Origin of Quantum Chromodynamics

The quantum chromodynamics (QCD) – a contemporary theory of strong interactions – emerged as a fusion of ideas formulated in studies of hadron spectroscopy and of deep inelastic scattering of electrons on protons. The key idea of the first research area is the SU(3) symmetry.

SU(3) symmetry

- In 1930s it was observed that nuclear forces are universal, they do not distinguish protons and neutrons. There emerged an idea of isospin and SU(2) symmetry of strong interactions.
- There was a doublet of nucleons of isospin $I = 1/2$, the triplet of pions of isospin $I = 1$, etc. The third component of isospin, which is related to electric charge, distinguishes protons from neutrons and pions of different electric charge.
- In 1960s many hadrons, in particular strange ones, were discovered and the SU(2) symmetry was extended to the SU(3) symmetry which includes strange hadrons. It was found that hadrons of the same spin and parity and similar masses can be grouped in multiplets corresponding to irreducible representations of the SU(3) group.
- The SU(2) group is a subgroup of the SU(3) group and thus the SU(3) symmetry implies the SU(2) isospin symmetry. Actually, the SU(2) is more accurate symmetry.
- Starting with the fundamental three-dimensional representation, one constructs irreducible representations of higher dimensions

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}, \tag{1}$$
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}, \tag{2}$$

  where $\mathbf{3}$ and $\bar{\mathbf{3}}$ denote the fundamental representation and its conjugate, while $\mathbf{8}$ and $\mathbf{10}$ denote the eight and ten dimensional representations.
- The lightest mesons of zero spin indeed form an octet shown in Fig. 1

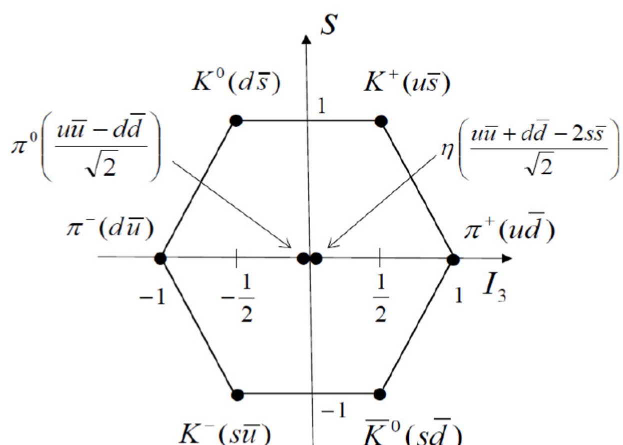

Figure 1: Octet of spinless mesons and their quark composition

- The lightest baryons of spin 1/2 form an octet and those of spin 3/2 a decouplet both shown in Fig. 2.
- Quarks u, d, s of spin 1/2 were proposed to correspond to a fundamental representation of the SU(3) group illustrated in Fig. 3.
- Once quarks are postulated to exist quark composition of hadrons belonging to the multiplets is determined as shown in Figs. 1 and 2.

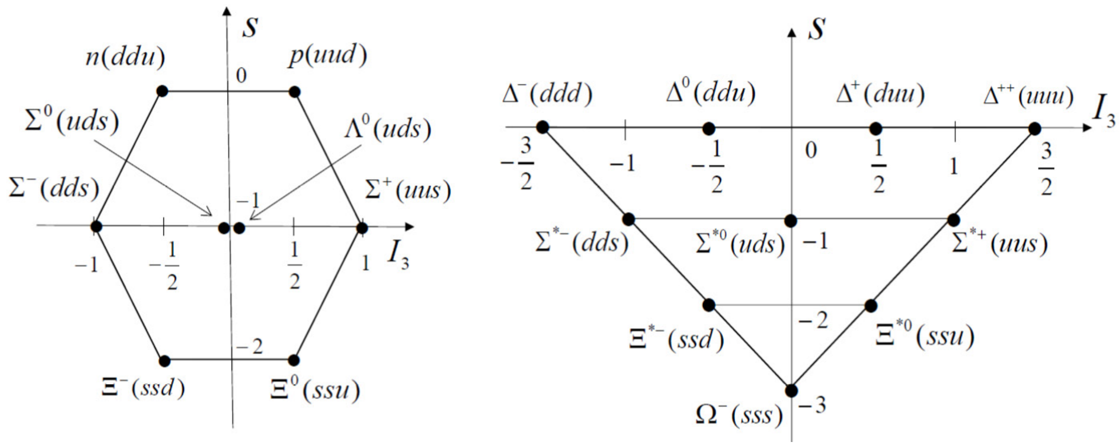

Figure 2: Octet and decouplet of spin 1/2 and 3/2 baryons and their quark composition

