Glasma as a fluid

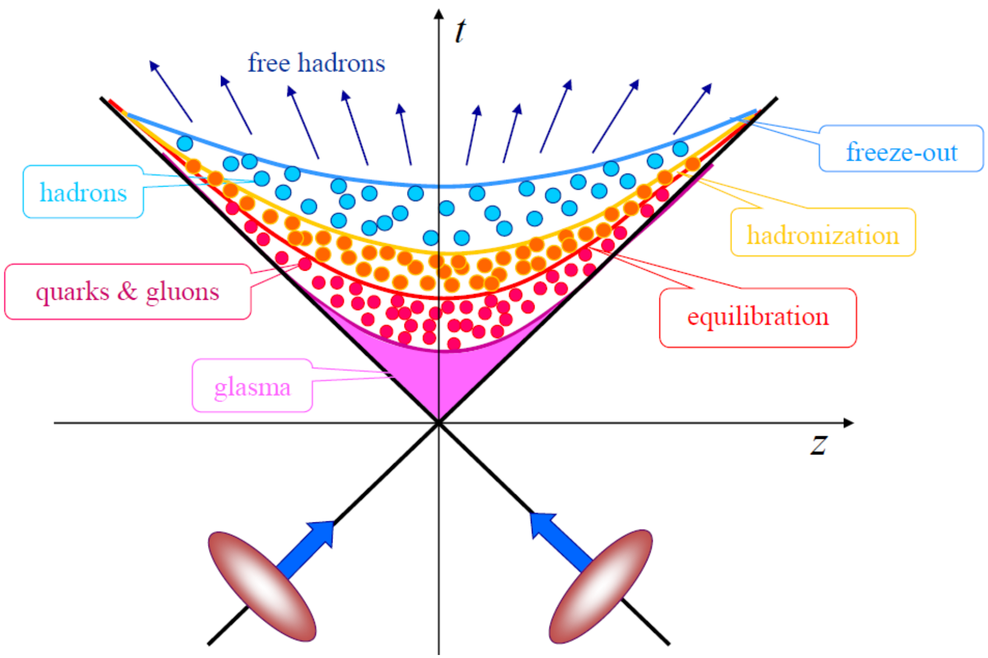
Stanisław Mrówczyński

National Centre for Nuclear Research, Warsaw, Poland

in collaboration with **Margaret Carrington &**

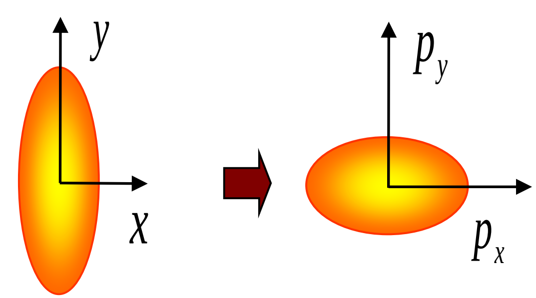
Alina Czajka, Wade Cowie, Bryce Friesen, Jean-Yves Ollitrault and Doug Pickering

Success of hydrodynamic description of heavy-ion collisions



Two conflicting requirements:

- Hydrodynamics needs local thermodynamical equilibrium.
- Hydrodynamic evolution must start very early, $\tau \sim 0.6$ fm/c.



Success of hydrodynamic description of heavy-ion collisions

Possible resolutions:

- Matter is strongly interacting and equilibrates fast.

  AdS/CFT: D.T. Son & A. O. Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

- Pre-quilibrium matter evolves along the hydrodynamic attaractor.

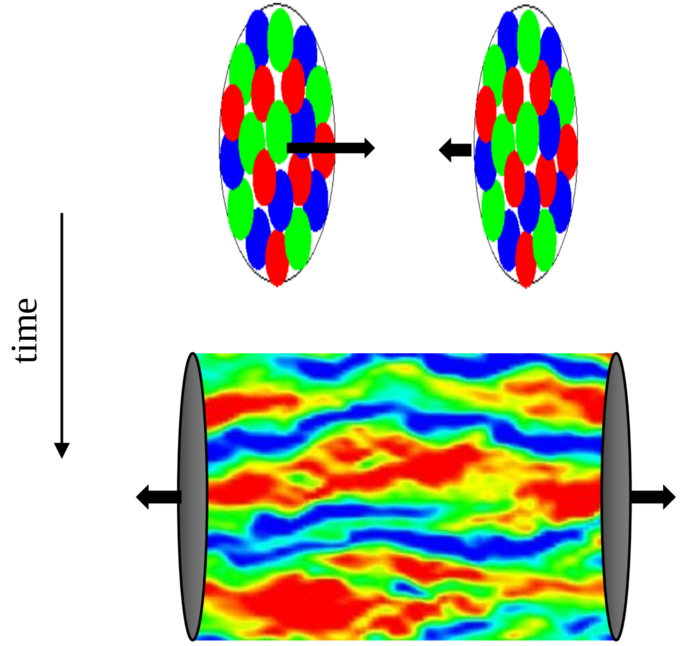
  M.P. Heller & M. Spaliński, Phys. Rev. Lett. **115**, 072501 (2015);
  J. Jankowski & M. Spaliński, Prog. Part. Nucl. Phys. **132**, 104048 (2023).

- Pre-quilibrium matter – glasma – behaves as a fluid.

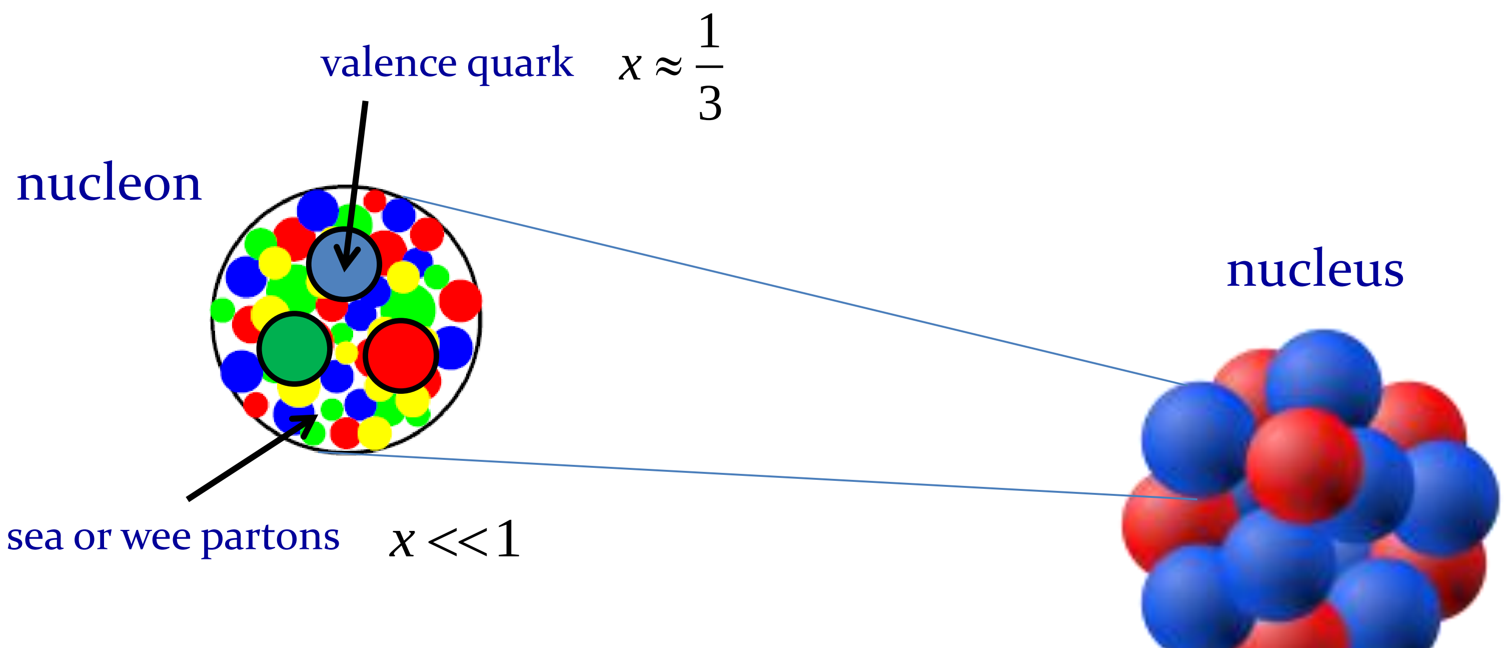
  M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110,** 054903 (2024),
  M. Carrington & St. Mrówczyński, arXiv:2505.07324

Glasma & Color Glass Condensate

Color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolves towards equilibrium.



