$\mathbf{u} \equiv \frac{\mathbf{q}}{E_q}, \quad \gamma_k \equiv \text{Im} \omega_-(\mathbf{k})$$



# Gauge dependence

Generic correlation function:  $L_{ab}(x, x') \equiv \langle H_a(x) K_b(x') \rangle$

Infinitesimal gauge transformation

$$H_a(x) \rightarrow H_a(x) + f_{abc} \lambda_b(x) H_c(x)$$

$$L_{ab}(x, x') \rightarrow L_{ab}(x, x') + f_{acd} \lambda_c(x) L_{db}(x, x') + f_{bcd} \lambda_c(x') L_{ad}(x, x')$$

colorless background

Actual correlation function:  $L_{ab}(x, x') \equiv \delta^{ab} L(x, x')$

$$L_{ab}(x, x') \rightarrow \left( \delta^{ab} + f_{acb} \lambda_c(x) + f_{bca} \lambda_c(x') \right) L(x, x')$$

$$L_{aa}(x, x') = \left( N_c^2 - 1 \right) L(x, x') - \text{gauge invariant!}$$

# Momentum broadening in equilibrium QGP

$$\hat{q} \equiv \frac{d\langle \Delta p_T^2(t) \rangle}{dt}$$

$$\begin{aligned} \langle \Delta p_T^2(t) \rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\{ \langle E_a^x(x(t_1)) E_a^x(x(t_2)) \rangle + \langle E_a^y(x(t_1)) E_a^y(x(t_2)) \rangle - \langle E_a^x(x(t_1)) B_a^y(x(t_2)) \rangle \right. \\ \left. + \langle E_a^y(x(t_1)) B_a^x(x(t_2)) \rangle - \langle B_a^y(x(t_1)) E_a^x(x(t_2)) \rangle - \langle B_a^x(x(t_1)) E_a^y(x(t_2)) \rangle \right. \\ \left. + \langle B_a^x(x(t_1)) B_a^x(x(t_2)) \rangle + \langle B_a^y(x(t_1)) B_a^y(x(t_2)) \rangle \right\} \end{aligned}$$

translational invariance

$$\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega(t-t') - \mathbf{k}(\mathbf{r}-\mathbf{r}'))} \langle E_a^i E_b^j \rangle_{\omega, \mathbf{k}}$$

fluctuation spectrum

$\mathbf{r}(t) = (0, 0, t)$  - parton's trajectory

$$\int_0^t dt_1 \int_0^t dt_2 e^{-i(\omega - k_z)(t_1 - t_2)} = \frac{4}{(\omega - k_z)^2} \sin\left(\frac{(\omega - k_z)t}{2}\right) \xrightarrow{t \rightarrow \infty} 2\pi t \delta(\omega - k_z)$$