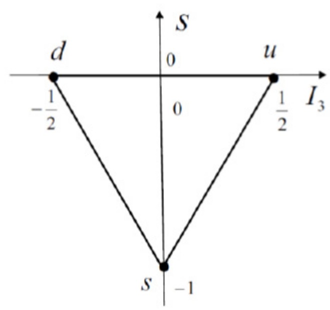

Figure 3: Triplet of quarks of spin 1/2

- Each quark carries 1/3 of a baryon number of a nucleon and the electric charges of u, d, s are $2/3, -1/3, -1/3$ of the elementary (positive) charge e.
- Existence of baryons $\Delta^{++} = (u, u, u)$ and $\Omega^- = (s, s, s)$ both of spin 3/2 suggests that there are internal degrees of freedom of quarks identified with color charges.

Elastic electron scattering on a proton

- Let us first consider an elastic electron scattering on a proton with one-photon exchange illustrated in Fig. 4 where a kinematics of the process is also described. The scattering is considered in the proton rest frame.
- The wavelength of the exchanged photon is

$$\lambda = \frac{2\pi\hbar}{|\mathbf{q}|}, \tag{3}$$

  where $\mathbf{q}$ is the momentum transfer.

- If the photon wavelength is much bigger than the proton radius of order 1 fm, which means $|\mathbf{q}| \ll 200$ MeV, the photon 'sees' the proton as a point-like (spin 1/2) object, the cross section is given by the formula

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\,\frac{E'}{E}\Big[\cos^2(\theta/2) + \frac{Q^2}{2M^2}\sin^2(\theta/2)\Big], \tag{4}$$

  where $\alpha \equiv \frac{e^2}{4\pi}$, M is the proton mass, $Q^2 \equiv -q^2 = -q_0^2 + \mathbf{q}^2$ and E, E' are initial and final proton energies.

  Exercise: Derive the cross section (4) of electron scattering on a point like proton of spin 1/2. The process is represented by a single Feynman diagram with a photon exchange. Perform averaging over initial spin states and summing over final ones.

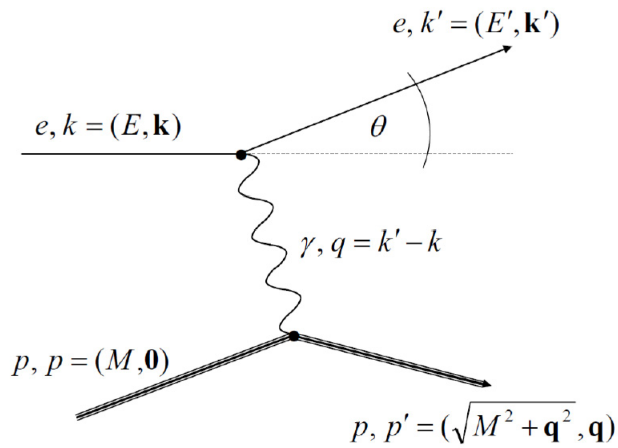

Figure 4: Elastic scattering of electron on a proton

- For a purpose of the further discussion, the cross section is written as

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[\cos^2(\theta/2) + \frac{Q^2}{2M^2}\sin^2(\theta/2)\Big]\,\delta\Big(\nu - \frac{Q^2}{2M}\Big), \tag{5}$$

where $\nu \equiv E - E' = q_0$. In case of elastic scattering, E' is uniquely determined by the scattering angle θ due to the energy and momentum conservation. One check it computing the square of the proton final-state four-momentum as

$$(M - q_0)^2 - \mathbf{q}^2 = (M + \nu)^2 - \mathbf{q}^2 = M^2 + 2\nu M - Q^2. \tag{6}$$

Since the proton is on mass-shell and the four-momentum square equals M^2, one finds $\nu = \frac{Q^2}{2M}$ which explains why the delta functions shows up in the cross section (5).