Scale separation between wee partons & valence quarks



In relativistic heavy-ion collsions

- Saturated wee partons – classical chromodynamic fields
- Valence quarks – classical sources of chromodynamic fields
- Saturation scale for A-A at LHC: $Q_s \approx 2\,\text{GeV} >> \Lambda_{\text{QCD}} \approx 0.2\,\text{GeV} \Rightarrow \alpha_s(Q_s) << 1$

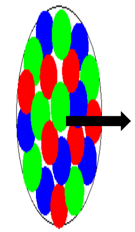
Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

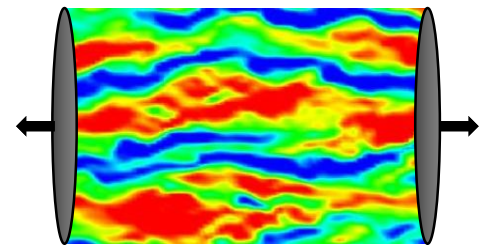
$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\pm}\delta(x^\mp)\rho_{1,2}(\mathbf{x}_\perp)$$



β_1^i β_2^i

pre-collision potentials

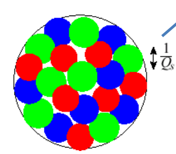


$\alpha, \alpha_\perp^i$

post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau, \mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau, \mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau, \mathbf{x}_\perp) \\ \qquad + \Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\frac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+A^- + x^-A^+ = 0$$

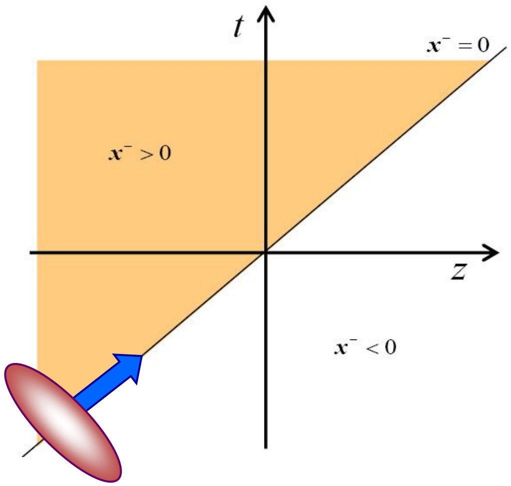
$$\beta_{1,2}^\pm = 0$$

Light-front variables

$$x^\pm \equiv \frac{t \pm z}{\sqrt{2}}$$

A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **52**, 3809 (1995).

Pre-collision potentials



$$j^\mu(x^-, \mathbf{x}_\perp) = \delta^{\mu +}\delta(x^-)\rho(\mathbf{x}_\perp)$$

Gauge condition: $\mathbf{A}_\perp(x^-, \mathbf{x}_\perp) = 0$

$$D_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \begin{cases} A^-(x^-, \mathbf{x}_\perp) = 0 \\ A^+(x^-, \mathbf{x}_\perp) = \delta(x^-)\Lambda(\mathbf{x}_\perp) \end{cases}$$

$$E^z = B^z = 0$$

$$\mathbf{E}_\perp, \mathbf{B}_\perp \sim \delta(x^-)$$

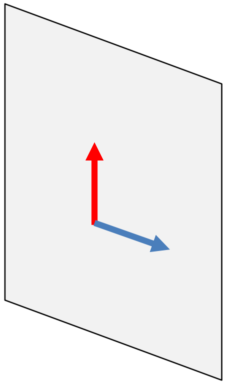
Poisson equation

$$\nabla_\perp^2 \Lambda(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp)$$

$$\Lambda(\mathbf{x}_\perp) = \int d^2x'_\perp\, G(\mathbf{x}_\perp - \mathbf{x}'_\perp)\rho(\mathbf{x}'_\perp)$$

$$G(\mathbf{x}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{\mathbf{k}_\perp^2 + m^2} = \frac{1}{2\pi}K_0(m|\mathbf{x}_\perp|)$$

IR regulator $m = \Lambda_{\rm QCD}$



$x^- = 0$

Pre-collision potentials cont.

Gauge transformation: $A^{\mu}(x^{-},\mathbf{x}_{\perp}) \quad \rightarrow \quad \beta^{\mu}(x^{-},\mathbf{x}_{\perp})$

gauge condition $A^{i}(x^{-},\mathbf{x}_{\perp})=0$

light-come gauge $\beta^{+}(x^{-},\mathbf{x}_{\perp})=0$

$$\beta^{\mu}(x)=U(x)A^{\mu}(x)U^{\dagger}(x)+\frac{i}{g}U(x)\partial^{\mu}U^{\dagger}(x)$$

$$U(x^{-},\mathbf{x}_{\perp})A^{+}(x^{-},\mathbf{x}_{\perp})U^{\dagger}(x^{-},\mathbf{x}_{\perp})+\frac{i}{g}U(x^{-},\mathbf{x}_{\perp})\partial^{+}U^{\dagger}(x^{-},\mathbf{x}_{\perp})=0$$

$$\begin{cases} U(x^{-},\mathbf{x}_{\perp})=P\exp\left(ig\int_{-\infty}^{x^{-}}dz^{-}A^{+}(z^{-},\mathbf{x}_{\perp})\right) \\ \beta^{i}(x^{-},\mathbf{x}_{\perp})=\frac{i}{g}U(x^{-},\mathbf{x}_{\perp})\partial^{i}U^{\dagger}(x^{-},\mathbf{x}_{\perp}) \end{cases}$$

pure gauge except $x^{-}=0$

J. Jalilian-Marian, A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\dots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

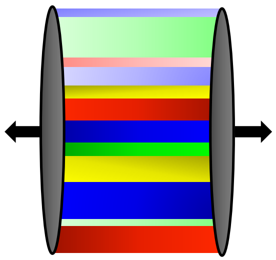
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0,0,E), \qquad \mathbf{B}_{(0)} = (0,0,B)$$

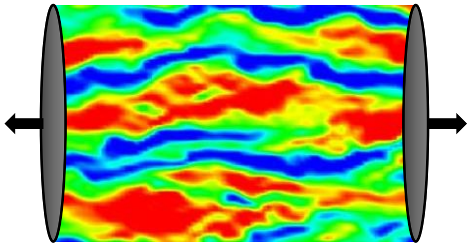
$$E^z_{(0)}(\mathbf{x}_\perp) = -ig[\beta^i_1(\mathbf{x}_\perp), \beta^i_2(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta^i_1(\mathbf{x}_\perp), \beta^j_2(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up



R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Energy-momentum tensor

- $$T^{\mu\nu} = 2\mathrm{Tr}[F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}]$$

- $$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} - ig[A^{\mu}, A^{\nu}]$$

The energy-momentum tensor is symmetric, gauge invariant and obeys

- $$\partial_{\mu}T^{\mu\nu} = 0$$

T^{00} - energy density

T^{0i} - energy flux, Poynting vector

T^{xx}, T^{yy}, T^{zz} - pressures

T^{ij} - momentum flux

Averaging over collisions

$$T^{\mu\nu} \sim \sum \partial^i \partial^j \beta^k \beta^l \ldots \beta^m \quad \Rightarrow \quad \left\langle T^{\mu\nu} \right\rangle \sim \sum \partial^i \partial^j \left\langle \beta^k \beta^l \ldots \beta^m \right\rangle$$

Wick theorem – Gaussian averaging

$$\left\langle \rho_a^k(\mathbf{x}_\perp) \rho_b^l(\mathbf{y}_\perp) \ldots \rho_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \right\rangle$$

Glasma graph approximation

$$\left\langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \ldots \beta_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \right\rangle = \sum \prod B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Basic correlator

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho_a^i(\mathbf{x}'_\perp)\rho_b^j(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle = g^2\mu(\mathbf{x}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

- System uniform in the transverse plane $\mu(\mathbf{x}_\perp) = \bar{\mu}$

color charge surface density

$$\bar{\mu} = g^{-4}Q_s^2$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|$$