# Momentum broadening in equilibrium QGP

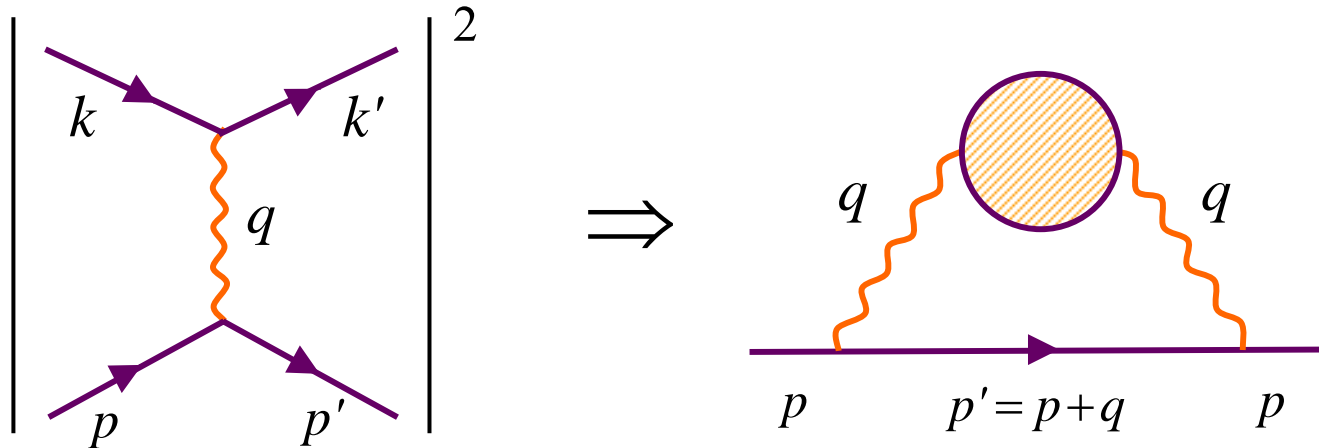
$$\hat{q} = g^2 C \int \frac{d^3 k}{(2\pi)^3} \left\{ \langle E_a^x E_a^x \rangle_{k_z, \mathbf{k}} + \langle E_a^y E_a^y \rangle_{k_z, \mathbf{k}} - \langle E_a^x B_a^y \rangle_{k_z, \mathbf{k}} + \langle E_a^y B_a^x \rangle_{k_z, \mathbf{k}} \right. \\ \left. - \langle B_a^y E_a^x \rangle_{k_z, \mathbf{k}} - \langle B_a^x E_a^y \rangle_{k_z, \mathbf{k}} + \langle B_a^x B_a^x \rangle_{k_z, \mathbf{k}} + \langle B_a^y B_a^y \rangle_{k_z, \mathbf{k}} \right\}$$

$$C_R \equiv C (N_c^2 - 1) = \begin{cases} \frac{1}{2} & \text{- quark} \\ N_c & \text{- gluon} \end{cases}$$

$$\hat{q} = 2g^2 C_R T \int \frac{d^3 k}{(2\pi)^3} \frac{k_T^2}{k_z \mathbf{k}^2} \left[ \frac{\text{Im } \varepsilon_L(k_z, \mathbf{k})}{|\varepsilon_L(k_z, \mathbf{k})|^2} + \frac{k_z^2 k_T^2 \text{Im } \varepsilon_T(k_z, \mathbf{k})}{|k_z^2 \varepsilon_T(k_z, \mathbf{k}) - \mathbf{k}^2|^2} \right]$$

$$\hat{q} \approx \frac{g^2}{2\pi} C_R m_D^2 T \ln(1/g)$$

# Momentum broadening in thermal field approach



$$\hat{q} = \frac{1}{4E_p} \text{Tr}[\not{p} \Sigma_w^>(p)]$$

$$q \sim gT, \quad k \sim T$$

Hard-Loop propagator

$$\Sigma_w^>(p) = g^2 C_F \int \frac{d^3 p'}{(2\pi)^3 2E'} q_T \gamma^\mu \not{p}' \gamma^\nu D_{\mu\nu}^<(q)$$

$$\hat{q} \approx \frac{g^2}{2\pi} C_R m_D^2 T \ln(1/g)$$

Moore & Teaney 2005; Baier & Mehtar-Tani 2008

# Momentum broadening in unstable QGP

Two-stream system

longitudinal electric fields only

$$\langle \Delta p_T^2(t) \rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\{ \langle E_a^x(x(t_1)) E_a^x(x(t_2)) \rangle + \langle E_a^y(x(t_1)) E_a^y(x(t_2)) \rangle \right\}$$

$$\langle \Delta p_T^2(t) \rangle \approx \frac{g^4}{4} C_R n \int \frac{d^3 k}{(2\pi)^3} e^{2\gamma_{\mathbf{k}} t} \frac{k_T^2 (\gamma_{\mathbf{k}}^2 + (\mathbf{k}\mathbf{u})^2)^3}{\mathbf{k}^4 (\omega_+^2 - \omega_-^2)^2 \gamma_{\mathbf{k}}^2 (k_z^2 + \gamma_{\mathbf{k}}^2)}$$

$$\hat{q} \sim e^{2\gamma t}$$

# Conclusions

- ▶ Unstable QGP is generically time dependent
- ▶  $\hat{q} \sim e^{2\gamma t}$