- To get the cross section (4) from (5) one takes the integral over E'. However, one should take into account that at fixed θ the variable Q^2 depends on E'. So, taking the integral one should remember about the formula

$$\int dx\,\delta\big(f(x)\big) = \frac{1}{f'(x_0)}, \tag{7}$$

where x_0 is the zeroth of the function $f(x)$ that is $f(x_0) = 0$.

<u>Exercise</u>: Derive the cross section (4) starting with the cross section (5).

- When the photon wavelength becomes comparable to the proton radius, the photon 'sees' a finite-size proton. If the scattering is still elastic, the formula (5) is modified as

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[W_2(Q^2)\cos^2(\theta/2) + 2W_1(Q^2)\sin^2(\theta/2)\Big]\,\delta\Big(\nu - \frac{Q^2}{2M}\Big), \tag{8}$$

where the dimensionless functions $W_1(Q^2)$ and $W_2(Q^2)$ are called the <u>form factors</u> describing proton electromagnetic structure that is the electric charge and magnetic moments distributions.

- When $W_2(Q^2) = 1$ and $W_1(Q^2) = \frac{Q^2}{4M^2}$, which occurs at small Q^2, we return to the cross section (5).
- When $Q^2 \gg M^2$ the form factors are found experimentally to depend on Q^2 as $W_2(Q^2) \sim Q^{-4}$ and $W_1(Q^2) \sim Q^{-2}$. Consequently, the cross section decays with Q^2 faster than Q^{-4}.

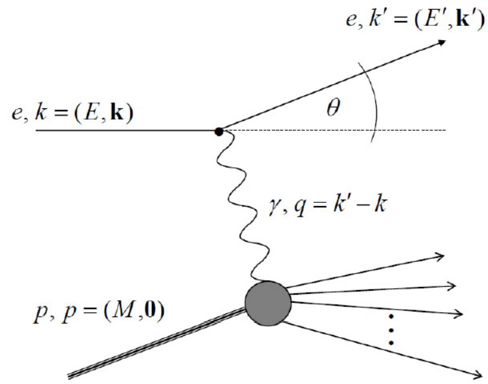

Figure 5: Deep inelastic scattering of electron on a proton

Deep inelastic scattering

- At a sufficiently high Q^2 the electron scattering, which is illustrated in Fig. 5, can be inelastic and the cross section is written as

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[W_2(\nu, Q^2)\cos^2(\theta/2) + 2W_1(\nu, Q^2)\sin^2(\theta/2)\Big], \tag{9}$$

where $W_1(\nu, Q^2)$ and $W_2(\nu, Q^2)$ are called the structure functions which depend not only on Q^2 but on ν as well.

- When

$$W_1(\nu, Q^2) = W_1(Q^2)\,\delta\Big(\nu - \frac{Q^2}{2M}\Big), \qquad W_2(\nu, Q^2) = W_2(Q^2)\,\delta\Big(\nu - \frac{Q^2}{2M}\Big), \tag{10}$$

we have the elastic scattering.

- When

$$W_1(\nu, Q^2) = \frac{Q^2}{4M^2}\,\delta\Big(\nu - \frac{Q^2}{2M}\Big), \qquad W_2(\nu, Q^2) = \delta\Big(\nu - \frac{Q^2}{2M}\Big), \tag{11}$$

we have the elastic scattering on a point-like proton.

- The inelastic scattering at $Q^2 \gg M^2$ is called deeply inelastic.
- Since the functions $W_1(Q^2)$ and $W_2(Q^2)$ decay fast as $Q^2 \to \infty$, one could think that $W_1(\nu, Q^2)$ and $W_2(\nu, Q^2)$ behave in a similar way. However, this is not the case.
- In 1968 James Bjorken formulated the hypothesis that when $Q^2 \to \infty$ and $\nu \to \infty$ at fixed variable

$$x \equiv \frac{Q^2}{2M\nu}, \tag{12}$$

which is called the Bjorken x, the functions $W_1(\nu, Q^2)$ and $W_2(\nu, Q^2)$ behave as

$$W_1(\nu, Q^2) = \frac{1}{M}\,F(x), \qquad W_2(\nu, Q^2) = \frac{2x}{\nu}\,F(x), \tag{13}$$

that is the functions $W_1(\nu, Q^2)$ and $\nu W_2(\nu, Q^2)$ depend not two variables ν and Q^2 but only on their combination x. This is the Bjorken scaling.