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) = \delta^{ab}\left(\delta^{ij}C_1(r) + \frac{r^i r^j}{\mathbf{r}^2}C_2(r)\right)$$

$$\begin{cases} C_1(r) \equiv \dfrac{m^2K_0(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\bar{\mu}\left(mrK_1(mr)-1\right)}{4\pi m^2\left(N_c^2-1\right)}\right]-1\right\} \\ C_2(r) \equiv \dfrac{m^3rK_1(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\bar{\mu}\left(mrK_1(mr)-1\right)}{4\pi m^2\left(N_c^2-1\right)}\right]-1\right\} \end{cases}$$

UV divergence

$$C_2(r) \xrightarrow[r\to 0]{} \infty$$

Regularization

$$C_2(r) \xrightarrow[r\to 0]{} C_2(Q_s^{-1})$$

J. Jalilian-Marian, A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Basic correlator cont.

- System nonuniform in the transverse plane

  Projected Woods-Saxon distribution

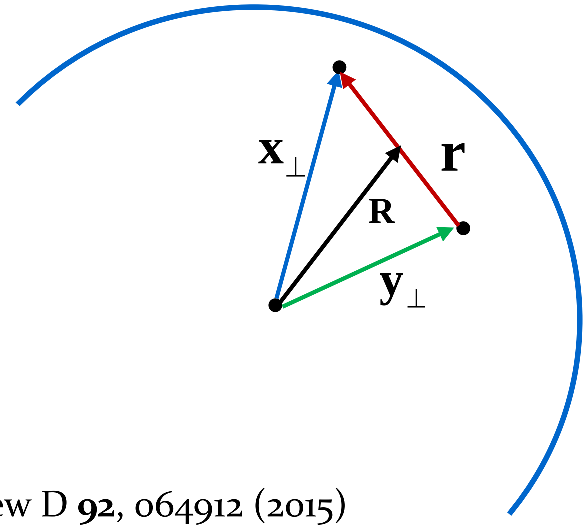
$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1+e^{R_A/a})} \int_{-\infty}^{\infty} \frac{dz}{1+\exp\left[\left(\sqrt{\mathbf{x}_\perp{}^2 + z^2} - R_A\right)/a\right]}$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp), \quad \mathbf{r} = \mathbf{x}_\perp - \mathbf{y}_\perp$$

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R}, \mathbf{r}) \approx \text{``}\textit{gradient expansion in } \mathbf{R}\text{''}$$

strong dependence on **r**

weak dependence of **R**



G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Numerical results

Pb-Pb collisions at LHC

$N_c = 3$

$g = 1$

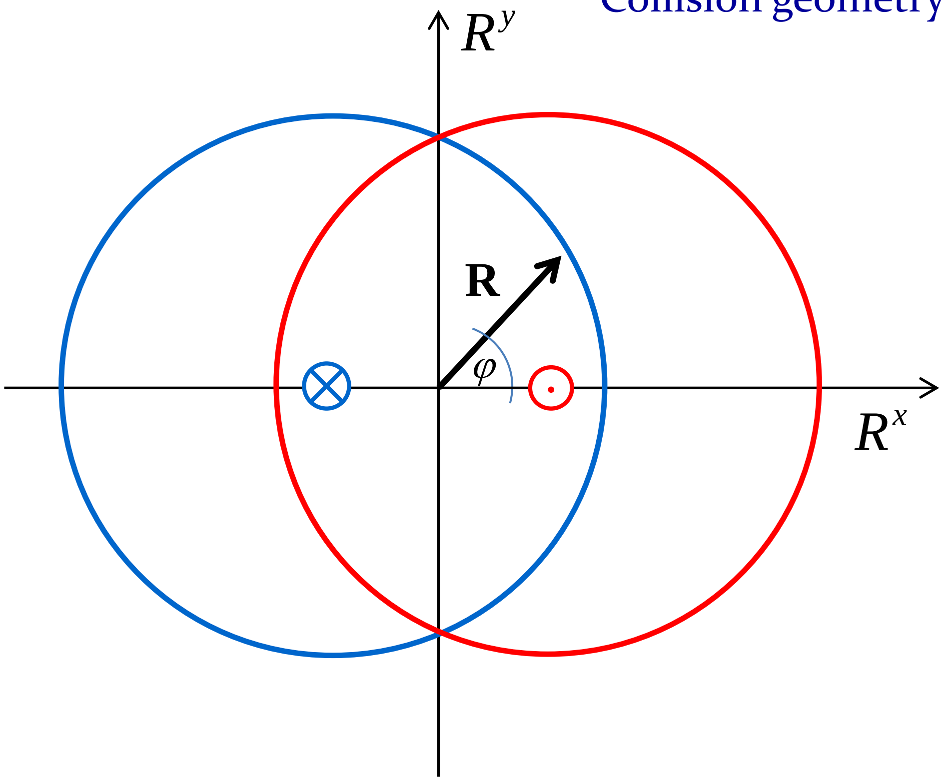
$Q_s = 2$ GeV

$m = 0.2$ GeV

$R_A = 7.4$ fm

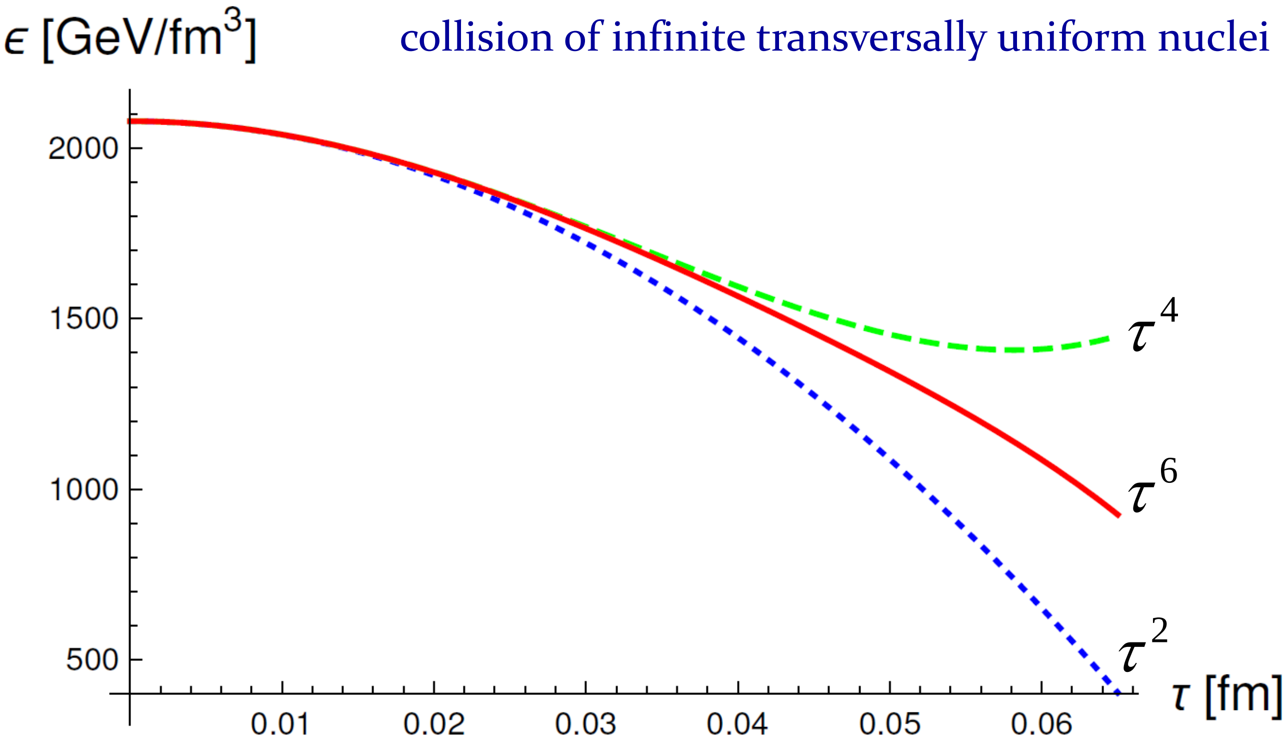
$a = 0.5$ fm

Collision geometry



Energy density

collision of infinite transversally uniform nuclei



$$\varepsilon_0 = 2080\,\frac{\text{GeV}}{\text{fm}^3} = \varepsilon_{\text{eq}} = \frac{\pi^2}{15}(N_c^2-1)T^4 \quad \Rightarrow \quad T \approx 1.3\,\text{GeV} \;>>\; \Lambda_{\text{QCD}}$$

