Higgs Mechanism

A gauge symmetry requires a masslessness of gauge bosons which in turn leads to the infinite range of the gauge forces. A Higgs mechanism explains how to combine a gauge symmetry with a finite mass of force carriers.

Spontaneous Symmetry Breakdown

- Let us start with the following Lagrangian density of a complex scalar field $\Phi(x)$

$$\mathcal{L}(x) = \partial^\mu \Phi^*(x) \partial_\mu \Phi(x) - \mu^2 \Phi^*(x)\, \Phi(x) - \lambda \big(\Phi^*(x)\, \Phi(x)\big)^2, \tag{1}$$

where the parameters $\lambda \in \mathbb{R}$ and $\lambda > 0$ while $\mu^2 \in \mathbb{R}$ but $\mu^2 > 0$ or $\mu^2 < 0$.

- The Lagrangian (1) is invariant under the global symmetry transformation

$$\Phi(x) \to e^{i\alpha} \Phi(x) \qquad \Phi^*(x) \to e^{-i\alpha} \Phi^*(x), \tag{2}$$

where $\alpha \in \mathbb{R}$.

- Let us define the potential $V \equiv \mu^2 \Phi^*(x)\, \Phi(x) + \lambda \big(\Phi^*(x)\, \Phi(x)\big)^2$ which is shown in the Fig. 1 for $\mu^2 > 0$ and $\mu^2 < 0$.

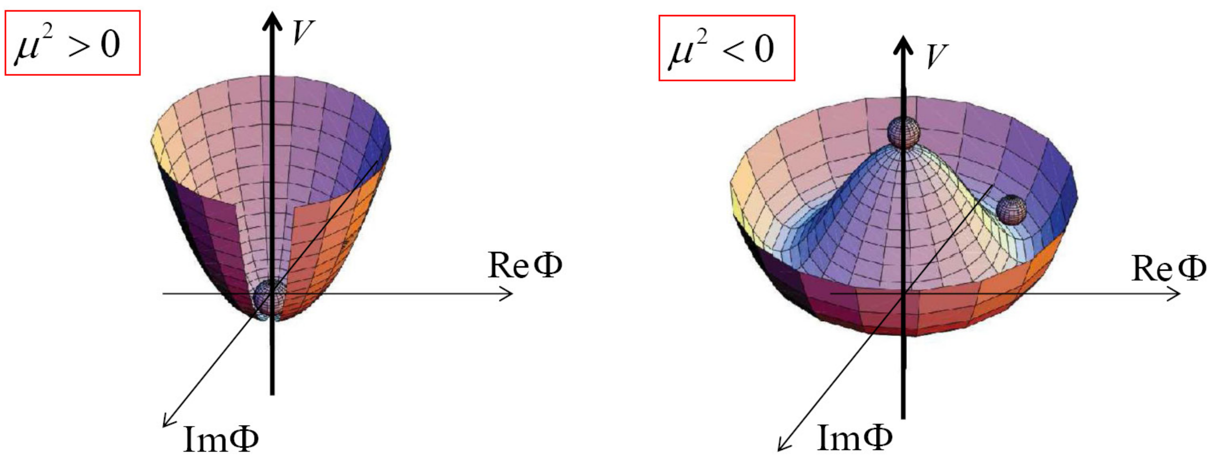

Figure 1: Effective potential for $\mu^2 > 0$ and $\mu^2 < 0$

- Further on we assume that $\mu^2 < 0$.
- If $\text{Im}\Phi = 0$, the potential, which is shown in Fig. 2, has the minimum at $\text{Re}\Phi = \pm\sqrt{-\frac{\mu^2}{2\lambda}}$.
- In general, the minimum of the potential V is determined by the equation:

$$(\text{Re}\Phi)^2 + (\text{Im}\Phi)^2 = -\frac{\mu^2}{2\lambda}. \tag{3}$$

- The Lagrangian density obeys the symmetry (2) and the minimum of the potential V given by Eq. (3) is also symmetric. However, if we choose one specific point like

$$\text{Im}\Phi = 0 \qquad \& \qquad \text{Re}\Phi = \sqrt{-\frac{\mu^2}{2\lambda}}, \tag{4}$$

the symmetry is broken, see Fig. 2.