- The Bjorekn scaling means that the deep inelastic scattering of electron on a proton actually occurs on point-like components of the proton.
- Measurements performed at the Stanford Linear Accelerator Center (SLAC) showed that the structure functions indeed satisfy the Bjorken scaling though approximately only.

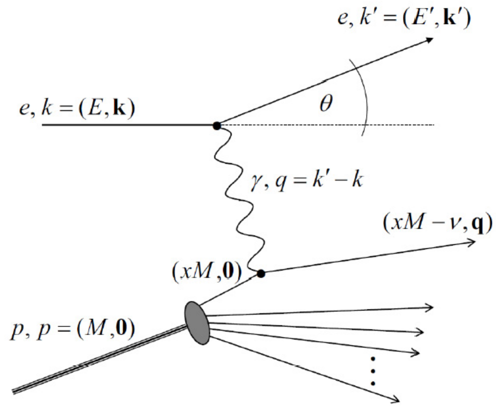

Figure 6: Deep inelastic scattering of electron on a proton made up of partons

- The 1990 Nobel Prize was awarded to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor "for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics".

Parton Model

- According to Richard Feynman, a proton is made up of point-like partons each carrying a fraction x of proton's momentum. Partons are of several types of different electric charges. The distribution of x of partons of i-type, which is denoted as $f_i(x)$, obeys the condition

$$\sum_i \int_0^1 dx\, x f_i(x) = 1, \tag{14}$$

which means that not the parton's number but their total momentum is fixed.

- To simplify our considerations we assume that in a proton rest frame the partons are at rest as well.
- We further assume that the deeply inelastic scattering of electron on proton occurs through the elastic scattering on point-like partons at rest which do not interact with surrounding partons. The process is illustrated in Fig. 6.
- The assumption that the scattering on a parton is elastic means that the square of parton's final-state four-momentum equals the square of parton's mass which is xM. Therefore,

$$(xM + \nu)^2 - \mathbf{q}^2 = x^2 M^2 \quad \Longrightarrow \quad x = \frac{Q^2}{2M\nu}. \tag{15}$$

In other words, the process kinematics determines the variable x. We also see that in the parton model the Bjorken variable x has a meaning of the fraction of proton's momentum carried by a parton.

- Assuming that the partons are spin 1/2 fermions of electric charge $\pm q_i e$, the cross section is

$$\frac{d\sigma}{dE' d\Omega} = \sum_i \int_0^1 dx\, x f_i(x)\, \frac{4\alpha^2 q_i^2 E'^2}{Q^4} \Big[\cos^2(\theta/2) + \frac{Q^2}{2x^2M^2} \sin^2(\theta/2)\Big]\, \delta\Big(\nu - \frac{Q^2}{2xM}\Big), \tag{16}$$

which gives

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4} \Big[\frac{x}{\nu} \cos^2(\theta/2) + \frac{1}{M} \sin^2(\theta/2)\Big] \sum_i q_i^2 x f_i(x), \tag{17}$$

where $x = \frac{Q^2}{2M\nu}$.

- Comparing the result (17) with the cross section (9), one finds

$$W_1(\nu, Q^2) = \frac{1}{2M} \sum_i q_i^2 x f_i(x), \qquad W_2(\nu, Q^2) = \frac{x}{\nu} \sum_i q_i^2 x f_i(x). \tag{18}$$

- The cross section (17) satisfies the Bjorken scaling with

$$F(x) = \frac{1}{2} \sum_i q_i^2 x f_i(x). \tag{19}$$

Towards QCD

- In QCD partons, which carry electric charges, are identified with quarks.
- Except electric charges, quarks carry color charges and 1/3 of the baryon number of a nucleon.
- Inter-quark interactions are mediated by spin 1 gluons which are also constituents of hadrons.
- QCD is asymptotically free theory that is the interaction vanishes when $Q^2 \to \infty$ which explains the Bjorken scaling.