weak coupling

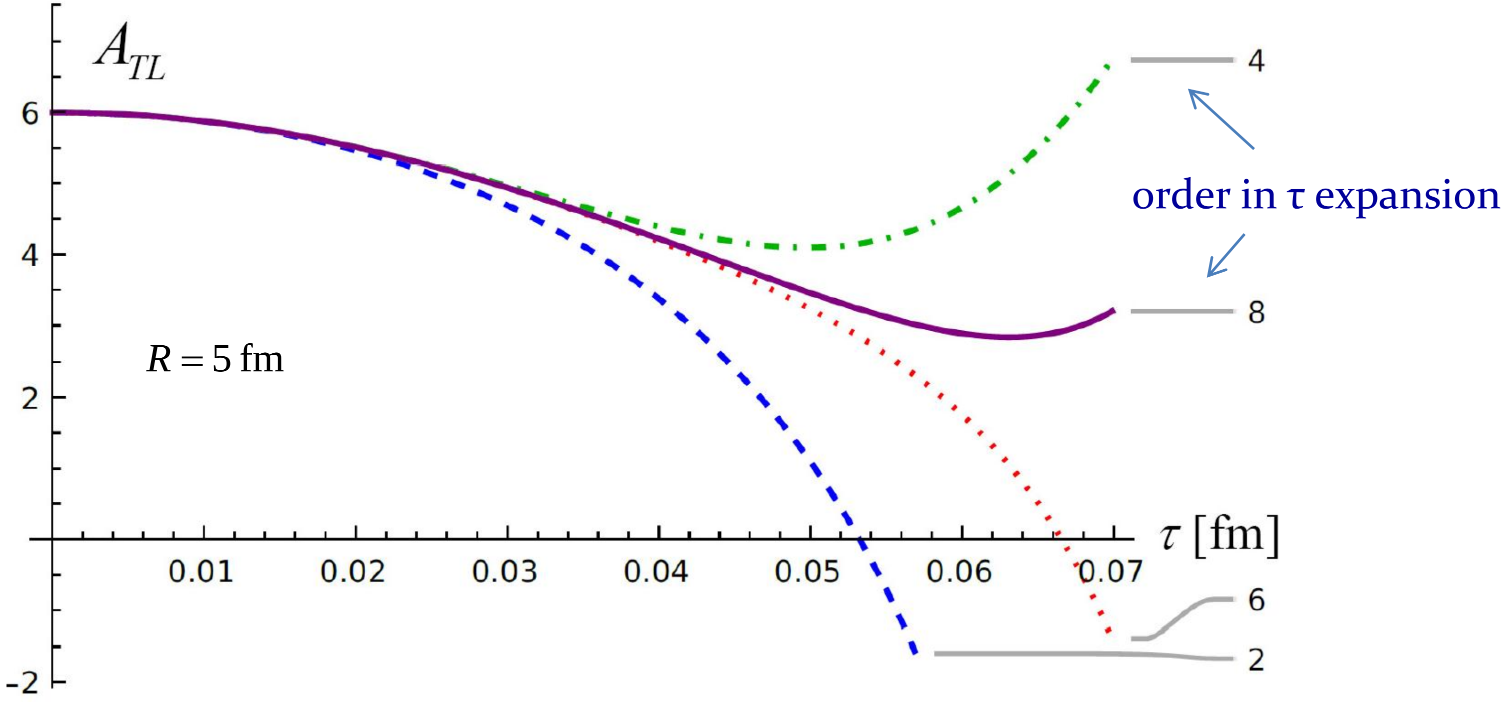
M. Carrington, A. Czajka & St. Mrówczyński, Eur. Phys. J. A **58**, 5 (2022)

Anisotropy

Central Pb-Pb collsions

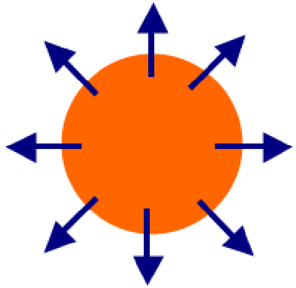
$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L} \qquad p_T \equiv \langle T^{xx} \rangle, \quad p_L \equiv \langle T^{zz} \rangle$$

$$\tau = 0 \quad \Rightarrow \quad p_T = -p_L = \varepsilon \quad \Rightarrow \quad A_{TL} = 6$$



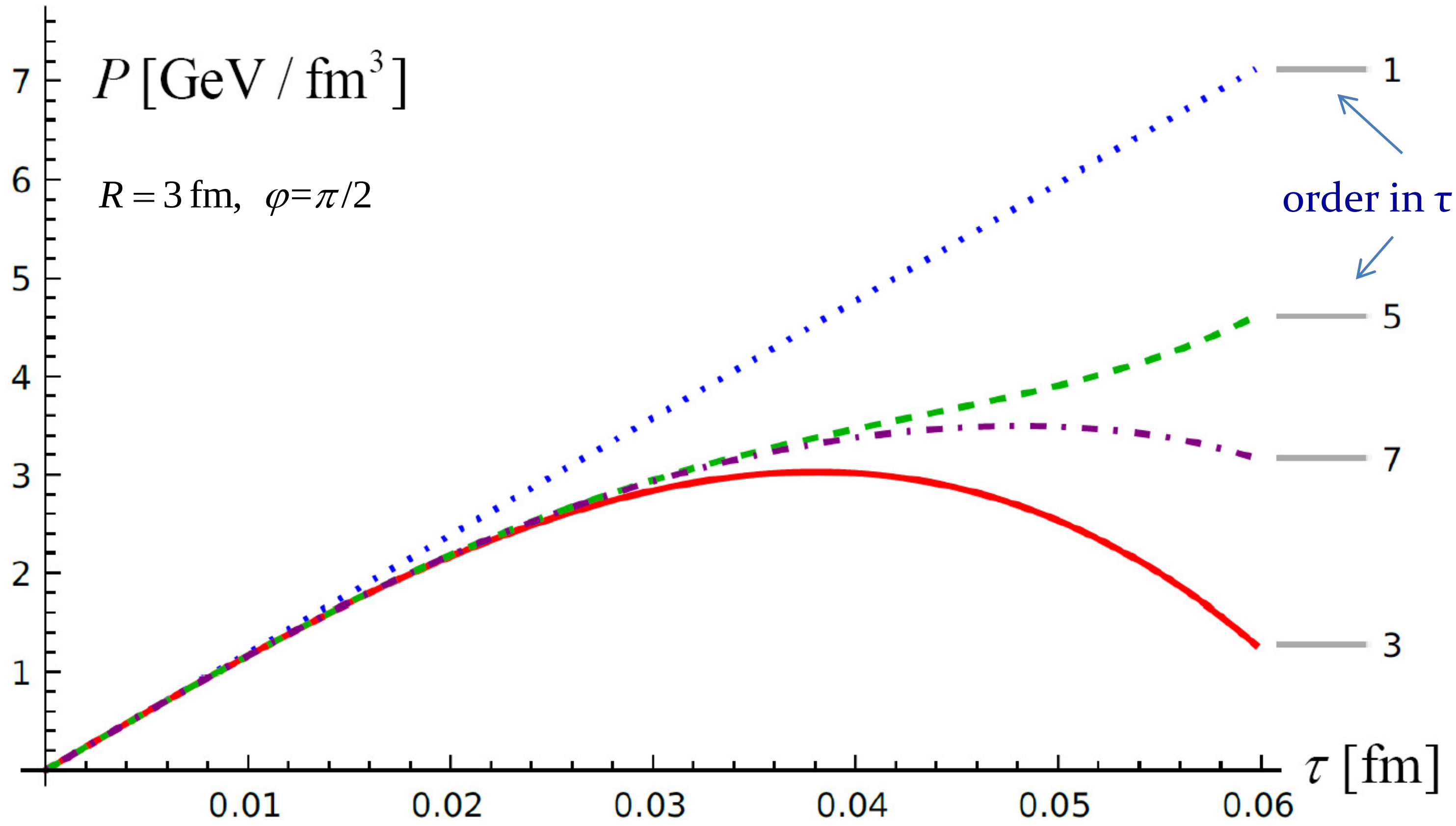
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow