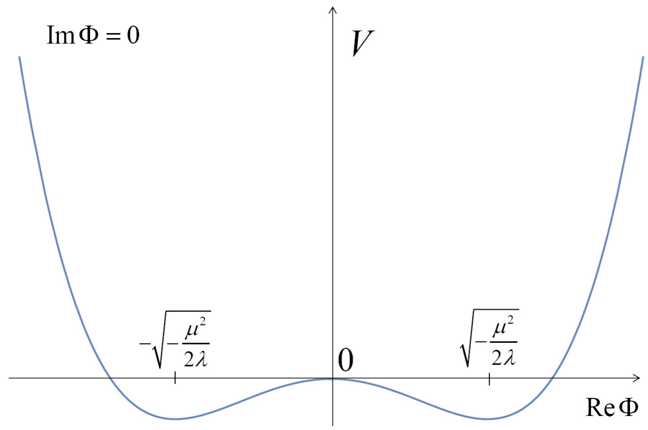

Figure 2: Effective potential for $\mu^2 < 0$ and $\text{Im}\Phi = 0$

- We define the new scalar real fields

$$\phi(x) \equiv \sqrt{2}\Big(\text{Re}\Phi(x) - \sqrt{-\frac{\mu^2}{2\lambda}}\,\Big), \qquad \chi(x) \equiv \sqrt{2}\,\text{Im}\Phi(x). \tag{5}$$

  The fields represent fluctuations around the chosen minimum (4) and $\chi(x)$ is called the Goldstone field. The coefficient is $\sqrt{2}$ is introduced to get the standard factor $\frac{1}{2}$ in front the kinetic term of the real field ϕ in the Lagrangian (7).

- The field $\Phi(x)$ expressed through $\phi(x)$ and $\chi(x)$ reads

$$\Phi(x) = \frac{1}{\sqrt{2}}\big(\phi(x) + v + i\chi(x)\big), \tag{6}$$

  where $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$

- Substituting the field Φ given by Eq. (6) into the Lagrangian (1), one finds the Lagrangian rewritten in terms of the fields ϕ and χ as

$$\mathcal{L}(x) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - g\phi(\phi^2+\chi^2) - \frac{\lambda}{4}(\phi^2+\chi^2)^2, \tag{7}$$

  where $m^2 \equiv 2\lambda v^2 = -2\mu^2$ and $g \equiv \lambda v$. As one observes, the field $\phi(x)$ is massive but the Goldstone field $\chi(x)$ is massless.

  Exercise: Derive the Lagrangian (7).

- We have found a special case of the Goldstone theorem which states that a massless (Goldstone) boson shows up when a continuous global symmetry is spontaneously broken.

Abelian Higgs Mechanism

- We extend the Lagrangian density (1) including the electromagnetic field. Thus, we get

$$\mathcal{L}(x) = \frac{1}{4}F^{\mu\nu}(x)\,F_{\nu\mu}(x) + \big(D^\mu\Phi(x)\big)^*D_\mu\Phi(x) - \mu^2\Phi^*(x)\,\Phi(x) - \lambda\big(\Phi^*(x)\,\Phi(x)\big)^2, \tag{8}$$

  where $D^\mu \equiv \partial^\mu + ieA^\mu(x)$.

- The Lagrangian (8) is invariant under the local gauge symmetry transformation

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x), \qquad \Phi(x) \to e^{-ie\Lambda(x)}\Phi(x). \tag{9}$$

- Since we assume that $\mu^2 < 0$, the symmetry is spontaneously broken.
- We define new fields: real scalar $H(x)$ and real vector $B^\mu(x)$ through the relations

$$\Phi(x) = \frac{1}{\sqrt{2}}\big(H(x) - v\big)e^{i\Lambda(x)/v}, \qquad A^\mu(x) = B^\mu(x) - \frac{1}{ev}\partial^\mu \Lambda(x). \tag{10}$$

- Substituting the fields Φ and A^μ expressed through H and B^μ into the Lagrangian (8), one finds

$$\begin{aligned}\mathcal{L}(x) = & \frac{1}{4}\, W^{\mu\nu}(x)\, W_{\nu\mu}(x) + \frac{1}{2}\, m_B^2 B^\mu B_\mu \\ + & \frac{1}{2}\,(\partial^\mu H)(\partial_\mu H) - \frac{1}{2}\, m_H^2 H^2 \\ + & \frac{e^2}{2}\, B_\mu B^\mu (H^2 + 2vH) - \lambda v\, H^3 - \frac{\lambda}{4}\, H^4,\end{aligned} \tag{11}$$

  where $W^{\mu\nu}(x) = \partial^\mu B^\nu - \partial^\nu B^\mu$, $m_B^2 \equiv e^2 v^2$ and $m_H^2 \equiv 2\lambda v^2 = -2\mu^2$. We note that the constant terms and linear in H are ignored in Eq. (11).

  Exercise: Derive the Lagrangian (11).

