Introductory remarks

What is and for what is statistical quantum field theory?

Statistical mechanics is usually formulated in terms of non-relativistic classical or quantum mechanics. Quantum field theory (QFT) is on the other hand typically applied to elementary particles which are relativistic. So, it is not obvious what for one needs statistical QFT which is a marriage of statistical physics and quantum field theory. However, there are at least three good reasons to develop and to study the theory.

- The first reason is obvious. In nature we encounter many-body systems which are both relativistic and quantum. One would enlist here matter which filled in an early Universe, some astrophysical objects observed today which are dense and/or hot, strongly interacting matter produced in relativistic heavy-ion collisions studied in terrestrial experiments. To describe such systems methods of statistical quantum field theory are unavoidable.
- The second reason is somewhat abstract. Ludwig Boltzmann (1844 - 1906) was presumably the first who formulated the program to derive principles of statistical mechanics from laws of microscopic underlying dynamics. Nikolay Nikolayevich Bogolyubov (1909 - 1992) succeeded to derive the equations of kinetic theory from the equations of classical Newtonian mechanics. Later on, an analogous derivation was presented starting with quantum mechanics. Since quantum field theory is currently the most fundamental formulation of microscopic dynamics, the laws of statistical physics should be derived from it.
- The third reason is pragmatic. Quantum field theory is so well developed that its methods, including e.g. Feynman diagrams or renormalization theory, are successfully applied to systems which are non-relativistic. Actually, non-relativistic statistical quantum field theory was invented already in late 1950s for the theoretical description of condensed matter. Therefore, some knowledge of statistical quantum field theory appears useful in studying various problems of theoretical physics.

Recommended textbooks

1. M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 2000).
2. J.I. Kapusta and Ch. Gale, *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, 2006).
3. L.P. Kadanoff and G. Baym, *Quantum Statistical Mechanics – Green's Function Methods in Equilibrium and Nonequilibrium Problems* (W.A. Benjamin, New York, 1962).

Lectures' content

After reminding basic notions of quantum field theory (QFT), thermodynamics and statistical mechanics, the imaginary-time or Matsubara and real-time or Keldysh-Schwinger formalisms of QFT will be introduced. The operator approach and that of functional methods will be used. Properties of various Green's and their equations of motion will be discussed. The perturbative expansion will be formulated. Various applications of statistical QFT will be presented, in particular a kinetic theory will be derived from QFT.

Basic notions

- Statistical physics deals with systems of many degrees of freedom, 'many' means of order of Avogadro number $N_A \approx 6 \cdot 10^{23}$.
- The goal of statistical physics is to provide a description of a many-body system in terms of microscopic dynamics. However, the description obtained is macroscopic, albeit to a different extent.
- Thermodynamics does not refer to microscopic concepts at all but it uses macroscopic notions like pressure, temperature, free energy etc. Hydrodynamics is also a fully macroscopic approach but kinetic theory and Gibbs statistical mechanics introduce microscopic quantities like scattering cross section of particles or inter-particle interacting potentials.
- Statistical physics splits into the physics of equilibrium and of non-equilibrium systems.
- According to the second principle of thermodynamics any isolated many-body system evolves towards the state of thermodynamic equilibrium. When the equilibrium is reached a system's state becomes stationary or time independent.
- Thermodynamics and Gibbs statistical mechanics deals only with equilibrium or thermal systems.
- Kinetic or transport theory is applicable to both equilibrium and non-equilibrium systems but its main objective is to describe an evolution towards equilibrium.

Units

- We use the natural system of units with $\hbar = c = k_B = 1$.