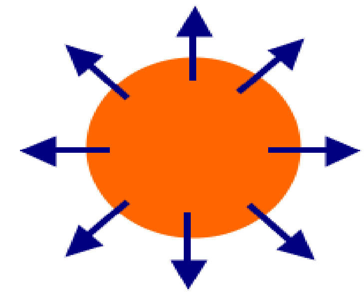
Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$



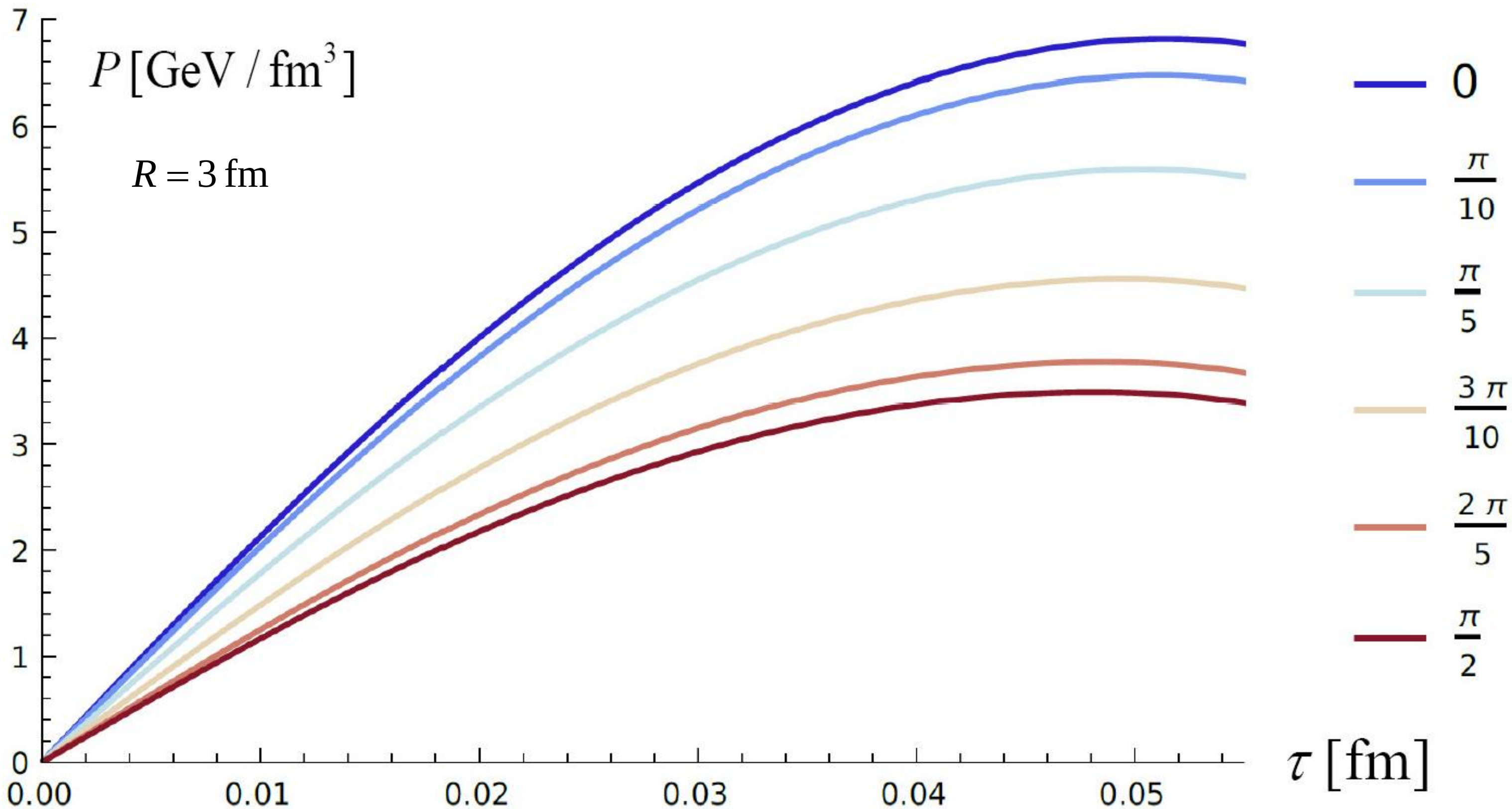
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow cont.



Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$

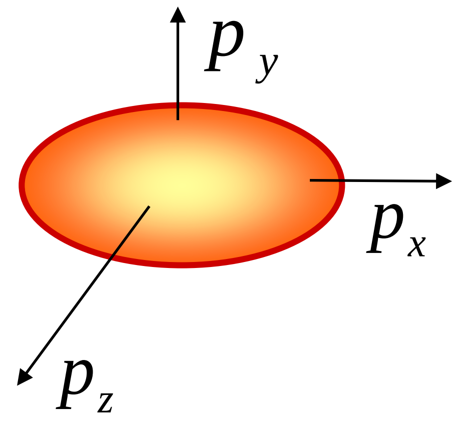


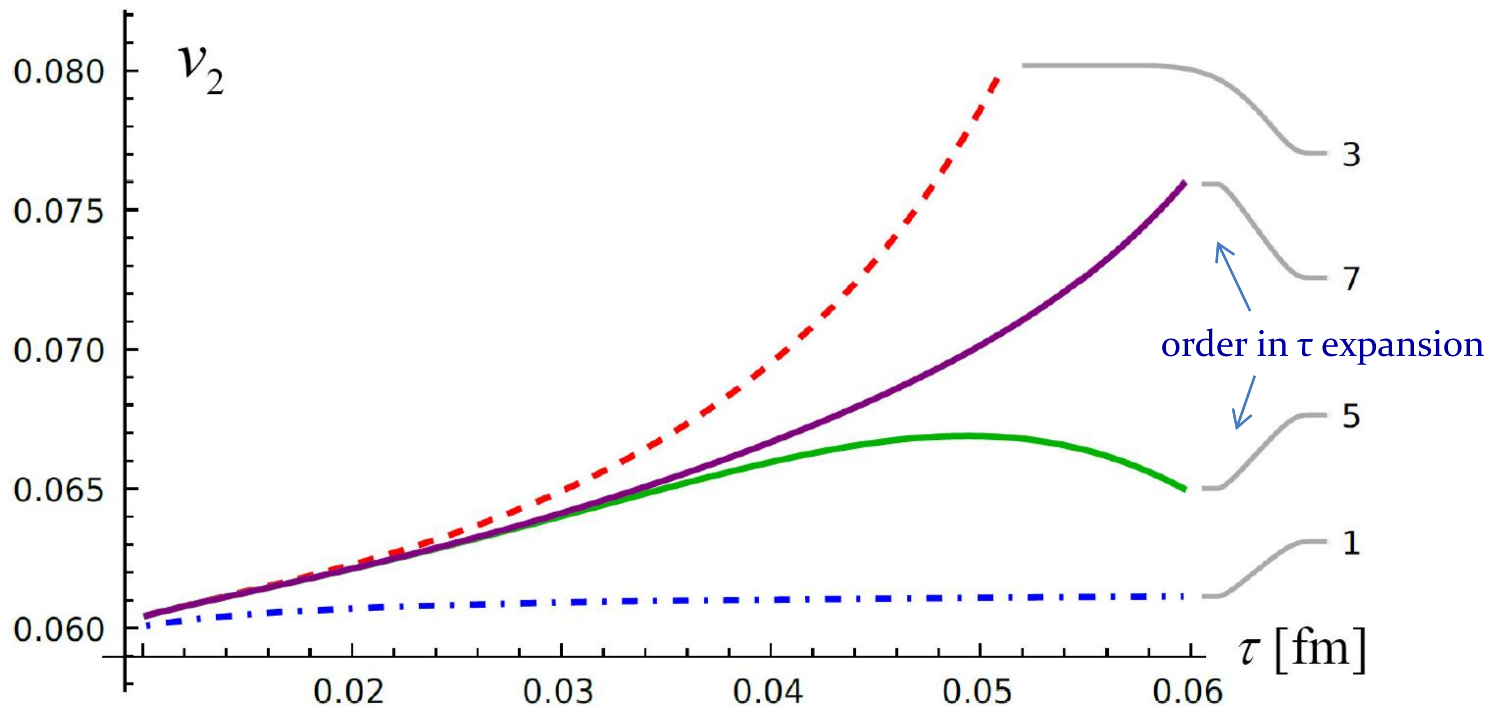
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Elliptic flow