- The Lagrangian (11) shows that the vector field becomes massive due to the spontaneous symmetry breakdown.
- The first line of Eq. (11) represents the Lagrangian density of the free massive vector field B^μ, the second line is the Lagrangian density of the free massive scalar real Higgs field H and the third line represents the self interaction of the Higgs field and the interaction of B^μ and H fields.
- There is no massless field (no Goldstone boson) in the Lagrangian (11).
- We note that the field $\Lambda(x)$ is entirely absent in Eq. (11) as the corresponding terms cancel each other. We see that the ground state of the theory is not gauge invariant but the procedure leading to the Lagrangian (11) is gauge independent. It means that we could get the Lagrangian (11) defining the fields $H(x)$ and $B^\mu(x)$ as

$$\Phi(x) = \frac{1}{\sqrt{2}}\big(H(x) - v\big), \qquad A^\mu(x) = B^\mu(x). \tag{12}$$

- Let us count the numbers of degrees of freedom before and after the spontaneous symmetry breakdown. In the Lagrangian (8) we have massless vector field of 2 degrees of freedom (two spin states) and the complex scalar field also of 2 degrees of freedom (particles and antiparticles). In the Lagrangian (11) we have massive vector field of 3 degrees of freedom (three spin states) and the real scalar field of 1 degree of freedom (truly neutral particles). The numbers of degrees of freedom are the same.
- Including in the Lagrangian (8) the spinor-field term

$$\overline{\psi}\,\Big(i(\partial^\mu + ieA^\mu)\gamma_\mu - m\Big)\,\psi, \tag{13}$$

  and applying the gauge transformation $\psi \to e^{-i\Lambda/v}\psi$ together with the redefinitions (10), one finds the following extra term in the Lagrangian (11)

$$\overline{\psi}\,\Big(i(\partial^\mu + ieB^\mu)\gamma_\mu - m\Big)\,\psi. \tag{14}$$

  Before the spontaneous symmetry breakdown the spinor field interacts with the massless field A^μ but after the spontaneous symmetry breakdown the field interacts with the massive field B^μ.

NonAbelian Higgs Mechanism

- The extension of the Higgs mechanism to the non-Abelian theories is straightforward. Further on we discuss the SU(2) case which is relevant for the Standard Model.
- The SU(2) analog of the Lagrangian (8) reads

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}[F^{\mu\nu}F_{\nu\mu}] + \left(D^\mu\Phi\right)^\dagger D_\mu\Phi - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2, \tag{15}$$

where $D^\mu \equiv \partial^\mu + igA^\mu$ and Φ is the two-component complex scalar field

$$\Phi = \begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}, \qquad \Phi^\dagger = (\Phi_1^*, \Phi_2^*). \tag{16}$$

- We note that $A^\mu = A^\mu_a\tau^a$ and the generators of the SU(2) group τ^a are $\tau^a = \frac{1}{2}\sigma^a$ where σ^a with $a = 1, 2, 3$ are the Pauli matrices.
- The Lagrangian (15) is invariant under the SU(2) gauge transformation

$$A^\mu \to UA^\mu U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger, \tag{17}$$

$$\Phi \to U\Phi. \tag{18}$$

- We choose the field which minimizes the potential as

$$\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad v \equiv \sqrt{-\frac{\mu^2}{\lambda}}. \tag{19}$$

- We introduce new fields: real scalar $H(x)$ and real vector $B^\mu(x)$ through the relations analogous of (10)

$$A^\mu = UB^\mu U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger, \qquad \Phi = \frac{1}{\sqrt{2}}U\begin{pmatrix}0\\ H - v\end{pmatrix}. \tag{20}$$

- Substituting the fields (20) into the Lagrangian (15), one finds

$$\frac{1}{2}\mathrm{tr}[F^{\mu\nu}F_{\nu\mu}] = \frac{1}{2}\mathrm{tr}[W^{\mu\nu}W_{\nu\mu}], \tag{21}$$

$$\mu^2\Phi^\dagger\Phi = \frac{\mu^2}{2}(H - v)^2, \tag{22}$$

$$\lambda(\Phi^\dagger\Phi)^2 = \frac{\lambda}{4}(H - v)^4. \tag{23}$$

- The only term, which is difficult to compute, is $\left(D^\mu\Phi\right)^\dagger D_\mu\Phi$. Using the fact that the covariant derivative transforms under the gauge transformations as