Pb-Pb collisions at b = 2 fm

$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$



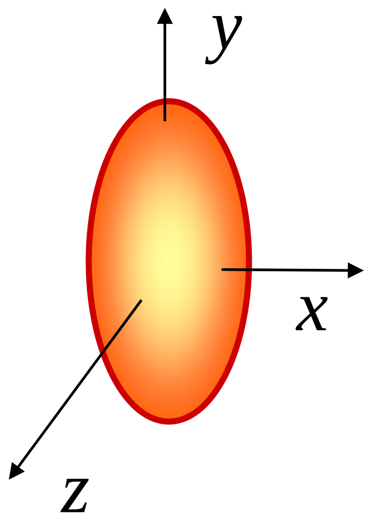


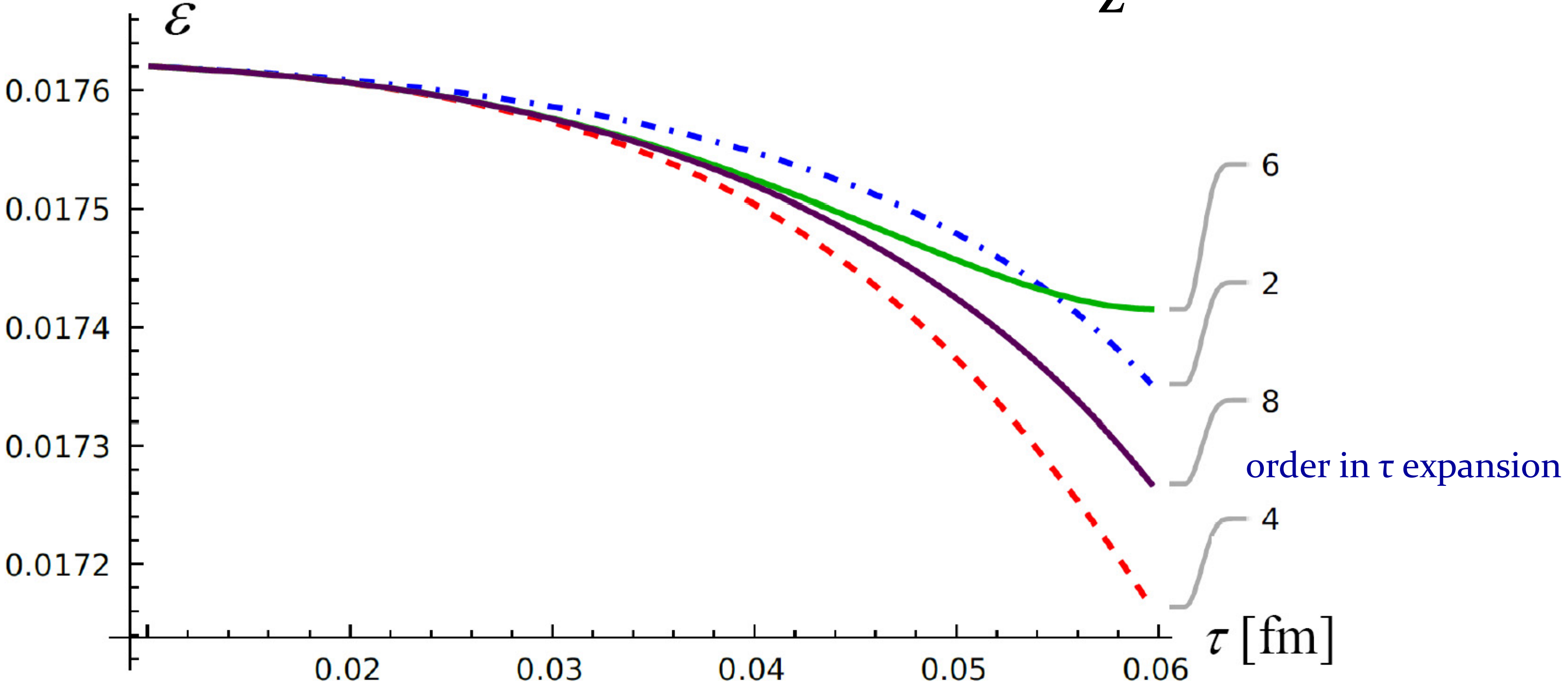
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Eccentricity

Pb-Pb collisions at b = 2 fm

$$\varepsilon = \frac{\int d^2R \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} T^{00}}$$

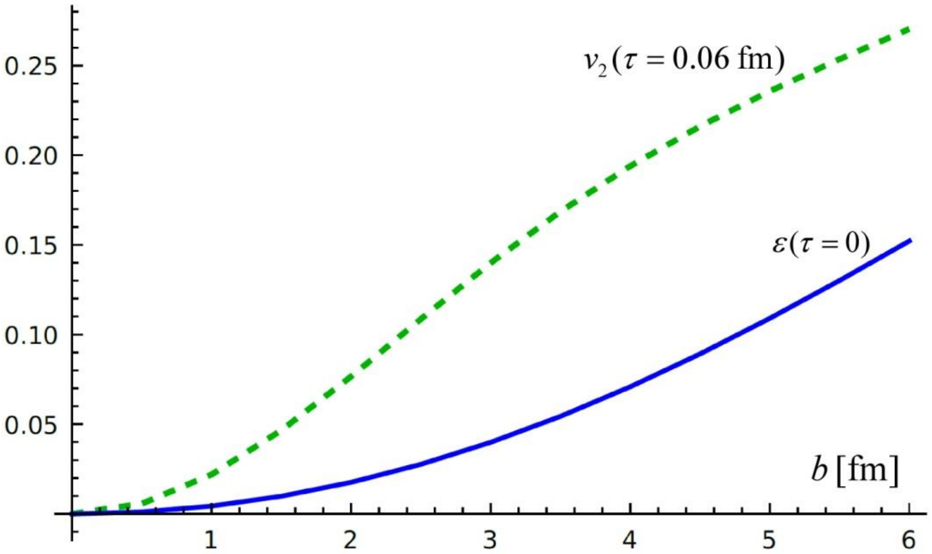


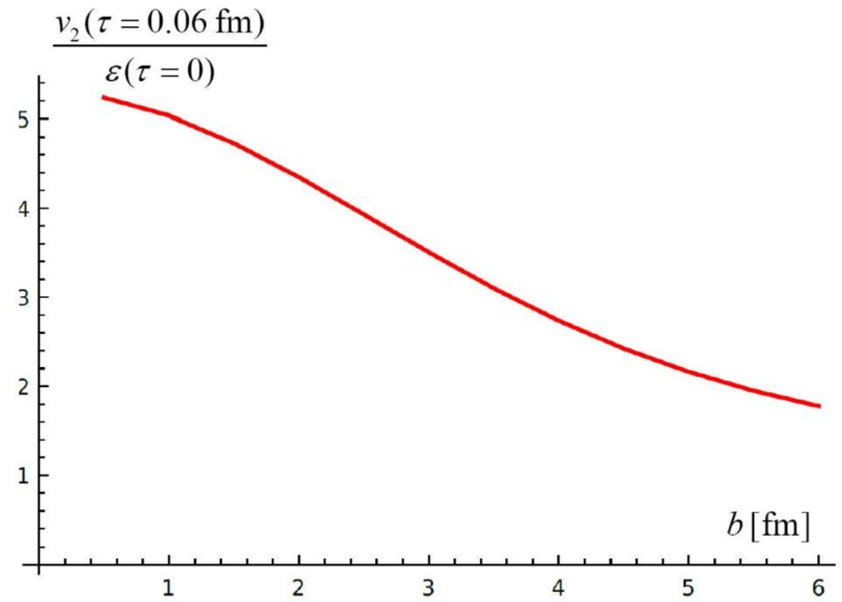


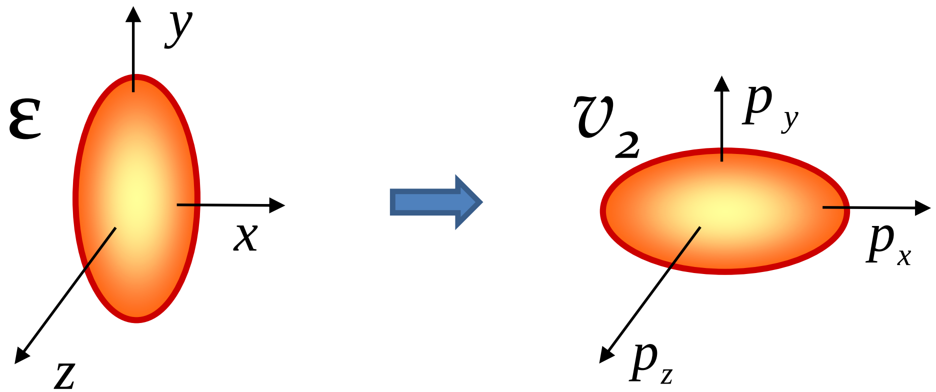
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hydrodynamic-like behavior