$$D^\mu \to UD^\mu U^\dagger, \tag{24}$$

one finds

$$\begin{aligned}\left(D^\mu\Phi\right)^\dagger D_\mu\Phi &= \frac{1}{2}(0, H - v)\big(\overleftarrow{\partial}_\mu + igB_\mu\big)\big(\partial^\mu - igB^\mu\big)\begin{pmatrix}0\\ H - v\end{pmatrix} \qquad (25)\\ &= \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \frac{ig}{2}(0, \partial_\mu H)B^\mu\begin{pmatrix}0\\ H - v\end{pmatrix} + \frac{ig}{2}(0, H - v)B_\mu\begin{pmatrix}0\\ \partial^\mu H\end{pmatrix}\\ &\quad + \frac{g^2}{2}(0, H - v)B_\mu B^\mu\begin{pmatrix}0\\ H - v\end{pmatrix}.\end{aligned}$$

Writing down the field B^μ as $B^\mu = \frac{1}{2} B^\mu_a \sigma^a$, where σ^a is a Pauli matrix, one finds that the second and third terms in Eq. (25) cancel each other because

$$(0, \partial_\mu H) B^\mu \begin{pmatrix} 0 \\ H - v \end{pmatrix} = \frac{1}{2} (\partial_\mu H)(H - v) B^\mu_a \; (0, 1) \sigma_a \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0, H - v) B_\mu \begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix}. \tag{26}$$

The last term in Eq. (25) is computed as

$$\frac{g^2}{2} (0, H - v) B_\mu B^\mu \begin{pmatrix} 0 \\ H - v \end{pmatrix} = \frac{g^2}{8} (H - v)^2 B_{a\mu} B^\mu_b \;\; (0, 1) \sigma^a \sigma^b \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{27}$$

Using the explicit form of the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{28}$$

one finds that

$$\frac{g^2}{2} (0, H - v) B_\mu B^\mu \begin{pmatrix} 0 \\ H - v \end{pmatrix} = \frac{g^2}{8} (H - v)^2 B_{a\mu} B^\mu_a. \tag{29}$$

So, the final result of the term $\left(D^\mu \Phi\right)^\dagger D_\mu \Phi$ is

$$\left(D^\mu \Phi\right)^\dagger D_\mu \Phi = \frac{1}{2} \, (\partial_\mu H)(\partial^\mu H) + \frac{g^2}{8} (H - v)^2 B_{a\mu} B^\mu_a. \tag{30}$$

- Combining the results (21, 22, 23) and (30), the new Lagrangian becomes

$$\mathcal{L} = \frac{1}{2} \mathrm{tr}[W^{\mu\nu} W_{\nu\mu}] + \frac{1}{2} \, (\partial_\mu H)(\partial^\mu H) + \frac{g^2}{8} (H - v)^2 B_{a\mu} B^\mu_a - \frac{\mu^2}{2} (H - v)^2 - \frac{\lambda}{4} (H - v)^4, \tag{31}$$

  which is rewritten as

$$\begin{aligned} \mathcal{L} \;\; = \;\; & \frac{1}{2} \mathrm{tr}[W^{\mu\nu} W_{\nu\mu}] + m_B^2 \, \mathrm{tr}[B^\mu B_\mu] \\ + \;\; & \frac{1}{2} \, (\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m_H^2 H^2 \\ + \;\; & g^2 (H^2 - 2vH)^2 \, \mathrm{tr}[B^\mu B_\mu] - \lambda v \, H^3 - \frac{\lambda}{4} H^4, \end{aligned} \tag{32}$$

  where $m_B^2 = g^2 v^2 / 4$ and $m_H^2 = -2\mu^2$. We note that the constant terms and those linear in H are ignored in Eq. (32).

- As in the Abelian case, the three lines of the Lagrangian (32) represent: the massive vector field, the massive scalar field and the interaction of the vector and scalar fields.

- In the Lagrangian (15) we have three massless vector fields of 2 degrees of freedom each (two spin states) and two complex scalar fields also of 2 degrees of freedom each (particles and antiparticles). So, we have ten degrees of freedom. In the Lagrangian (32) we have three massive vector fields of 3 degrees of freedom each (three spin states) and the real scalar field of 1 degree of freedom (truly neutral particles). The number of degrees of freedom is ten and it is the same as before the spontaneous symmetry breakdown.

- We note that analogously to the Abelian case, the procedure leading to the Lagrangian (15) is independent of the gauge transformation matrix U. Therefore, one obtains the Lagrangian (15) defining the fields $H(x)$ and $B^\mu(x)$ as

$$A^\mu = B^\mu, \qquad\qquad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H - v \end{pmatrix}. \tag{33}$$

- The Higgs mechanism plays a key role in the Standard Model as will be evident in subsequent lectures.