Pb-Pb collisions





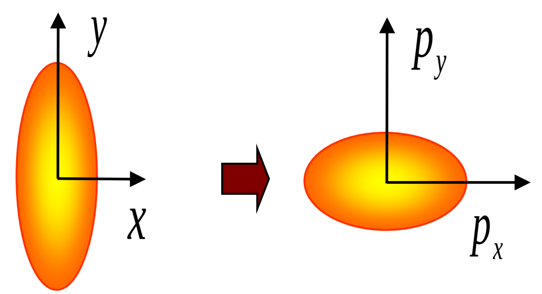


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Equation of universal flow

$$T^{tx} \approx -\frac{1}{2} t \frac{\partial T^{tt}}{\partial x}$$

Energy flow is generated by gradient of energy density.



Assumptions:

- $\partial_\mu T^{\mu\nu} = 0$
- Energy momentum tensor is mostly diagonal.
- The system is boost-invariant.

Glasma in ultrarelativistic collisions

$$T^{\mu\nu}(t=0) = \text{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$$

short-time evolution

J. Vredevoogd & S. Pratt, Phys. Rev. C **79**, 044915 (2009);

M. Carrington, St. Mrówczyński &J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Equation of universal flow cont.

Derivation in Milne coordinates $\tau \equiv \sqrt{t^2 - z^2}, \quad \eta \equiv \frac{1}{2} \ln \frac{t+z}{t-z}$

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\rho} T^{\rho\nu} + \Gamma^\nu_{\mu\rho} T^{\mu\rho} = 0$$

$\nu = x$

$$\left(\frac{\partial}{\partial \tau} + \frac{1}{\tau}\right) T^{\tau x} + \frac{\partial T^{xx}}{\partial x} + \underbrace{\frac{\partial T^{yx}}{\partial y}}_{\approx 0 \text{ mostly diagonal } T^{\mu\nu}} + \overbrace{\frac{\partial T^{\eta x}}{\partial \eta}}^{= 0 \text{ boost invariance}} = 0$$

$\eta \approx 0 \quad \Rightarrow \quad z = 0 \ \& \ \tau = t$

$$\left(\frac{\partial}{\partial t} + \frac{1}{t}\right) T^{tx} + \frac{\partial T^{xx}}{\partial x} = 0$$

$T^{xx} \approx T^{tt} \approx \text{const} \quad \& \quad T^{tx}(t=0) = 0$

$$T^{tx} = -\frac{1}{2} t \frac{\partial T^{tt}}{\partial x}$$

Universal flow
in proper time expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \tau^n T^{\mu\nu}_n$$

$$T^{tx}_{n+1} = -\frac{1}{2} \tau \frac{\partial T^{tt}_n}{\partial x}$$

$n = 1, 2, \dots 7$

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Mapping on hydrodynamic $T^{\mu\nu}$

$$T^{\mu\nu}_{\text{glasma}}(\tau, \mathbf{x}_T) \quad vs. \quad T^{\mu\nu}_{\text{hydro}}(\tau, \mathbf{x}_T)$$

Eigenvalue problem:

$$\boxed{T^{\mu\nu}_{\text{glasma}} w_\nu = \lambda w^\mu}$$

Ideal hydrodynamics

$$\boxed{T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}}$$

$$T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p = \frac{1}{3}\varepsilon$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu$$

Anisotropic hydrodynamics

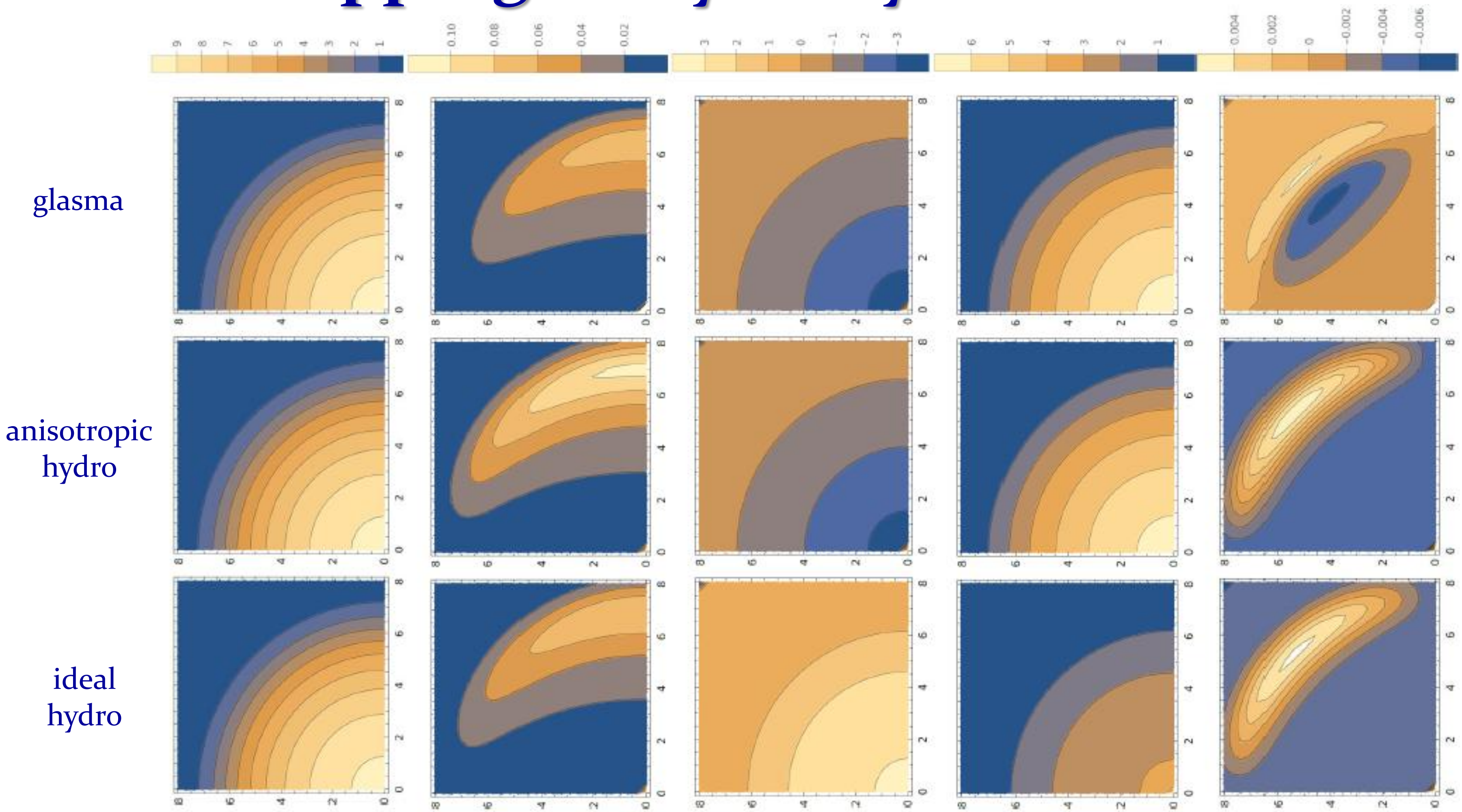
$$\boxed{T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p_T) u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L) z^\mu z^\nu}$$

$$T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p_L = \varepsilon - 2 p_T$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu, \qquad T^{\mu\nu}_{\text{hydro}} z_\nu = -p_L z^\mu$$

W. Florkowski & R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)
M. Martinez & M. Strickland, Nucl. Phys. A **848**, 183 (2010)

Mapping on hydrodynamic $T^{\mu\nu}$

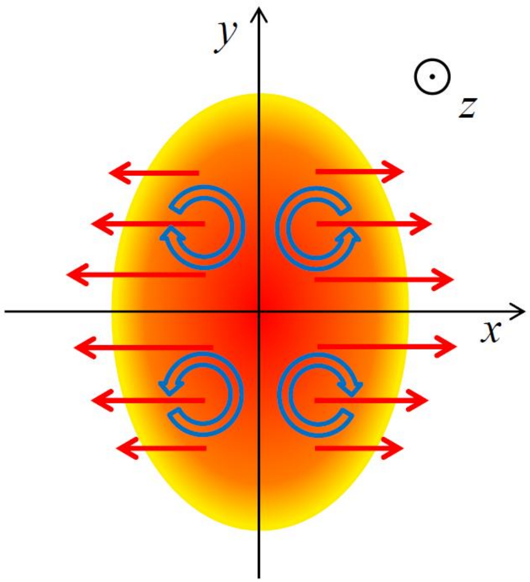


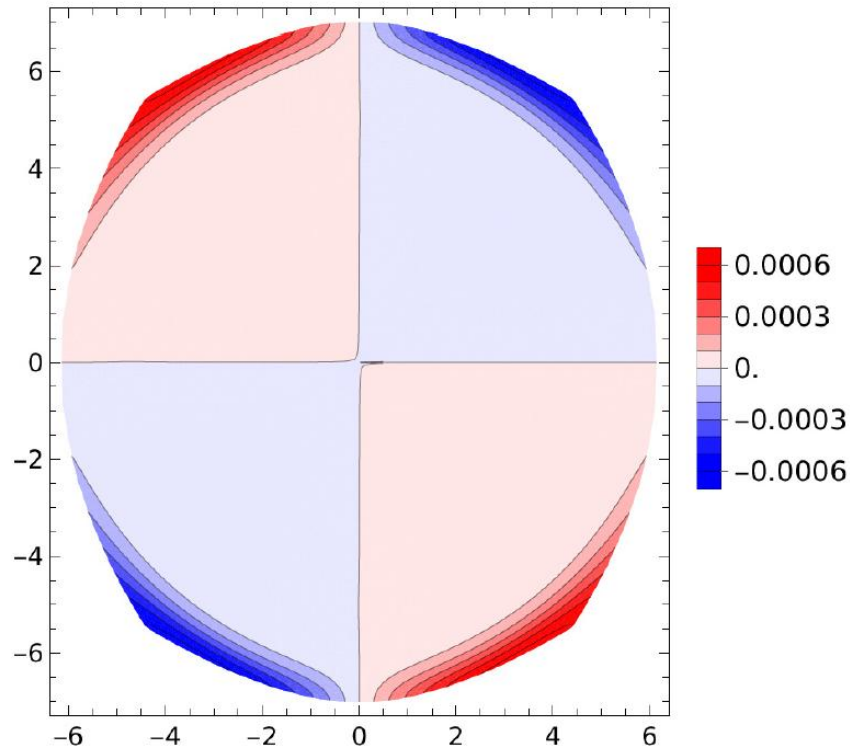
$\tau = 0.06$ fm $\quad T^{tt} = \varepsilon \quad T^{tx} \quad T^{zz} \quad T^{xx} \quad T^{xy}$

$b = 0,\ z = 0$

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Vorticity





$$V^i(\vec{r}) \equiv \frac{T^{0i}(\vec{r})}{T^{00}(\vec{r})}$$

ω^z at $b = 2$ fm & $\tau = 0.06$ fm in τ^8 order

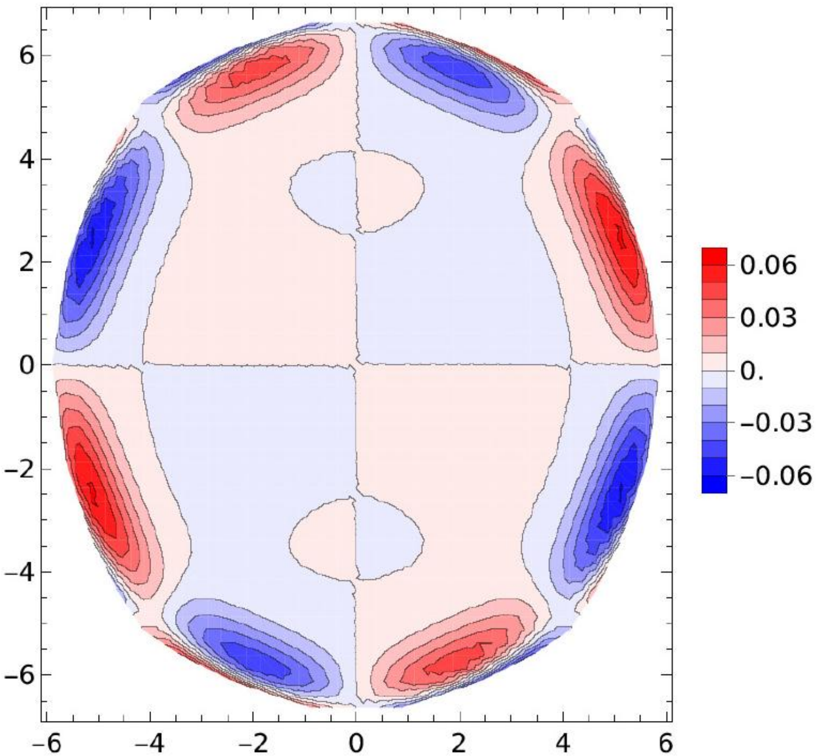
$$\omega(\vec{r}) = \nabla \times \vec{V}(\vec{r})$$

$$\omega_{\rm th}(\vec{r}) = \nabla \times \frac{\vec{u}(\vec{r})}{T(\vec{r})}$$

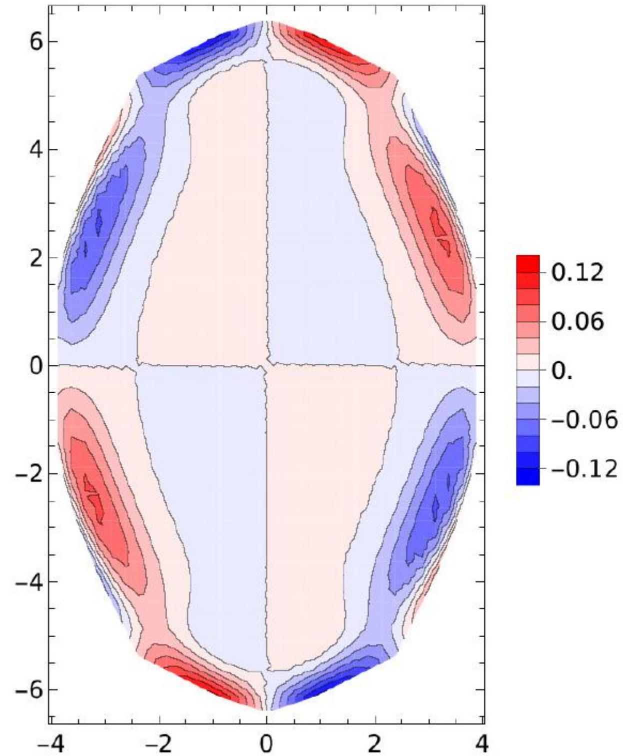
M. Carrington & St. Mrówczyński, arXiv:2505.07324

Local angular momentum

$$\frac{dL^z(\vec{r}_0)}{d\eta} = -\tau \int_{\Delta^2} d^2 r \left((r^y - r_0^y) T^{0x} - (r^x - r_0^x) T^{0y} \right)$$



$b = 2$ fm



$b = 6$ fm

M. Carrington & St. Mrówczyński, arXiv:2505.07324

Conclusion

Glasma behaves as a fluid.